Passive Height Stability and Trajectory Repeatability of a Quadrotor Maneuvering in Ground Effect with Regulated Voltage Bus

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Abstract

Passively stable aerodynamic systems are able to regulate their flight without need for additional sensing or actuation. Passive rotor-surface interaction provides a mechanism for repelling a rotorcraft away from the ground and surrounding objects. However, the vertical dynamics of a helicopter are very sensitive to battery voltage, which makes developing these systems more challenging. There is a need to maintain a constant motor bus voltage so that the response of the system is consistent.

We employ a step-down buck regulator to maintain a constant voltage for the entire drive system of a miniature quadrotor. This allows us to better study the power efficiency of a quadrotor in ground effect, demonstrate robust passive height stability and determine the influence of voltage variability on trajectory repeatability.

1 Introduction

Miniature hovering robots have substantial potential for use flying in confined 3D spaces such as duct-work for maintenance and inspection tasks. The small-size and inaccessibility of these environments limits the scope of sense and avoid equipment that can be employed for safe navigation. While flight control of robot aircraft conventionally employs active feedback to maintain the desired trajectory, some designs (most notably fixed-wings) use passive stabilisation techniques that do not require additional sensing and actuation. We aim to develop a miniature multirotor craft that uses passive aerodynamic feedback to stabilise position and avoid obstacles and surfaces. Of particular interest are two key capabilities: consistent aerodynamic performance and self-stabilisation of altitude in ground effect.

Quadrotors are predominantly powered by lithium polymer cells which, while providing excellent energy density and discharge rates, suffer from non-constant output voltages [Tuta Navajas and Roa Prada, 2014] — ranging from 4.2 V when fully charged, down to <3 V when depleted. The throttle must be continuously adjusted to maintain constant height. Furthermore, diminishing battery voltage will reduce the available current draw to affect motor torque for dynamic rotor response. Podhradsky *et al* found that sagging battery voltage increased motor command response latency and gave rise to a noticeable reduction in the agility of the craft [Podhradsky *et al*, 2013].

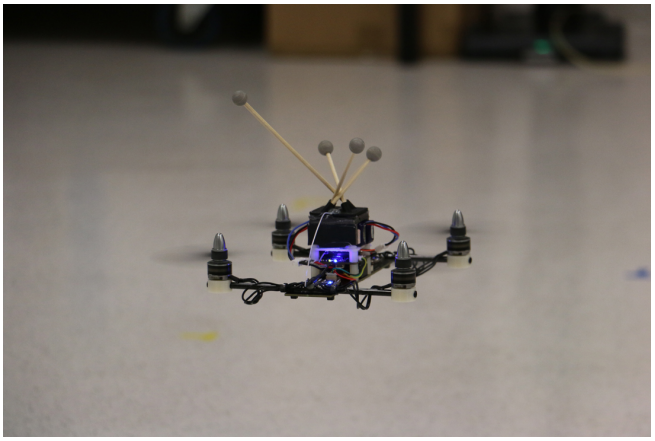

Figure 1: Mini quadrotor with voltage regulator

Compensators designed to overcome this problem employ either endogenous techniques such as battery voltage models, or exogenous controllers based on sensor feedback. Podhradsky *et al* use a characteristic discharge curve for constant thrust control algorithm within the control loop. The curve for each battery pack tested followed the same general negative linear gradient, but were more pronounced for smaller packs than larger packs. As each curve is different, this approach cannot adjust for small inconsistencies in the discharge curves between different models of battery [Podhradsky *et al*, 2013]. Efe *et al* developed an active control solution which measures instantaneous battery voltage to maintain constant thrust [Efe, 2012]. The findings demon-

strated that accounting for differential battery voltage within the control loop had desirable performance effects.

Bangura *et al* at ANU regulate aerodynamic power via onboard measurements of rotor speed and current for each motor [Bangura *et al*, 2014]. These were used to estimate usage of electrical power and thus the total contribution to aerodynamic power, resistive loss within the motor and mechanical power being produced by the motor. Their testing focused on validating the effectiveness of the aerodynamic controller at rejecting wind disturbances in axial and planar direction. The controller was able to resist induced changes in translational lift due to imposed horizontal and axial airflow disturbances at altitudes well outside of the ground effect. The controller was able to achieve a finer degree of thrust manipulation than state-of-the-art motor control based on regulating rotor RPM.

For our application, thrust consistency and regularity in dynamic response are an important part of a research apparatus for developing fully passive systems. In preliminary experiments, it was found that varying voltage over the course of several tests made results impossible to compare. For this reason, we seek to develop an onboard power regulation system that maintains constant battery voltage, even during active maneuvers. This will allow us to better study the effects of coupled airstream-vehicle-object dynamics when flying in cluttered environments.

Passive height control exploits the increase in thrust due to ground effect when flying close to the ground. This phenomenon is well known [Leishman, 2002; Prouty, 1995; Sharf *et al*, 2014]. Kushleyev *et al* found that micro rotors within 1 rotor radii to the ground required 15 per cent less power to hover [Kushleyev *et al*, 2013]. By setting the throttle slightly below that required to hover in free air, the aircraft will descend until the increase in thrust exactly balances gravity. However, the dynamics of hovering thrust are complicated by the coupled rotor speed mechanics and inflow dynamics of the rotor system.

From Bangura *et al*, we expect that simple voltage regulation will provide for stable altitude dynamics, and facilitate more consistent quadrotor dynamic performance when flying trajectories [Bangura *et al*, 2014]. We have developed a quadrotor platform that integrates a compact switching step-down voltage regulator capable of sourcing sufficient current to drive all four motors simultaneously, with minimal voltage ripple. This obviates the need for explicit height above ground sensing to stabilise altitude.

In this paper, we describe a quadrotor drive system with regulated voltage, and its performance in ground effect hovering and maneuvering compared to a matching, unregulated quadrotor. In section 2 we outline a regulator circuit for powering an entire quadrotor drive system and its design considerations. In section 3 we describe the vertical dynamics of a quadrotor in ground effect and present a simulation demonstrating its stability. We present experiments tracking the power required to hover in ground effect with regulated voltage for varying throttle settings. In section 4, we present experiments measuring the repeatability of flight motions for a regulated and unregulated quadrotor, and show that the regulated aircraft gives more consistent performance. A brief conclusion and discussion of future work completes the paper.

2 Platform Development

We developed a quadrotor platform for ground effect flight, equipped with onboard power regulation to provide constant voltage to the motor Electronic Speed Controllers (ESC). The current draw of the entire quadrotor in hover is 3.5 A; We designed a step-down buck regulator capable of supplying 6 A with a large output inductor to give a low out voltage ripple of 50 mV(pp) at full load. The ESCs receive a constant 8.1 V, even during periods of higher instantaneous current demand. The primary voltage regulator was combined with a 5 V regulator for avionics, power measurement sensors on the input and output of the regulator, and power and control signal distribution to the motor drivers and flight controller. The Printed Circuit Board (PCB) and circuit diagram are shown in figures 2 and 3 respectively.

The PCB also forms the structure of the airframe. Two parallel woven carbon fibre tubes were attached using high strength double-sided tape and cable ties. Woven carbon fibre was selected for its high torsional strength, as required in quadrotors [Davis *et al*, 2013]. The brushless motors were attached to these arms using 3D printed ABS mounts. The complete quadrotor is shown in figure 1.

The drive system consisted of four 100 mm 4x4.5 propellers each driven by a brushless 1306-3100 kV motor and KISS 12 A ESC. The main flight control board was a Sparky2 running Taulabs code. Two 2 cell 300 mAh LiPo batteries supplied the power. In the regulated voltage configuration the batteries were connected in parallel and regulated down to a constant 8.1V. In the unregulated voltage configuration the batteries were connected in series to the ESCs directly, giving a varying voltage of 8.4V to 6.6V at cell exhaustion.

3 Altitude Stability and Ground Effect Interaction

The vertical dynamics of a rotor have often been modeled with tacit assumptions about constancy of power, thrust or rotor speed, but this is not the case for a rotor

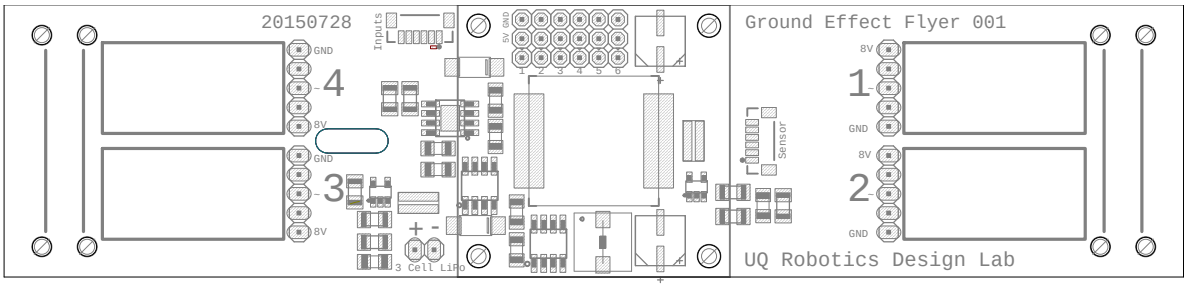

Figure 2: Ground effect flyer PCB layout at 1:1 scale

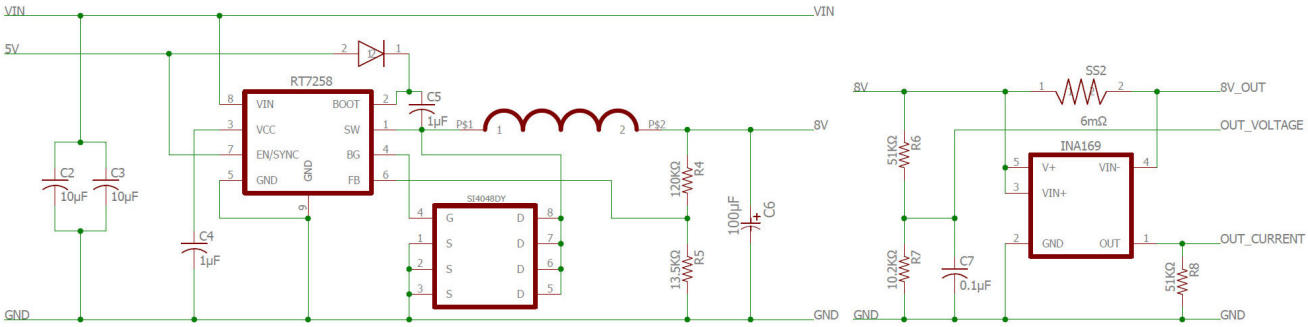

Figure 3: Schematic of the ground effect flyer power regulation circuit (left) and motor voltage sensor (right)

moving vertically close to ground. The rotor has numerous dynamic states:

- Power bus voltage
- Drive circuit current draw
- Rotor angular velocity
- Inflow velocity
- Airfoil induced angle
- Height above ground
- Vertical velocity

Of these, bus voltage and current draw are usually measured, and rotor angular velocity is often available integrated into the motor's electronic speed controller. Rotor inflow velocity and airfoil induced angle are only rarely measured; height above ground and vertical velocity are sometimes available with added sensors such as GPS. Consequently, rotor velocity is often closely regulated, with longer control loops around aircraft position and velocity. The relationships between rotor speed, vertical motion and air flow mechanics are typically treated as instantaneous and abstracted into thrust generation. These unsensed and uncontrolled states must be intrinsically stable for systems where the only feedback is passive aerodynamic interaction.

3.1 Rotor Coupled Dynamics Model

The vertical motion of the rotor is give by simple Newtonian mechanics:

$$m\ddot{z} = -mg + T \tag{1}$$

where m is the rotor system mass (eg. a quarter of a quadrotor), z is rotor height above ground, g is acceleration due to gravity and T is the applied thrust.

We model the basic motor electrodynamics with a simple linear system:

$$V = \omega\lambda + iR \tag{2}$$

$$\mathrm{I}_r\dot{\omega} = \lambda i - Q \tag{3}$$

where V is the battery voltage, ω is the rotor angular velocity, λ is the motor flux-linkage coefficient, i is the drawn electrical current, R is the combined drive circuit resistance (motor plus battery), I_r is the rotor rotational inertia and Q is the rotor drag torque. Lithium polymer cells can exhibit impulse responses with peak and relaxation behaviour [Pounds *et al*, 2009]; it is expected that fast voltage regulation will eliminate this dynamic.

We use a BEM (Blade Element Method) model for free-air thrust and drag, with non-constant inflow velocity [Prouty, 1995], using airfoil lift-slope and variable inflow angle in place of a non-dimensionalised thrust co-

efficient:

$$T_{FA} = \frac{a\sigma}{4}\left(\theta - \frac{v_i}{\omega r} - \frac{\dot{z}}{\omega r}\right)\rho A r^2 \omega^2 \quad (4)$$

$$Q = \frac{(a\sigma)^{3/2} + 2b\sigma}{16}\left(\theta - \frac{v_i}{\omega r} - \frac{\dot{z}}{\omega r}\right)^{3/2}\rho A r^3 \omega^2 \quad (5)$$

where T_{FA} is the equivalent free-air thrust, a and b are the rotor airfoil lift and drag slopes, σ is the rotor disc solidity ratio, θ is the fixed geometric rotor tip angle, v_i is the induced velocity, r is the rotor radius, ρ is the density of air and A is the rotor disc area. The term $\theta - v_i/\omega r - \dot{z}/\omega r$ is collectively the variable airfoil angle of attack; vertical velocity term $\dot{z}/\omega$ gives rise to passive inflow damping.

We use the Cheeseman and Bennett model for scaling the thrust generated in ground effect [Cheeseman and Bennett, 1955; Leishman, 2002]:

$$T_{GE} = \frac{1}{1 - (\frac{Br}{4z})^2} T_{FA} \quad (6)$$

where T_{GE} is the thrust generated in proximity to the ground, r is the rotor radius and z is rotor height above ground, and B is a rotor radius scaling parameter — typically taken as $B = 1$ for helicopters.

The inflow velocity is problematic — is difficult to measure during flight, and depends on the generated thrust, ground effect, lateral flight motion and ground surface geometry. Carpenter and Fridovich associate a temporal delay with the air's response, which they call 'apparent mass' [Carpenter and Fridovitch, 1953], but we consider this negligible on the scales of micro drones and do not treat it as a dynamic state. Bangura and Mahony solve for induced velocity through induced aerodynamic power [Bangura and Mahony, 2012]. We instead use the induced velocity-thrust relationship to facilitate the incorporation of ground effect [Prouty, 1995, p4]:

$$v_i = \sqrt{\frac{T}{2\rho A}} \quad (7)$$

Note that the inflow velocity depends in the thrust generated, which itself is effected by the magnitude of rotor inflow.

Solving the complete electromechanical non-linear dynamical system explicitly is challenging, and simulating it requires resolving the recursive inflow velocity calculation.

Leishman notes that ground effect can be assumed to influence induced power only [Leishman, 2002, p260]. The free air thrust value T_{FA} can therefore be used for computing the inflow angle correction in rotor drag, and the drag mechanic does not inform the calculation for v_i.

Table 1: Micro Quadrotor Simulation Parameters

I_r	Rotor inertia	0.001	kgm^2
m	Quarter mass	0.2/4	kg
θ	Blade tip angle	$6.8\frac{pi}{180}$	rad
σ	Rotor solidity ratio	0.1273	
r	Rotor radius	0.05	m
A	Rotor area	0.0157	m^2
a	Airfoil lift slope	5.5	rad^{-1}
b	Airfoil drag slope	0.1215	rad^{-1}
$lambda$	Flux linkage coefficient	0.0031	NmA^{-1}
R	Drive circuit resistance	0.02	Ω

We link the BEM and momentum theory thrust inflow relation to get a closed-form description for T_FA:

$$T_{FA} = \frac{1}{4}\left(\frac{K\omega r}{\sqrt{2\rho A}} - \left(\frac{(K\omega r)^2}{2\rho A} + 4K(\theta\omega r - \dot{z})\omega r\right)^{\frac{1}{2}}\right)^2 \quad (8)$$

where $K = \frac{a\sigma}{4}\rho A$ is a constant coefficient. This then yields a straight-forward expression for v_i from (7) to apply to (5):

$$v_i = \frac{\left((a\sigma\omega r)^2 - 32a\sigma\omega r(\dot{z} - \theta\omega r)\right)^{1/2} - a\sigma\omega r}{16} \quad (9)$$

3.2 Vertical Stability

We implemented the system in Matlab Simulink and simulated a quadrotor with constant voltage flying close to the ground (see figure 4). The quadrotor has the parameters of the regulated micro quadrotor in Section 2, as given in Table 1. The simulation initial conditions are $z_0 = 0.5$, $\dot{z}_0 = 0$, $\omega_0 = 1150$, with $V = 7.4$ at 49 per cent throttle setting.

It can be seen that the vertical velocity decays to zero and the height above ground converges to a constant value. The rotor speed and current draw are approximately constant. Small variation in inflow velocity and thrust caused by ground effect can be seen. When the quadrotor moves close to the ground, the inflow velocity decreases and thrust increases significantly. This shows that supplying a constant voltage to the quadrotor will allow passive height stability to be achieved. It is useful to note that, for small motions around the equilibrium hover height in ground effect, the behaviour of the system is approximately linear.

We replicated the simulation in experiment with the test platform and found similar large-scale behaviour (see Fig. 5), including the asymmetrical sharp 'bounce' as the aircraft approaches the ground. The physical experiment exhibited a slower oscillation period than the simulated result (2.3 s compared to 0.6 s), possibly due to parameter estimation error or unmodelled damping (eg. airframe drag).

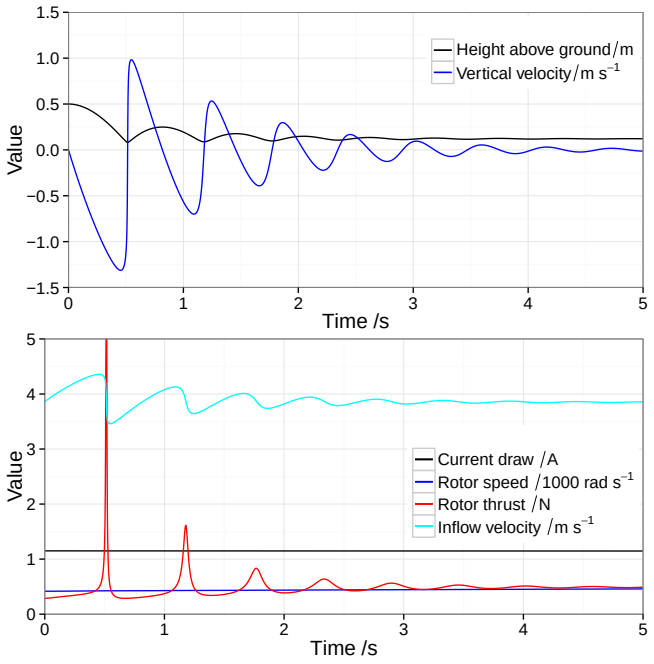

Figure 4: Simulation results for a rotor in ground effect with a constant voltage

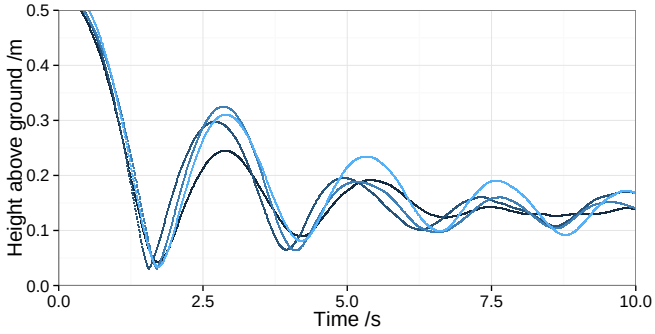

Figure 5: Experimental results for quadrotor in ground effect with a constant voltage

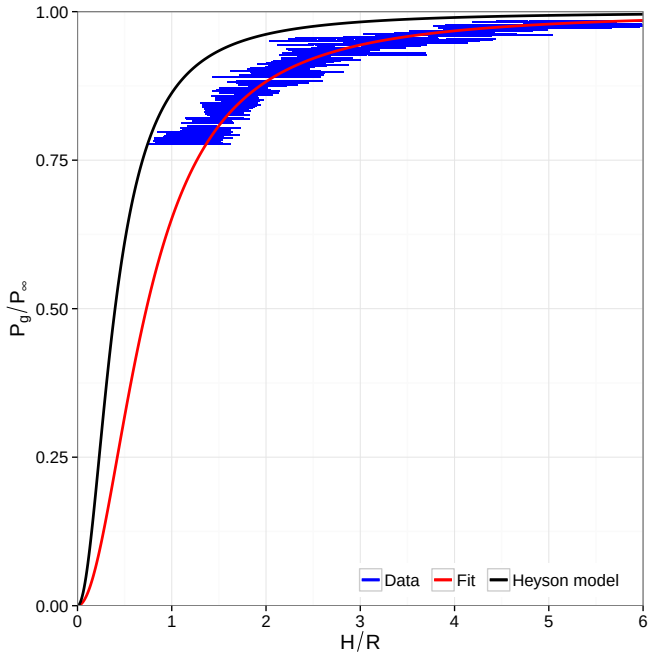

Figure 6: Experimental results for normalised hover power vs height with model

3.3 Ground Effect Model Comparison

The regulated drive voltage of the micro quadrotor testbed provides a useful platform for calibrating the aerodynamic ground effect performance of the aircraft. This has previously been done for full-size helicopters [Cheeseman and Bennett, 1955; Heyson, 1977], but not for quadrotors [Bangura and Mahony, 2012].

The regulated quadrotor was commanded to maintain lateral position over flat ground with fixed throttle, and allowed to come to equilibrium hover altitude over 30 seconds. In each experiment, the throttle was decreased from 44.8 per cent to 41.6 per cent in steps of 0.2 per cent. Total power to the motors was measured along with vertical position of the quadrotor. From this, we calculated the power ratio required to hover, compared to free air, as a function of normalised hover height relative to rotor radius (see figure 6).

The results follow the general trend for a helicopter presented by Heyson [Heyson, 1977]. However, it is clear that ground effect has a greater effect on the quadrotor relative to its rotor size. We believe this is due to the rotor wake interaction from multiple rotors in proximity, entraining airflow in the central volume of the vehicle. Adjusting for fit, we find that a correction factor of $B = 2.5$ — reflecting the equivalent distance from the outside rotor tip to the centre of the quadrotor times the rotor radius — gives a compelling fit. We posit that this is a more appropriate dimension for estimating ground effect for multirotors.

Futhermore, ground effect is commonly held to be negligible for helicopters beyond 2 to 3 rotor radii from the ground. However, we found that with a quadrotor the ground effect zone extends up 5 to 6 rotor radii, a result which has also been observed by Sharf *et al* [Sharf *et al*, 2014]. We believe that this is due to the concentrated stream tubes of small rotors.

4 Dynamic Repeatability with Changing Bus Voltage

Aerodynamic effects, such as ground effect, can be subtle and are especially prone to disturbances. Consistent repetitions of each experiment are required to form a good understanding of theses dynamic effects. A high gain controller is not a suitable solution to obtain consistency as the measurable impact of any aerodynamic effects will be masked by corrective action of the controller. Instead we investigated the dynamic repeatability offered by regulating supply voltage.

To test the quadrotor trajectory tracking consistency and height response over an uneven surface, an experimental arena was setup with a small ramp (see figure 7). The ramp is 1 by 1.6 m with a peak of 0.12 m. The quadrotor was commanded to follow the same pre-

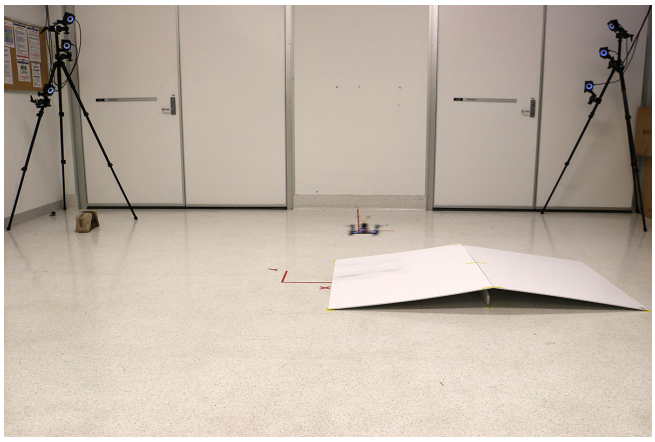

Figure 7: Experimental arena with uneven ground surface

generated trajectory, for multiple laps, until the battery was exhausted or the quadrotor collided with the ramp. Four experiment configurations were tested, they are outlined in Table 2. The z and y axis performance during these experiments is shown in figures 8 and 9 respectively.

Robot position and orientation was tracked at 120 Hz by an Optitrack Prime13 motion capture system. A PID position controller was run on the ground computer to generate desired angle control signals. The control signals were transmitted to the quadrotor using an Spektrum DX10t transmitter and DSMX satellite receiver at a rate of 45 Hz.

The results for the z axis follow expected trends. The addition of a regulator improves feedback height control flight consistency (see figures 8(a) and 8(c)). Initially it appears that the addition of a regulator in the constant throttle configuration decreases performance (see figures 8(b) and 8(d)). However, the unregulated experiment had a significantly shorter duration, 25 vs 40 laps. It started out flying laps at the top of the height range, touching the ground at several points in the final laps and eventually impacting the ramp. Conversely, the regulated configuration maintained flight without contacting the ground or slope for the duration of the experiment.

From the y axis, with height control, it can be seen that the regulated voltage quadrotor offers greater consistency than the unregulated configuration, as shown by the smaller standard deviation and range (see figures 9(a) and 9(c)). This was expected as the regulator should lead to uniform motor response. Constant throttle gave an unexpected result: in the case of the regulated quadrotor it lead to greater lateral consistency than all other configurations (see figure 9(b)). Despite the large variation observed in the z axis.

This improvement in lateral consistency observed is thought to be due a coupling between the height, position and on-board controllers. By removing the height controller, the lateral position controller is able to achieve greater repeatability as the motors are only changing speed in relation to the attitude commands without reducing throttle headroom to affect height control (see figure 10). This allows the motors to respond more quickly as the changes in speed required will be smaller. The regulator buffers the power bus, allowing the voltage sag due to instantaneous current demand to be resisted.

Table 2: Configurations for path following experiments

Exp.	Supply Voltage	Throttle Control	Laps
(a)	regulated	height feedback	41
(b)	regulated	fixed value (44.4%)	40
(c)	unregulated	height feedback	46
(d)	unregulated	fixed value (46.8%)	25

Motor voltage can be expressed as a proportion of supply voltage:

$$V = V_S[h_0 + f(x, y, z)] \quad (10)$$

where, V is motor voltage, V_S is the supply voltage, h_0 is voltage fraction to hover and $f(x, y, z)$ is the control system from position to motor command.

With a regulated supply the motor voltage will be independent of battery voltage (V_B), since $V_S = C$. However, with an unregulated supply the motor voltage is now dependant on battery voltage, since $V_S = V_B$. This means that for the same vehicle position a decreased battery voltage will lead to a decreased motor command, since h_0 and $f(x, y, z)$ are independent of battery voltage. As a result motor response and vehicle reaction will differ depending on battery charge.

5 Conclusion and Future Work

In this paper, we described a quadrotor with full regulation of drive system power, and used it to test ground effect performance and dynamic repeatability of regulated micro quadrotors as building blocks for passively stable hovering platforms. We presented a model of vertical rotor dynamics that does not make inflow velocity assumptions, and showed that it is stable in ground effect via simulation.

Of particular note, we demonstrated that the conventional helicopter ground effect model is not a perfect fit for quadrotors. We propose that instead of using rotor radius for a quadrotor, instead the distance from the outside rotor tip to the vehicle centre should be used. Furthermore, the effective height above ground at which ground effect was prevalent was found to be higher for the quadrotor than the Cheeseman and Bennet model would suggest.

We found that voltage regulation of the quadrotor drive system improved the consistency of trajectory paths, compared to unregulated flight, for both controlled and uncontrolled vertical dynamics. Interestingly, it was found that the best tracking regularity was achieved for the case where active height control was disabled. We believe this is due to the improved dynamic control headroom available without the constant corrections for altitude.

5.1 Future Work

The results in this paper are building blocks for developing platforms that can hover and avoid obstacles without need for active sensing. The constant voltage flight properties of the ground effect quadrotor will be put to use in exploring passive interactions with walls and surfaces. In particular, the ease of obtaining useful ground effect performance data that differs from common aerodynamic theory makes a detailed exploration of the phenomenon a natural extension to this work.

While the simulation results of the vertical dynamics indicate good stability characteristics, we would like to have analytical bounds on the conditions under which stability is guaranteed. In future work we will explore Lyapunov stability for this system.

The unanticipated improvement in performance for the regulated platform without height control raises interesting questions about the tradeoffs being made in attempting to slavishly follow prescribed trajectories, rather than accepting consistent performance for less strictly prescribed paths. This may have ramifications for developing more robust flight control schemes for maneuvering through complex environments.

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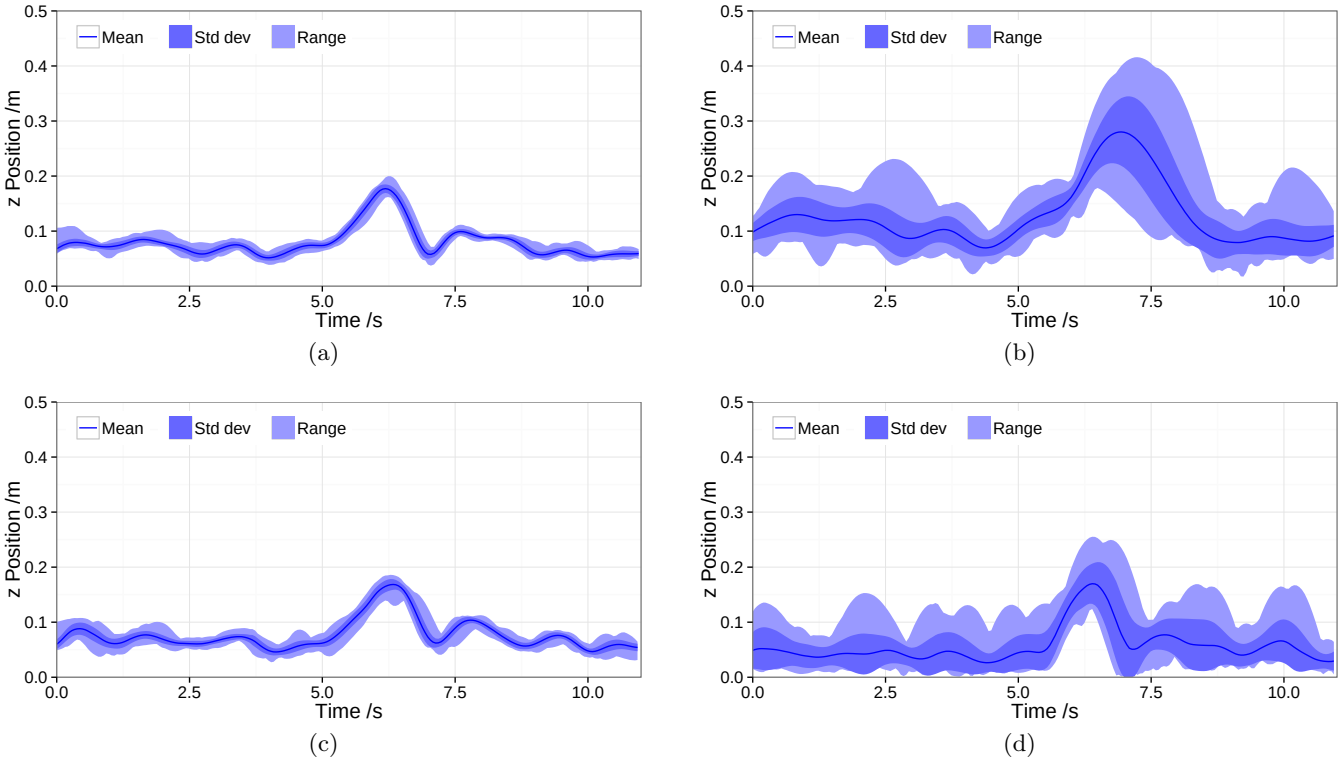

Figure 8: z axis path following consistency, configurations a–d as outlined in Table 2

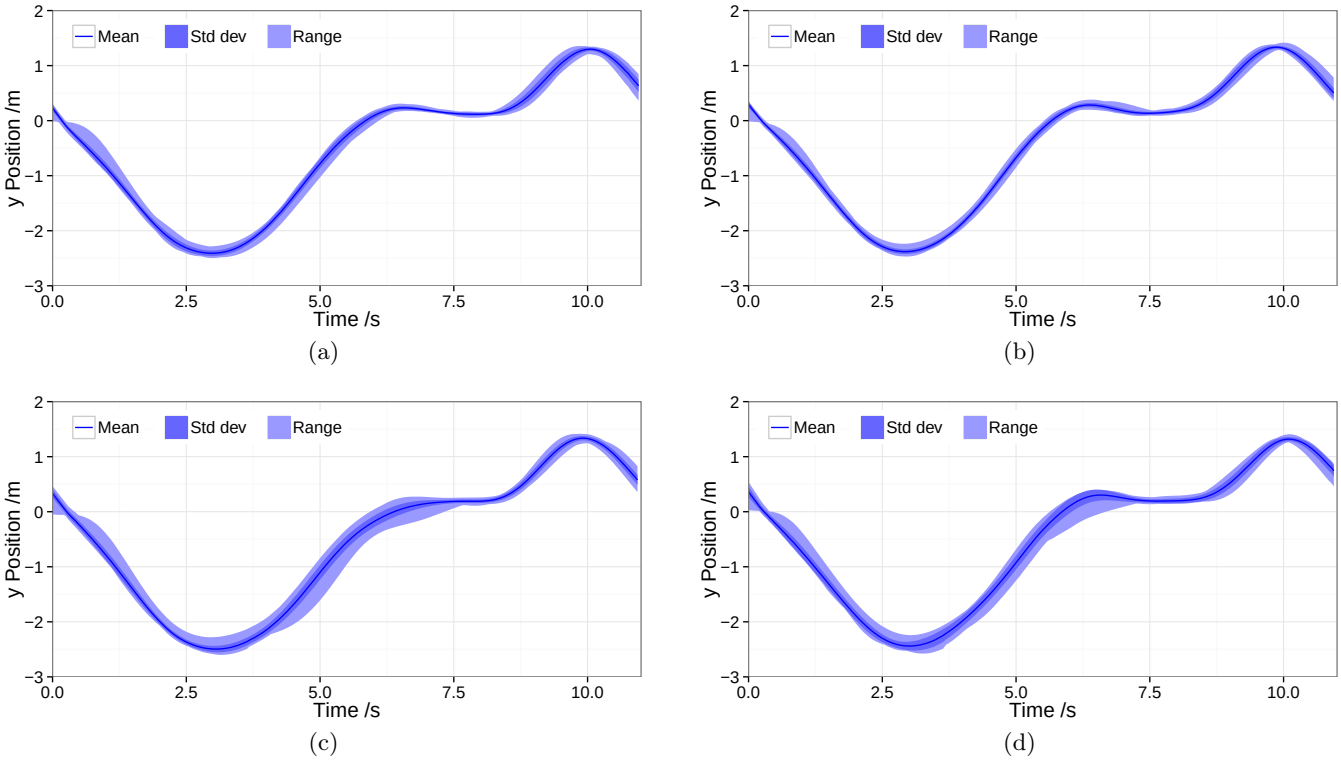

Figure 9: y axis path following consistency, configurations a–d as outlined in Table 2

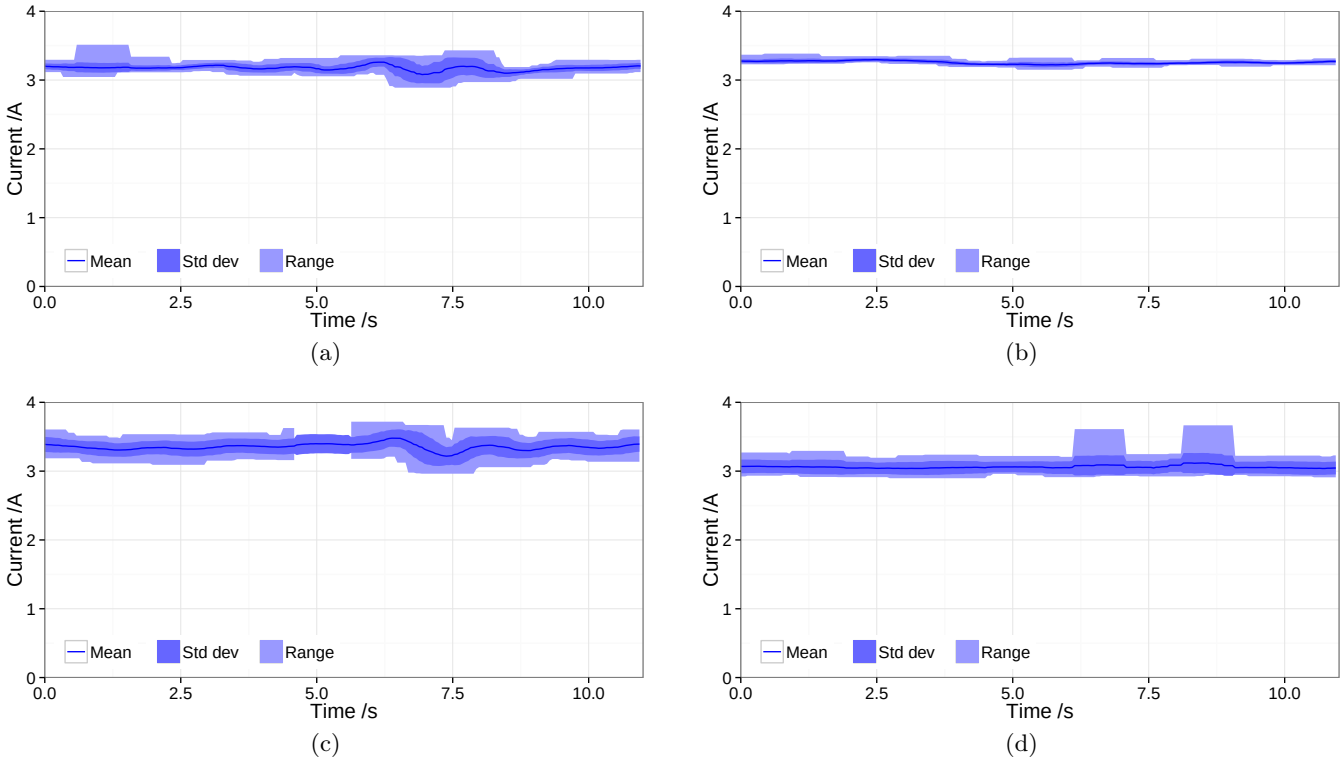

Figure 10: Supply current during path following, configurations a–d as outlined in Table 2