

The collision avoidance control algorithm of the UAV formation flight

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Abstract

A collision avoidance control algorithm for a multi-UAV system based on bidirectional network connection structure is proposed in this paper, which can effectively avoid the collision of between UAVs, between the UAV and the obstacle, achieve the UAV cooperative formation flight and complete the mission. In order to avoid the collision well, the proposed consensus-based algorithm and the “leader-follower” control strategy are simultaneously applied to the formation control so that we can ensure the convergence of the formation. The three UAVs composed of the triangular formation as the control object, the leader flight path as expected path, the followers track the leader to maintain the triangle formation flight. And the safety distance of between the UAVs, between the inside of the UAV formation and the obstacle keep the safety distance. Each of the UAVs has the same forward velocity and heading angle rate in the horizontal plane as well as they keep the relative distance constant in the vertical direction. This paper proposes a multiple UAVs consensus-based collision avoidance algorithm based on the artificial potential field method. The simulation experiments are performed on multiple UAVs to validate the proposed control algorithm and have some reference significance for engineering application.

1 Introduction

A multi-UAV formation system has more significant advantages than a single UAV, such as the long the time of flight, the big combat radius, the wide range of investigation, the high combat efficiency and strong search ability and so on in the military field. And spraying pesticides, aerial photography, terrain survey and air refueling in the civilian field. Therefore, it has attracted much more attention from many scholars and becomes a hot topic of research[1], especially focusing on the path planning[2-4], cooperative formation control, information

sharing and fusion[5-7], obstacle avoidance as well as other aspects of research, which has been made some obvious achievements. Also, a multi-UAV formation system is only a small part of application in engineering practice. However, most of them are based on theoretical level of research. The security problem of the UAV formation flight is one of the key factors, especially the obstacle avoidance research is particularly important. The obstacle avoidance problem for a multi-UAV formation system has been studied well in recent years, and the proposed many control algorithms have been developed for the problems. The obstacle avoidance control algorithm is roughly grouped into two categories that rule based approaches and optimization based approaches. One of the rule based approaches is an artificial potential field based approach[8-9]. An artificial potential field is not only applied to the UAV obstacle avoidance problems, but has been extensively applied to autonomous robot navigation, and then its basic theory is that the UAVs move along the negative gradient of the composite of the potential fields. However, the optimization-approaches is a model predictive control(MPC)[10]. The main control algorithm in process of the formation control is applied in the consensus-based control theory to design the controller well. The consensus-based algorithm for the UAV cooperative formation control is a kind of distributed control method, which has the advantage of having network structure flexibility[11-14], and achieves multi-channel compound control obstacle avoidance. The key problem of the multi-agent system obstacle avoidance control is how to apply a consensus-based algorithm to cope with it well [15-17], it will greatly simplify the complexity of the problem we studied. Therefore, a consensus-based algorithm and the artificial potential-based method are applied to the formation so that they can effectively avoid obstacle in the three dimensional space.

This paper is organized as follows. In Section 2, we state the building model for the three UAVs formation system. Firstly, with regard to a three UAVs formation system with bidirectional network links. Secondly, with regard to a single UAV, it is decoupled the linear system of the horizontal lateral direction and the vertical direction

to design the collision avoidance algorithm well. In Section 3, we proposed the control methods for the problems defined in Section 2, including the “leader-follower” control scheme, the artificial potential-based method, and the collision avoidance schemes of design. Section 4 presents experimental results to validate the proposed control algorithm to achieve the collision avoidance effectively, including the setup initial condition and the analysis of simulation experimental results. Finally, the concluding remarks are stated in Section 5.

2 Modeling a Multi-UAV system

Suppose that there are $N(N=3)$ UAVs with the same motion characteristics, including $N-1$ followers and a leader, which is composed of a UAV formation system. With regard to a multi-UAV system with bidirectional network links between the UAVs, that is, the between the any two UAVs of the formation system exchange information with each other. This network can be mathematically described using graph theory. So the UAV formation system of the network topology, as shown in figure 1:

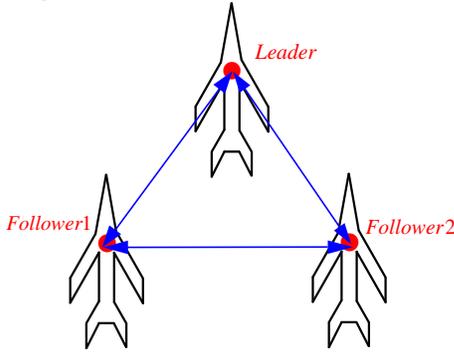


Fig. 1 The UAV formation system of the network topology

A graph G consists of a set of the nonempty finite sets $V = V(G)$ and a set of the V disordered pairs. Where V is a set of nodes, $|V|$ is the order of graph G , E is a set of edges and the disordered number in E is usually expressed in the form of the line to connect the two nodes, called edge. Also, $|E|$ is the number of sides of graph G [18]. In this paper, we use a graph $G = (V, A)$ to establish the information interaction relationship of between n UAVs, where $V = \{v_1, v_2, v_3, \dots, v_N\}$ is a set of nodes, and $A \in V \times V$ is a set of edges. The edge (v_i, v_j) in E represents that there is a network path from the UVA i to the UAV j , that is, the UAV j can obtain and use information from the UAV i . A directed tree is a graph G , including all of nodes. Every node acts as one parent node except for itself, called a root node. In a directed graph, the root node cannot be directly connected to the parent node, can be directly connected with other nodes. However, in the undirected graph, the edge (v_i, v_j)

notes that UVA i and the UAV j can exchange information with each other, in other words, (v_i, v_j) and (v_j, v_i) are the same. Let $A \in R^{N \times N}$, $D \in R^{N \times N}$ and $L \in R^{N \times N}$ be an adjacency matrix, a degree matrix, and a graph Laplacian matrix. Where A represents the relationship of between nodes in graph theory, and based on the graph theory, the neighbor matrix $A = [a_{ij}]$ is given by

$$a_{ij} = \begin{cases} 1, & (v_i, v_j) \in A \\ 0, & (v_i, v_j) \notin A \end{cases} \quad (1)$$

From the above equation, if the UVA i is directly obtaining information from the UAV j , so $a_{ij} = 1$, otherwise $a_{ij} = 0$. The degree matrix

D represents an in-degree matrix given by

$$D = \text{diag}(\text{deg}(v_1), \text{deg}(v_2), \dots, \text{deg}(v_N)) \quad (2)$$

where $\text{deg}(v_i)$ is the communication sum number of v_i node with other nodes. The graph Laplacian matrix

L is defined as[3]:

$$L = [l_{ij}] \in R^{n \times n}, l_{ij} = \sum_{i \neq j} a_{ij}, l_{ij} = -a_{ij} (i \neq j) \quad (3)$$

From the equation above, the matrix L has the following two properties:

$$(1) l_{ij} < 0, i \neq j, \sum_{j=1}^n l_{ij} = 0, i = 1, 2, \dots, n$$

Definition $\mathbf{1}_n$ denotes a N -dimensional column vector of all elements that are 1, and $\mathbf{0}_n$ denotes an N -dimensional column vector of all elements that are 0. According to the definition of the Laplace matrix, the following equation holds: $L\mathbf{1}_n = \mathbf{0}_n$ [20].

(2) $L = D - A$, if a graph G has a directed spanning tree, the matrix L has a single eigenvalue at zero, and all nonzero eigenvalues of the graph Laplacian have positive real part.

3 Collision avoidance algorithm

In this section, we further study how to avoid obstacle for the formation system based on the artificial potential field method and two kinds of obstacle schemes to validate the proposed control algorithm, which can achieve the obstacle avoidance between the UAVs.

Aiming at the obstacle avoidance of the UAV formation system in the three dimensional space, the control algorithm is proposed to effectively avoid obstacle in the vertical direction and in the horizontal plane. In the paper, a consensus-based control algorithm is applied to avoid the collision in the vertical direction only to take evasive action. However, the triangular formation system can fly as the control command, and avoid the collision between them effectively. This section mainly study how to avoid the collision about the obstacle of moving using

the consensus-based algorithm in the vertical direction. For formation flight in the vertical direction, the controlled objectives are roughly grouped into two categories. The one category is that the three UAVs consist of the formation system as a control objective, the other is that each of the UAVs act as a control object. The control objective cooperative formation flight is realized using the consensus-based algorithm while keeping the geometric configuration of the formation. However, each of the UAVs can achieve the formation flight using the “leader-follower” control strategy. The leader provides the directly connected two followers with its own posture and velocity to ensure that the follower can track the leader keeping the triangle formation. In the paper, the desired control algorithm has an advantage in bidirectional connection between the UAVs in topology, and the information transmission between them is smooth, which prevent information from clogging duo to the overload of information. The control law for UAV i is given by

$$\begin{aligned} \tilde{T}_{total_i}(t) = & -\sum_{j=1}^{N+1} a_{ij} \left[\sum_{k=0}^1 \gamma_k (\hat{h}_i^{(k)} - \hat{h}_j^{(k)}) \right] \\ i \in & \{1, 2, \dots, N+1\} \end{aligned} \quad (4)$$

$$\hat{h}_j^{(k)} = h_j^{(k)} - h_{h_j}^{(k)}, \quad j \in \{1, 2, \dots, N+1\}, \quad k \in \{0, 1\} \quad (5)$$

where the meaning of symbol $N+1$ denotes a leader, and $\gamma_k \in \mathbb{R}$, $k \in \{0, 1\}$ are control gains, $h_j^{(k)} \in \mathbb{R}$, $k \in \{0, 1\}$ are the state of the UAV j , $h_{h_j}^{(k)} \in \mathbb{R}$, $k \in \{0, 1\}$ are the desired relative state of between the UAV j and the leader in the vertical direction. As shown in Eq. (1), a_{ij} indicates whether there is information acquisition between the UAV i and the UAV j , that is, when there is information acquisition between the UAVs, a_{ij} is set to one, otherwise a_{ij} is set to zero.

In the paper, the proposed control algorithm for the UAV formation system with collision avoidance capability is as follows:

$$\tilde{T}_i = f_{form_i} + f_{ca_i}, \quad i \in \{1, 2, \dots, N\} \quad (6)$$

where f_{form_i} is the control algorithm for formation in the vertical direction, f_{ca_i} is the obstacle avoidance of the artificial potential field.

Theorem : Suppose that the linear model of the UAV formation system comprises a leader and $N (\geq 2)$ UAVs as expressed in Eq. (6) and that assumptions (1)-(3) are satisfied. Also, the Eq. (6) with the positive control gains $\gamma_k, k \in \{0, 1\}$ is satisfied with each of the UAVs, then all of the states of the UAVs in the vertical direction will asymptotically converge to the desired flight states.

Proof: By applying control algorithm (6), we can get

$$\begin{aligned} \dot{h}_i^{(1)} = & -\sum_{j=1}^{N+1} a_{ij} \left[\sum_{k=0}^1 \gamma_k (\hat{h}_i^{(k)} - \hat{h}_j^{(k)}) \right] + f_{ca_i} \\ i \in & \{1, 2, \dots, N\} \end{aligned} \quad (7)$$

where we define the new state vector

$$\bar{h}^{(k)} = \begin{bmatrix} h_1^{(k)} & h_2^{(k)} & \dots & h_N^{(k)} & 0 \end{bmatrix}^T \in \mathbb{R}^{(N+1)} \quad \text{and}$$

$$\hat{h}^{(k)} = \begin{bmatrix} \hat{h}_1^{(k)} & \hat{h}_2^{(k)} & \dots & \hat{h}_{N+1}^{(k)} \end{bmatrix}^T \in \mathbb{R}^{(N+1)}, \quad k \in \{0, 1\}, \quad \text{and then}$$

the new force vector $\bar{f}_{ca} = \begin{bmatrix} f_{ca}^T & 0 \end{bmatrix}^T \in \mathbb{R}^{(N+1)}$. Using new defined symbols, we can rewrite (13) as:

$$\dot{\bar{h}}^{(1)} = -\gamma_0 L \hat{h}^{(0)} - \gamma_1 L \hat{h}^{(1)} + \bar{f}_{ca} \quad (8)$$

where the matrix L is the graph laplacian of the multiple UAV system, the state vector $\hat{h}^{(k)}$ consists of the states of follower and commands from the leader. In the paper, the following identities concerning the rows of the graph laplacian L hold, as follows in Eq. (9):

$$\begin{aligned} a_i(N+1) = & \sum_{j=1}^{N+1} a_{ij} - a_{i1} - a_{i2} - \dots - a_{iN} \\ a_{ii} = & 0 \quad i \in \{1, 2, \dots, N\} \end{aligned} \quad (9)$$

According to the $d_{N+1}^{(k)} = 0$, the $k \in \{0, 1\}$, and the Eq. (9), so the Eq. (8) is expressed as:

$$\begin{aligned} \dot{h}^{(1)} = & -\gamma_0 M h^{(0)} - \gamma_0 M h^{(0)} + \gamma_0 M \tilde{h}_{N+1}^{(0)} + \gamma_1 M \tilde{h}_{N+1}^{(0)} \\ & + \gamma_0 M d_h^{(0)} + \gamma_0 M d_h^{(0)} + f_{ca} \end{aligned} \quad (10)$$

where the matrix $M \in \mathbb{R}^{N \times N}$ is defined as (11), the state vector $h^{(k)}$ and $\tilde{h}_{N+1}^{(k)}$ are respectively

$$h^{(k)} = \begin{bmatrix} h_1^{(k)} & h_2^{(k)} & \dots & h_N^{(k)} \end{bmatrix}^T \quad \text{and} \quad \tilde{h}_{N+1}^{(k)} = 1_N \otimes h_{N+1}^{(k)} \in \mathbb{R}^N.$$

Also \otimes is the kronecker product, and $1_N = [1 \ 1 \ \dots \ 1]^T \in \mathbb{R}^N$. Here, the matrix M is similar to the graph laplacian, but not equal to the matrix.

$$M = \begin{bmatrix} \sum_{j=1}^{N+1} a_{1j} & -a_{12} & \dots & a_{1N} \\ a_{21} & \sum_{j=1}^{N+1} a_{2j} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{Nj} & a_{Nj} & \dots & \sum_{j=1}^{N+1} a_{Nj} \end{bmatrix} \quad (11)$$

According to the matrix M , the Eq. (12) is simplified in a matrix vector as:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} = & \begin{bmatrix} 0_N & I_N \\ -\gamma_0 M & -\gamma_1 M \end{bmatrix} \begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} + \begin{bmatrix} 0_N \\ f_{ca} \end{bmatrix} \\ & + \begin{bmatrix} 0_N & I_N \\ \gamma_0 M & \gamma_1 M \end{bmatrix} \begin{bmatrix} \tilde{h}_{N+1}^{(0)} \\ \tilde{h}_{N+1}^{(1)} \end{bmatrix} + \begin{bmatrix} 0_N & I_N \\ \gamma_0 M & \gamma_1 M \end{bmatrix} \begin{bmatrix} d_h^{(0)} \\ d_h^{(1)} \end{bmatrix} \end{aligned} \quad (12)$$

where $I_N \in \mathbb{R}^{N \times N}$ is a N -dimensional unit matrix, $0_N \in \mathbb{R}^N$ is a N -dimensional zero vector.

To validate the stability of the Eq. (12), the homogeneous Eq. (18) is expressed as :

$$\frac{d}{dt} \begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_N & I_N \\ -\gamma_0 M & -\gamma_1 M \end{bmatrix} \begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_N \\ f_{ca} \end{bmatrix} \quad (13)$$

Here, we construct the following Lyapunov candidate function V , it consists of the total energy of the multi-UAV system. The expression is as shown:

$$V = \frac{1}{2} \dot{h}^T \dot{h} + \frac{1}{2} \gamma_0 h^T M h + U_c \quad (14)$$

The time derivative of the function V is given by:

$$\dot{V} = \dot{h}^T (\ddot{h} + \gamma_0 M h) + \dot{U}_c = -\gamma_1 \dot{h}^T M \dot{h} + \dot{h}^T f_{ca} + \dot{U}_c \quad (15)$$

To avoid obstacle well between the leader and follower, artificial force is proposed, as follow in Eq. (16):

$$f_{ca_i} = -\nabla h U_c, \quad i \in \{1, 2, \dots, N\} \quad (16)$$

From (22), we can get the Eq. (23):

$$\dot{U}_c = \dot{h}^T \nabla U_c = -\gamma_1 \dot{h}^T f_{ca} \quad (17)$$

Hence, from (21) and (23), we can get

$$\dot{V} = -\gamma_1 \dot{h}^T M \dot{h} \quad (18)$$

The Lyapunov candidate function V represents the energy function of the artificial field between the UAV formation system. Through the study of the function V , we can get the UAV formation system with collision-avoidance capability to achieve the control of the ground station effectively and the purpose of avoiding obstacle. The graph G has a directed spanning tree if the assumptions (1)-(3). Hence, from the property of the graph Laplacian, L has a single eigenvalue at zero, and the otherwise have positive real part. According to the property and the relationship between the L and the M , we can obtain the matrix M is positive definite. In addition, the artificial potential field parameter K_h is positive value, we can get $U_c \geq 0$. Therefore, when the control gain γ_0 and k_h are positive simultaneously, we get $V \geq 0$, and then when the control gain γ_0 and k_h are zero simultaneously, we get $V = 0$. And also, when only $h = 0$, $\dot{h} = 0$ and $U_c = 0$, we can also get $V = 0$. Note that when there is no overlap of the safety regions between the UAVs, we can get $U_c = 0$.

For the derivative of the Lyapunov candidate function \dot{V} , we can get $\dot{V} \leq 0$ when the control gain γ_1 is positive and $M > 0$. We can get $\dot{V} = 0$ if $h = 0$, $\dot{h} = 0$ and $U_c = 0$, otherwise, $\dot{V} \neq 0$. The asymptotic stability can be solved by Lyapunov theorem involving LaSalle's principle[21].

When there is no overlap in the safety region, the particular solution of the Eq. (29) can be given by

$$\begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} = \begin{bmatrix} \tilde{h}_{N+1}^{(0)} \\ \tilde{h}_{N+1}^{(1)} \end{bmatrix} + \begin{bmatrix} d_h^{(0)} \\ d_h^{(1)} \end{bmatrix} \quad (19)$$

This validity can be confirmed by substituting the Eq. (19) for the Eq. (12), so it's no need to explain it. Because the leader provide the desired commands of each of followers, not the desired input commands, we can use

$\tilde{h}_{N+1}^{(2)} = 0$ and $d_h^{(2)} = 0$. Note that $\tilde{h}_{N+1}^{(2)} = 0$ and $d_h^{(2)} = 0$ are the input of the leader and the input error of between the leader and the follower.

In conclusion, the general solution of the non-homogeneous differential the Eq. (13) is the sum of between the general solution of the homogeneous equation and the particular solution. Hence, the general solution of Eq. (13) asymptotically converges to the (20) when control gains γ_k , $k \in \{0, 1\}$ and the artificial potential field parameter k_h are selected to be small positive.

$$\begin{bmatrix} h^{(0)} \\ h^{(1)} \end{bmatrix} \rightarrow \begin{bmatrix} \tilde{h}_{N+1}^{(0)} \\ \tilde{h}_{N+1}^{(1)} \end{bmatrix} + \begin{bmatrix} d_h^{(0)} \\ d_h^{(1)} \end{bmatrix}, \quad \text{and } t \rightarrow \infty \quad (20)$$

From the (20), we can get the result of convergence to the command from the leader. According to the element of the first row block in (9), it is proved that every UAV has the capability of the collision avoidance and can asymptotically converge to the desired trajectory for the triangle formation flight.

4 Simulation experiment and analysis

In this section, we validate the proposed the consensus-based control algorithm using the simulation experiment for application to a multi-UAV platform. $k_h = 5.5$

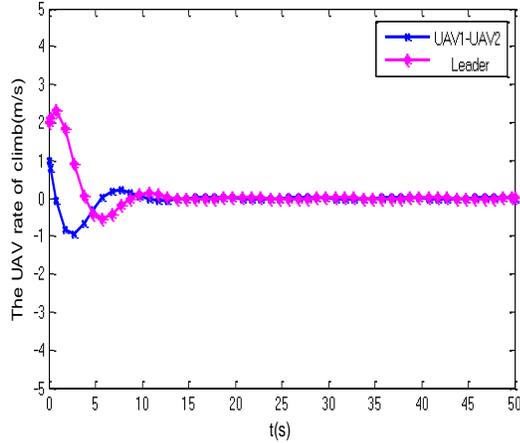
4.1 The setup initial condition

The relative the ground velocity of the UAV and the moving velocity of the obstacle are 56 m/s, the mass of each of the AUVs is 90kg, the pitch rate limit for UAV is 10deg/s, the yaw rate limit for UAV is 12 deg/s. To satisfy the building model and simulation of all of the condition in vertical direction, we set up the control gains $\gamma_0 = 1.5$, $\gamma_1 = 3$ and the control parameter for collision $k_h = 5.5$. And the altitude of safety region for every UAV is 3m, the radius of safety of region is 3.5m, and the altitude gap of between them is no more than 1.2m, the relative distance of between them is no more than 10m. In order to validate the proposed control algorithm, the simulation experiments can be grouped into two categories, the one is carried out without adding the collision avoidance algorithm, the other is carried out with adding the collision avoidance algorithm. The UAV cooperative formation flight control algorithm has been published in the literature, and has a good effect on the collision avoidance [8]. By the comparison and analysis of two group simulation experiments, the proposed control algorithm can not only improve the control method of published literature, but achieve the effect of the collision avoidance well.

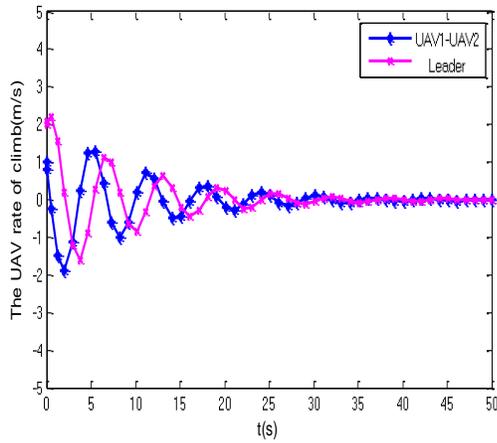
4.2 Simulation results

According to the initial conditions and assumptions of the simulation, we show the simulation results, as shown in fig.2 to 4. The fig. (a) and (b) of the following figures represent respectively the simulation result of the without

the control algorithm and the simulation result of adding the control algorithm.

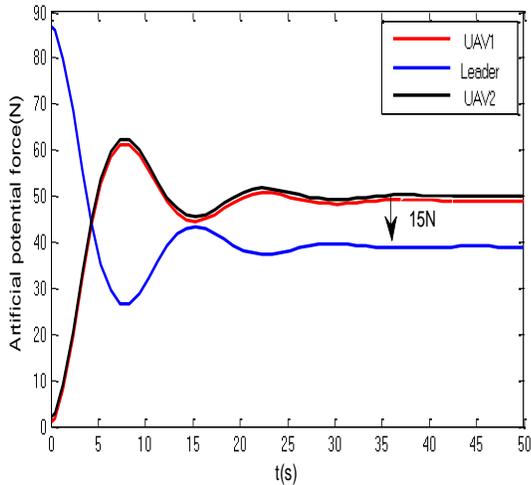


(a) The time response without collision avoidance algorithm

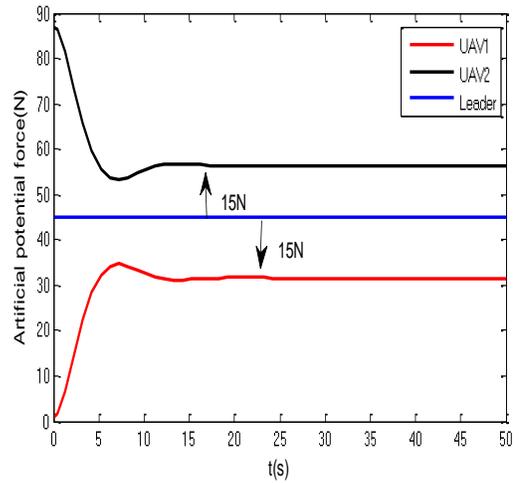


(b) The time response with collision avoidance algorithm

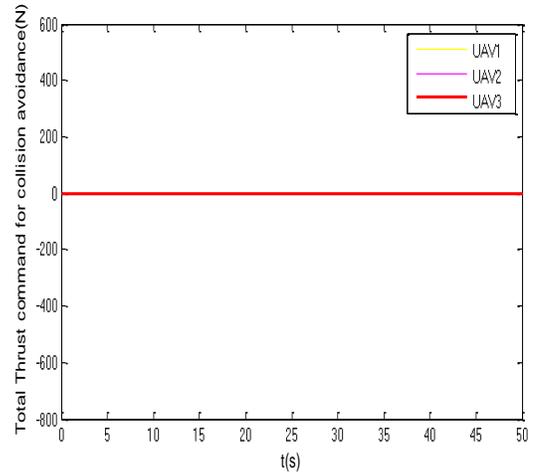
Fig. 2 The rate of climb of the UAV



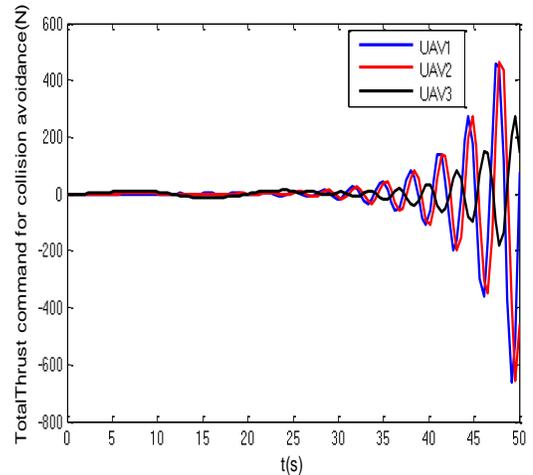
(a) The time response without collision avoidance Algorithm



(b) The time response with collision avoidance algorithm
Fig. 3 The rate of climb of the UAV



(a) The time response without collision avoidance Algorithm



(b) The time response with collision avoidance algorithm
Fig. 4 The rate of climb of the UAV

The fig. 2(a) shows that the rate of climb has the same of between the two followers and a leader from 1s to 12s, and the trend of curves decreases firstly and then increases slowly, and eventually remains constant value, in other words, the three UAVs keep the triangle

formation flight. As we can be seen from fig. 2(b), the two curves both present a continuous oscillation trend from 3s to 30s, and eventually converge to the constant value after 30s. The UAV formation system takes evasive maneuver if the UAV formation system encounter the obstacle, and then recover the triangle formation flight immediately.

The fig. 3 (a) shows the whole process of the UAV formation system, including from taking off to the assembly formation, through the encountered the obstacle to the loose formation, and then the assembly triangular formation. The curve of the leader presents a trend of the decrease firstly and then increase slowly, however, the trend of the two followers is opposite from 1s to 30s. Because the relative distance of between the followers and a leader is increase if the UAV formation system encountered the obstacle. And also, the relative distance decrease firstly and then keep constant, while the artificial potential field presents a trend of the decrease firstly and then increase slowly. From the fig. 3 (b), we can obtain that the desired constant value of artificial potential field of between the two followers and a leader is 45N. By adding the collision avoidance algorithm, the relative distance of any two UAVs in the formation remains constant, and the artificial potential force of between the two followers and the a leader are equal to 15N, which can achieve the purpose of the collision avoidance.

To ensure the UAV formation as the geometric configuration of the triangle and the desired trajectory flight, the total thrust of every UAV provided is equal and their direction is consistent, so the curve presents a horizontal trend. From the fig. 4(b), the three UAVs cooperatively flight from 1s to 30s, the UAV encounter the obstacle after 30s, and the curve presents an oscillation divergence trend. When the UAV formation encounter the obstacle and avoid the collision successfully, the relative distance of the inside of follower in the formation and the moving obstacle, presents a trend of the increase firstly and then decrease slowly. At the same time, the artificial potential force presents a trend of the decrease firstly and then increase slowly.

Based on the above analysis, the proposed control strategies can effectively avoid the collision between the UAVs, between the UAV formation system and the obstacle through the comparative analysis of simulation experiment, and can achieve the purpose of the collision avoidance in theory. The main ideas of collision avoidance can be divided into two categories. The one is that the collision avoidance algorithm combined with artificial potential field is applied to avoid the collision in the vertical direction, the other is that the artificial potential field is applied to avoid the collision in the horizontal plane.

5 Conclusion and future research

Aiming at the three UAVs formation system and the obstacle as the controlled objectives, a linear mathematical model for the UAV formation system is built by using the graph theory, and the UAV formation

cooperative control the collision avoidance algorithm is proposed in the three dimensional space. Lyapunov theory was applied to analyze the stability of the proposed for the collision avoidance. The simulation experiments are carried out to validate the convergence and validity of the desired control algorithm under the condition of building model and simulation hypothesis. The improved control algorithm of the artificial potential field and "leader-follower" control strategy is applied to the complicated collision avoidance in the three dimensional space, and it can be simplified as the collision avoidance control in the horizontal plane and the control in vertical direction. The relative distance of between any two UAVs in the formation is less than in the safety region in the vertical direction, it must take evasive maneuver to avoid the collision. And the artificial potential field method is applied to avoid the collision in the UAV formation in the horizontal plane. When the relative distance of between any two UAVs in the formation is less than in the safety region, it will produce the repulsive force, otherwise, it will produce the attractive force. Eventually the UAV formation system reaches an equilibrium state and forms the triangle formation flight. However, the paper is only theoretical research, no real experiment. Because the real experiment need to take many factors into consideration, including the moving of the four objectives, many uncontrollable factors and the involving more knowledge. Hence, the proposed algorithm is applied in actual project, which is the next major research topic.

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