Loop Shaping Design Procedure for Quadrotor Control with Weights Designed by Resolving a Constrained Non-linear Optimization Problem

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Abstract

This paper presents robust controller designs for resolving the attitude/altitude tracking problem of a decoupled quadrotor with parametric uncertainties. A new method is developed for selecting the weights associated with the robust Loop Shaping Design Procedure (LSDP). Over the conventional LSDP technique, an outer framework is constructed which solves the problem of weight selection by resolving a constrained non-linear optimization problem. Two new algorithms have been developed for the one and two degrees of freedom (DOF) designs. The algorithms substantially reduce the development time in terms of weight selection and the robust controllers developed, provide acceptable levels of performance and stability margins. The robustness analysis and comparison of the model-controller system shows the 1 DOF configuration to be superior as to the 2 DOF design. The results based on numerical simulations are presented.

1 Introduction

Unmanned aerial vehicles (UAVs) are increasingly becoming a favourite topic for research and has been recognized by the industry as a solution to many problems across a wide variety of disciplines [Canis, 2015]. Among them the quadrotor is a popular variety of rotorcraft drones that have attracted much attention both in the industry and in the academia. This study attempts to resolve the tracking attitude/altitude problem in the continuous domain using the Loop Shaping Design Procedure (LSDP) [Glover and McFarlane, 1989] developed under the robust control framework. The weight designs used are developed by resolving a constrained non-linear optimization problem. The design can be considered as a variant to the Method Of Inequalities (MOI) approach developed in [Whidborne *et al.*, 1995] where the weight designs are obtained by resolving a Moving Boundary problem (MBP). The one and two Degree of Freedom (DOF) designs will be explored in the context of the decoupled SISO model of a quadrotor.

The tracking attitude/altitude problem of the quadrotor have been intensively researched in past two decades [Kendoul, 2012]. Most of the strategies involved in these studies stop short of providing guaranteed stability margins in the presence of model uncertainty. Robust control resolves this problem by incorporating the uncertainty into controller design such that the developed controller will guarantee robust stability and performance.

Model uncertainty of the quad-rotor has been tackled using the robust control framework under the linear approach mainly using three different techniques: the Mixed Sensitivity Optimization, LSDP and μ synthesis. In this study LSDP based controllers are developed to resolve the tracking problem. LSDP based robust flight controllers for VSTOL aircraft first appeared in [Hyde, 1991] and have since been used in a number of controller design applications. A static $\mathcal{H}_\infty$ LSPD based controller for a fly-by-wire is discussed in [Prempain and Postlethwaite, 2005] where a non convex problem based on sufficient conditions alone is resolved using LMI solvers and result in a lower order controller. LSDP has been further used in the development of flight controllers for a tri-rotor UAV in [Mohamed and Lanzon, 2012] and for quadrotors in [Chen and Huzmezan, 2003] and [Rich *et al.*, 2013].

The LSDP weights used to modify open loop plant characteristics, in all the above studies have been chosen by trial and error methods based on intuition and flying experience, except for the final study where the weights are derived from an LQG based controller designed for the craft. This study attempts to automate this weight selection procedure by introducing an constrained optimization framework that can reduce the time spent on selecting the weights manually. This outer framework involves a cost function as well as linear and non-linear constrains which make sure that the defined performance standards are strictly met while developing the controller. The [Glover and McFarlane, 1989] algorithm used here for controller development attempts to maximize the stability margin against co-prime factor uncertainty in the model.

The paper is organised as follows. The quadrotor non-linear model is developed in section 2. In section 3, the theory behind the robust controller development is briefly explained. Experimental results are presented in section 4 followed by conclusion in section 5.

2 System Modelling

The quadrotor is an under-actuated mechanical system with four rotors that could be positioned either in the 'X' or '+' formation which acts as control inputs, and 6 degrees of freedom (DOF); these being (x, y), altitude (z) measured in the inertial frame and attitude (ϕ, θ, ψ being the roll, pitch and yaw angles respectively) which are measured in body frame. The diametrically opposite rotors rotate in the same direction and those adjacent to one another rotate in opposite directions. The notations and model presented are based on [Thomas, 2017].

Figure 1 shows rotors 1 and 3 rotating clockwise and rotors 2 and 4 in the counter clockwise direction. The direction of the corresponding torques generated are shown by circular indicators. The lift generated by each of the rotors are indicated by f_i, where i represents rotor number, 1 -4.

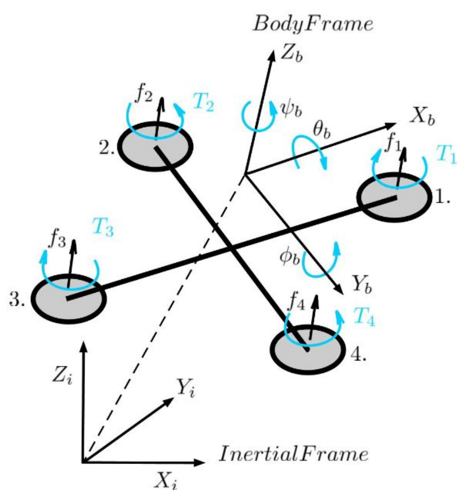

Figure 1: The quadrotor orientation and nomenclature

As seen in Figure 1 the two different frames of reference- the body frame attached to the quadrotor and fixed inertial frame of reference are used to develop the non-linear equations of motion of the quadrotor. Quadrotor rotational and translational dynamics that are highly coupled can be decoupled by defining the input vector as follows and linearising the plant at the hover point (see Section 4). With f_i representing the thrust produced by the i^{th} rotor, the input is given by:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{1}$$

The equations of motion can be either derived using the Newton-Euler or Euler-Lagrange approach and those concerned with attitude/altitude control are:

$$\begin{aligned}
\dot{\phi} &= p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\
\dot{\theta} &= q\cos(\phi) - r\sin(\phi) \\
\dot{\psi} &= q\,\frac{\sin(\phi)}{\cos(\theta)} + r\,\frac{\cos(\phi)}{\sin(\theta)} \\
\dot{p} &= \frac{I_{YY} - I_{ZZ}}{I_{XX}}\,q\,r - \frac{k_r p}{I_{XX}} + \frac{L u_1}{I_{XX}} \\
\dot{q} &= \frac{I_{ZZ} - I_{XX}}{I_{YY}}\,p\,r - \frac{k_r q}{I_{YY}} + \frac{L u_2}{I_{YY}} \\
\dot{r} &= \frac{I_{XX} - I_{YY}}{I_{ZZ}}\,p\,q - \frac{k_r r}{I_{ZZ}} + \frac{d\,u_3}{I_{ZZ}} \\
\ddot{z} &= -g + \frac{1}{m}\left(\cos(\theta)\ \cos(\phi)\right) u_4 - \frac{k_t}{m}\,\dot{z}
\end{aligned} \tag{2}$$

where p, q and r represents the angular velocities about body x, y and z axis measured in the body frame, m the mass of the quadrotor, g the acceleration due to gravity, L the distance between the centre of the rotor and centre of mass of the quadrotor, d ratio between drag and thrust coefficients of the blade, k_r and k_t being the translational and rotational drag coefficients and finally I_{XX}, I_{YY} and I_{ZZ} the moment of inertias along x, y and z axis.

3 Loop Shaping Design Procedure

In LSDP, the plant representation allows for both poles and zeros to cross over to the right hand complex plane. LSDP employs the co-prime factor representation [Glover and McFarlane, 1989] of the plant in which the plant is represented as $G = M_l^{-1} N_l$ where the subscript 'l' indicates a left co-prime factorisation of the plant. In the following section both the 1 DOF approach using the pre-compensator and the 2 DOF controller designs are explained in context of the outer cost function minimization approach. Both designs could be appropriated for SISO plant models.

3.1 1 DOF LSDP with Pre-compensator

In the co-prime factor representation of the uncertain plant the perturbations carry no weights, and the controller for the resulting plant $G_S = \{(M_S + \Delta_{M_S})^{-1}(N_S + \Delta_{N_S}) : ||[\Delta_{M_S}\ \Delta_{N_S}]||_\infty \leq \epsilon\}$ attempts to minimize the following cost function

$$\gamma_K = \left\| \begin{bmatrix} K_s \\ I \end{bmatrix} (I - G_S K)^{-1} M_S^{-1} \right\|_\infty \quad (3)$$

Here $\Delta_{M_S}, \Delta_{N_S}$ are stable unknown transfer functions that amount to the uncertainty in nominal plant model G. The subscript 'S' represents the shaped plant. In order to specify the performance requirements the open loop singular values are modified using pre and post compensator weights W_1 and W_2. A pre-compensator is employed to eliminate any steady state error (see Figure 2). Weights W_1 and W_2 used to shape the open loop plant takes certain general formats. W_1 usually would be a low pass filter- high gains at lower frequencies, a slope of about -1 at required bandwidth and greater roll off rates at high frequencies. The form could be either the conventional

$$W_1 = \frac{s/M_s + \omega_B^*}{s + \omega_B^* A} \quad \text{or simply} \quad \frac{w_1(s + w_2)}{s + w_3} \text{ with } w_2 > w_3$$

Here M_s represents the peak sensitivity, ω_B^* the approximate bandwidth requirement and A the upper bound on sensitivity 'S' of the system. By constraining $w_2 > w_3$ we are essentially limiting w_P to the high-gain low pass filter regime. Higher order weights can also be used in case a steeper roll off rate is required at higher frequencies.

Both W_2 and K_P take the form of constants, with the final shaped plant of the form $W_2 G W_1$. The weight parameters as well as the pre-compensator can be obtained by developing an outer framework which minimizes a particular closed loop performance objective or combination of it. The procedure to form this framework is:

Algorithm # 1 Single DOF Weight Design Procedure

1. For the given plant G define the form of weights W_1, W_2 and K_P and choose initial values for the parameters. Choose the value of χ which represents the factor with which $\gamma_{\min}$ will be multiplied to obtain the suboptimal LSDP $\mathcal{H}_\infty$ norm. Here $\gamma_{\min}$ is defined as the inverse of the maximum attainable stability margin for the given shaped plant G_s [Glover and McFarlane, 1989],

$$\gamma_{min} = \left\{1 - ||[N_S \ M_S]||_H^2 \right\}^{-1/2}$$

where $|| \cdot ||_H$ denotes the Hankel norm.

2. Define the cost function $\mathcal{J} = \mathcal{W}_1 \cdot \text{IAE}_\text{y} + \mathcal{W}_2 \cdot \text{IAE}_\text{u} + \mathcal{W}_3 \cdot \texttt{order}(\text{K})$ and initialize the weights $\mathcal{W}_1, \mathcal{W}_2$ and $\mathcal{W}_3$. IAE_y represents the absolute integral error between the setpoint and the output, while the weight on IAE_u functions as a handle on input usage. The order of the controller 'K' can also be limited by incorporating it to the cost function as a penalty as shown.
3. Implement the constrained non-linear optimization problem.This can be done using any constrained optimization routines such as the NOMAD [Le Digabel, 2011] search algorithm (in this study the OPTI MATLAB toolbox [Currie, 2015] has been used for the implementation). Lower and upper bounds should be specified for the parameters of W_1, W_2 and K_P (which form the linear constraints) and γ (which forms the non-linear constraint). Here γ represents the LSDP $\mathcal{H}_\infty$ norm.
4. Depending upon robustness requirements, vary the forms of weights W_1 and W_2 to reach the required performance levels. Calculate the highest obtainable stability margin $1/\gamma_{\min}$. If this value is less than .25, vary the weights W_1, W_2 as well change the upper and lower limits of the linear constraints and repeat step (iii) again.
5. Alongside the satisfactory performance weights and pre-compensator the robust controller is obtained.

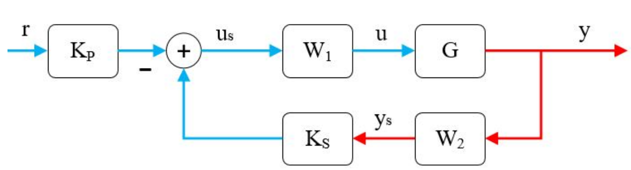

Figure 2: LSDP Controller with Pre-Compensator

The order of the controller calculated in step 2 refers to the number of poles of the developed controller for SISO models. The controller K_s along with the weights W_1 and W_2 are then used to develop the final feedback controller $K = W_1 K_s W_2$.

3.2 2 DOF LSDP Control Design

When strict model matching becomes a primary motive in the controller design, the 2 DOF LSDP [Hoyle *et al.*, 1991] should be used.

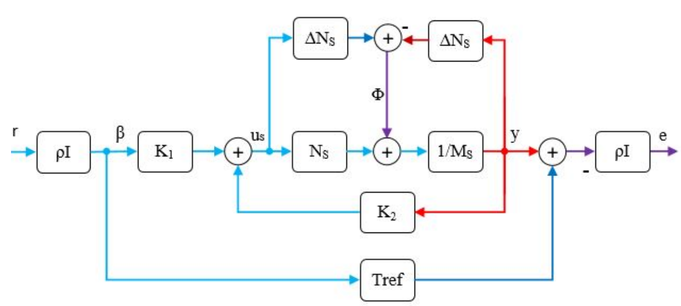

Figure 3: LSDP 2DOF Controller

The controller $[K_1 \ \ K_2]$ here aims to minimize the $\mathcal{H}_\infty$ norm of the transfer function matrix from $[r \ \phi]^T$ to $[u_S \ y \ e]^T$ in the Figure 3 for the shaped plant $G_S = GW_1$ with the co-prime factor representation $M_S^{-1} N_S$. With $G_s = [A_s B_s; C_s D_S]^T$ and $T_\text{ref} = [A_r B_r; C_r D_r]^T$ this transfer function in the standard robust control layout can be represented by the following plant matrix $P =$

$$\left[\begin{array}{cc|cc:c} A_s & 0 & 0 & (B_S D_S^T + Z_S C_S^T) R_S^{-1/2} & B_S \\ 0 & A_R & B_R & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I \\ C_S & 0 & 0 & R_S^{-1/2} & D_S \\ \rho C_S & -\rho^2 C_R & -\rho^2 D_R & \rho R_S^{-1/2} & \rho D_S \\ \hdashline 0 & 0 & \rho I & 0 & 0 \\ C_S & 0 & 0 & R_S^{-1/2} & D_S \end{array}\right]$$

being the system matrix from $[r \ \phi \ : u_s]^T$ to $[u_S \ y \ e \ : \beta \ y]^T$. Here $R_S = I + D_S D_S^T$, $S_S = I + D_S^T D_S$ and Z_S can be obtained by solving the Matrix Riccati Equation

$$(A_S - B_S S_S^{-1} D_S^T C_S) Z_S + Z_S (A_S - B_S S_S^{-1} D_S^T C_S)^T - Z_S C_S R^{-1} C_S Z_S + B_S S_S^{-1} B_S^T = 0$$

(Refer to [Glover and McFarlane, 1989] and [Zhou *et al.*, 1996] chapter 18 for derivation). The controller K_2 aims for internal robust stability while K_1 enables model matching by ensuring

$$||(I - GK_2^{-1} GK_1 - T_{ref})||_\infty \leq \gamma \rho^{-2}$$

where T_{ref} is the closed loop performance we are trying to achieve and ρ is a parameter selected by the control engineer to choose between robustness and model matching (usually a value between 1 and 3). Weight W_1 generally takes the form of a low pass filter (similar to 1DOF counterpart) while W_i takes the form of a constant. To develop the outer framework the following procedure is to be followed.

Algorithm # 2 Two DOF Weight Design Procedure

1. For the given plant G define the form of weights W_1,W_i ρ and choose initial values for the parameters. Choose a simple plant model T_{ref} that reflects desired closed loop response.
2. Define the cost function $\mathcal{J} = \mathcal{W}_1 \cdot \mathrm{IAE_y} + \mathcal{W}_2 \cdot \mathrm{IAE_u} + \mathcal{W}_3 \cdot$ `order`(K) and initialize these weights.
3. Implement the constrained non-linear optimization problem. Lower and upper bounds should be specified for the parameters of W_1, W_i and T_{ref} (which form the linear constraints) and γ (which forms the non-linear constraint). Here γ represents the LSDP $\mathcal{H}_\infty$ norm.
4. Depending upon robustness requirements vary the forms of weights W_1 and W_i (scaling factor) to reach the required performance levels. If the value of $1/\gamma$ is less than .25, increase the vary weight $\mathcal{W}_1, \mathcal{W}_2$ as well change the upper and lower limits of the linear constraints and perform step (iii) again.
5. Alongside the satisfactory performance weight the robust controller K = $[K_1 \ K_2]$ is obtained.

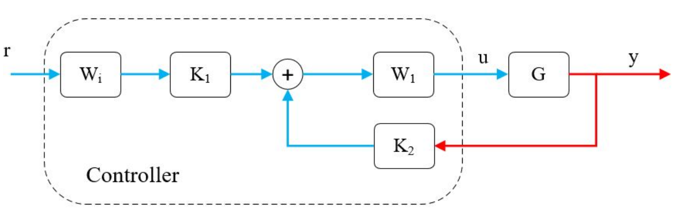

Figure 4: LSDP 2DOF Controller

Once designed the final controller is implemented as in Figure 4.

4 Experimental Results

The experimental results presented in the section are based on the quadrotor model developed using parameters from [Bouadi *et al.*, 2011; Thomas, 2017]. The non-linear equations in (2) based on these parameters are converted to the state space format, the equations can be linearised at state vector equal to 0 and input vector equal $\mathbf{u} = [0 \ 0 \ 0 \ m \cdot g]^T$. With the state vector defined as $\mathbf{x} = [\phi \ \theta \ \psi \ z \ p \ q \ r \ \dot{z}]^T$ and output vector $\mathbf{y} = [\phi \ \theta \ \psi \ z]^T$, linearisation at the mentioned values essentially decouples the plant model, ie we generate a plant model with y_i dependant only on u_i where i can take a value from 1 to 4.

The Jacobian matrices from the non-linear model are calculated using the MATLAB Symbolic Toolbox and once the decoupled state space models are received they are converted to the following transfer functions:
From u_1 to y_1 and u_2 to y_2 :

$$\frac{65.31s^6 + 56.76s^5 + 10.16s^4 + 0.4785s^3}{s^8 + 1.015s^7 + 0.282s^6 + 0.02996s^5 + 0.001066s^4} \quad (4)$$

From u_3 to y_3 :

$$\frac{1.514s^6 + 1.418s^5 + 0.3163s^4 + 0.02067s^3}{s^8 + 1.015s^7 + 0.282s^6 + 0.02996s^5 + 0.001066s^4} \quad (5)$$

From u_4 to y_4 :

$$\frac{-18.12s^6 - 6.684s^5 - 0.7945s^4 - 0.02991s^3}{s^8 + 1.015s^7 + 0.282s^6 + 0.02996s^5 + 0.001066s^4} \quad (6)$$

An amount of ±20% uncertainty in inertial matrix parameters, ±30% uncertainty in mass and ±50% uncertainty in translational and rotational drag coefficients are considered while conducting performance tests.

4.1 1 DOF Design Weights and Step Responses

The controllers are designed using Robust Control MATLAB Toolbox. The outer framework cost function which results in the associate controller weight designs are performed using the NOMAD [Le Digabel, 2011] algorithm provided by OPTI Toolbox. The design weights and the pre-compensator associated with the controllers are noted below.

Roll/Pitch Controller Weights

$$W_1 = \frac{1.513s + 12.83}{3.731s + 102.2}, \, W_2 = 2.5670, \, K_p = 0.4224$$

Yaw Controller Weights

$$W_1 = \frac{2.499s + 18.78}{6.153s + 62.42}, \, W_2 = 4.3127, \, K_p = 2.5000$$

Altitude Controller Weights

$$W_1 = \frac{1.76s + 10.74}{8.692s + 69.43}, \, W_2 = 4.1807, \, K_p = 1.3346$$

The value of χ is taken to be 1.1. The time responses to a unit step input for all the uncertain plants within this range is presented next. A unit step input (load) disturbance is introduced at $t = 5$ s such that it enters the system at the output of the plant model.

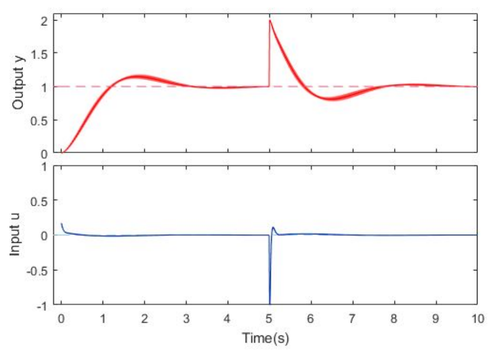

Figure 5: Roll/Pitch 1 DOF controller response

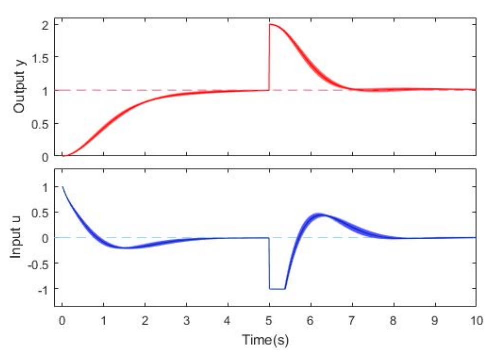

Figure 6: Yaw 1 DOF controller response

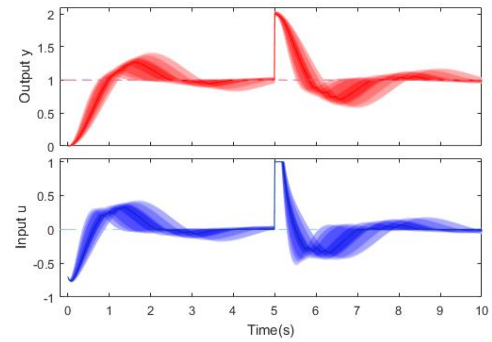

Figure 7: Altitude 1 DOF controller response

The darker intensity in the plot corresponds the performance of those plants which are closer to the nominal plant. We have satisfactory responses for roll, pitch and altitude models, for the single DOF controller configuration although the response for the altitude controller for the uncertain plants isn't tight indicating weaker robustness qualities.

4.2 2 DOF Design Weights and Step Responses

The controller weights are developed following the procedure explained in **Algorithm #2**. The outer framework that resolves constrained minimisation problem generates the weight parameters at the end of the routine as well as optimises parameters of the preferred model matching transfer function. These are noted below.

Roll/Pitch Controller Weights

$$W_1 = \frac{18.93s + 1.988}{9.506s + 1}, \, T_{\text{ref}} = \frac{1}{1.797s + 1}, \, W_i = 2.3247$$

Yaw Controller Weights

$$W_1 = \frac{10.79 + 1.16}{9.322s + 1}, \, T_{\text{ref}} = \frac{1}{1.502s + 1}, \, W_i = 3.5096$$

Altitude Controller Weights

$$W_1 = \frac{2.154s^2 + 1.887s + 18.18}{s^2 + 1.148s + 12.12}, \, T_{\text{ref}} = \frac{1}{1.44s + 1}, \, W_i = 2.4506$$

The time domain responses are presented from figures 8 to 10. All the plants in the uncertain plant space are subjected to a unit step input and a unit input(load) disturbance at the 10^{th} second entering the system at the output of the plant model. Here ρ was chosen as 1.

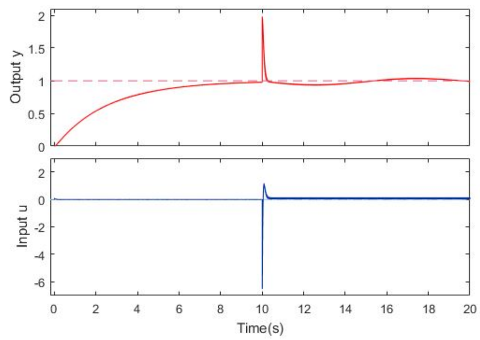

Figure 8: Roll/Pitch 2 DOF controller response

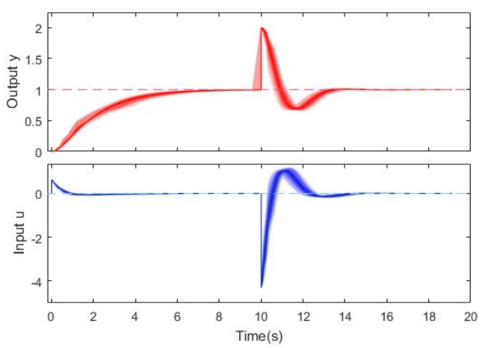

Figure 9: Yaw 2 DOF controller response

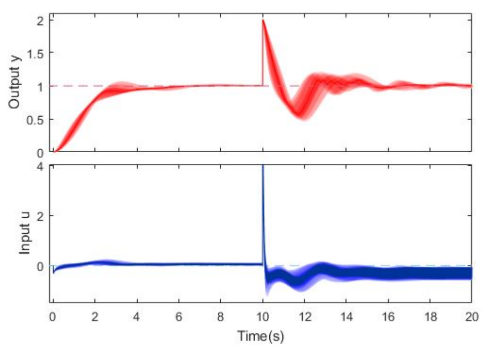

Figure 10: Altitude 2 DOF controller response

As evident from Figures 8 to 10, the response compared to one DOF configuration is rather sluggish as the designed controllers exhibit weaker disturbance rejection capabilities. Both the pitch/roll controller and the altitude controller responses takes relatively longer times to settle compared to their single degree of freedom counter parts. However the 2 DOF controllers exhibit zero overshoots while both 1DOF roll/pitch controller and altitude controller exhibits slight overshoots in the corresponding time responses. For the controller designs presented the process of weight selection automation has both removed the ambiguity resulting in manually selecting the weights, as well as, decreased the net time spent on deciding the parameters.

4.3 Performance Comparison

Figure 11 shows the closed loop sensitivity and complimentary sensitivity function of the designed controllers. The peak sensitivity M_S which relates to the disturbance rejection and reference tracking capabilities fall below 2 dB for all the designed controllers. Similarly the values of complimentary sensitivity function M_T which relates to noise rejection also falls below 2 dB. The values indicate that the designed controllers achieve acceptable levels of performance.

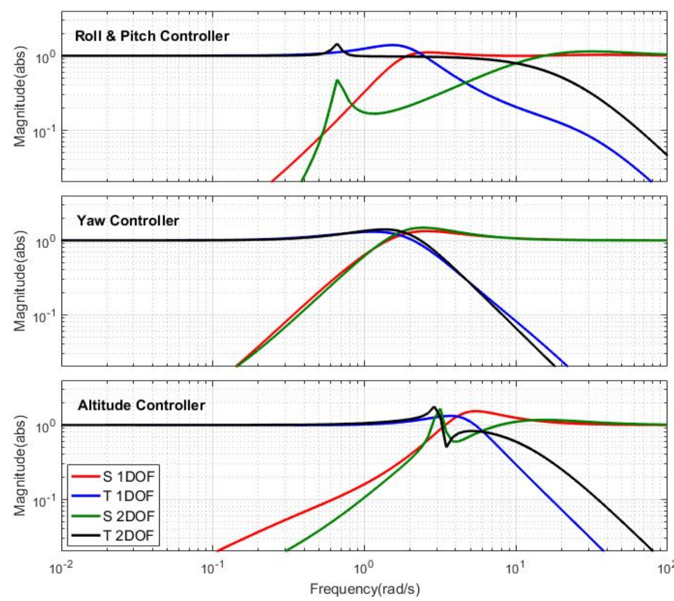

Figure 11: Sensitivity and Complimentary Sensitivity function for Closed loop SISO Roll model- Controller system, for 1 and 2 DOFs.

The exact values performance specifications are listed in Table 1. The 1 DOF freedom controller exhibits better performance in terms of M_S and M_T than the 2 DOF configuration, which had been inferred previously from the time responses.

The robust performance margin which is the inverse of the $\mathcal{H}_\infty$ cost function γ indicates factor by which the controller can accommodate additional co-prime factor uncertainty in the plant model. Values above 0.25 generally indicates acceptable robustness based on practical experience [Skogestad and Postlethwaite, 2007]. As seen from the Table 1 DOF controllers deliver better performance margins than their 2 DOF counterpart. The controller orders are also noted in the table. 1 DOF designs offer smaller controller orders compared the 2DOF configuration.

Table 1. Performance Comparison

Model	DOF	M_S (dB)	M_T (dB)	Robust Perf. Margin	Order
Roll/Pitch	1	1.111	1.395	0.4547 ✓	6
	2	1.145	1.461	0.2592 ✓	7
Yaw	1	1.334	1.307	0.405 ✓	5
	2	1.482	1.404	0.25 ✓	7
Altitude	1	1.536	1.329	0.4084 ✓	4
	2	1.801	1.916	0.2746 ✓	8

5 Conclusion

The one and two degree of freedom loop shaping design procedure based robust controllers have been developed for resolving the attitude and altitude tracking problem of a quadrotor UAV. A new framework has been introduced through two algorithms to both automate and formalize the process of controller weight selection. The algorithms resolve a non-linear constrained optimization problem developed over the conventional LSDP method to achieve this. Acceptable levels of performance have been obtained in both configurations for the attitude and altitude models. The 1 DOF controllers have been seen to offer better performance characteristics compared to the 2 DOF configuration as seen in the comparison test. The obtained robust performance margins guarantee acceptable levels of robustness against co-prime factory uncertainty in the model. Automating the weight selection procedure has reduced the net time spent on designing the controller while simultaneously ensuring acceptable levels of performance standards.

6 Acknowledgement

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