Distributed Formation Building with Obstacle Avoidance for a Team of Wheeled Mobile Robots

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Abstract

The problem of formation building with obstacle avoidance for a team of mobile robots is considered. The algorithm of global formation building is combined with a local obstacle avoidance algorithm. We propose a distributed motion coordination control algorithm so that the robots collectively move in a desired geometric pattern from any initial position while avoiding the obstacles on their routes. We use standard kinematic equations for the robots with hard constraints on their linear and angular velocities. Furthermore, there is no leader in the team and each robot applies the distributed control algorithm using the consensus variables rule based on the local information. Moreover, an obstacle avoidance technique based on the information from the range sensors is used. A mathematically rigorous proof of the proposed control algorithm is given and the effectiveness of algorithm is illustrated via computer simulations.

1 Introduction

The research about distribution control of a team of mobile robots has become a new challenging area for researchers in recent years; see e.g [Sabattini *et al.*, 2013], [Harmati and Skrzypczyk, 2009], [Farrokhsiar and Najjaran, 2012], [Turpin *et al.*, 2012] and references therein. The distinguished difference between general distributed control approach and the approaches that are used for a team of mobile robots is that in the second case, there is no dynamic coupling among the robots, meaning that the robots do not directly affect each other. Based on the distributed control approach, each robot uses the information provided by its nearest neighbouring robots in order to update its linear and angular velocities at discrete time instants. There are a number of articles on this topic that using their purposed distributed control algorithm, the robots will eventually move with the same heading and speed; see e.g. [Hong *et al.*, 2006], [Yu and Wang, 2008], [Liu and Jiang, 2013].

A more challenging problem is to apply a distributed control algorithm to force the robots to move so that they finally build a desired geometric pattern. Furthermore, formation building in the existence of obstacles is even a more difficult problem. In this paper, we consider the problem of distributed control of a team of autonomous mobile robots in which the robots finally move with the same direction and speed in a desired geometric pattern while avoiding the obstacles.

Many of the articles presented in this area consider a simple linear model for the motion of the robots without constraints on the control inputs, see [Dong, 2011], [Kwon and Chwa, 2012], [Guo *et al.*, 2010]. In particular, these simple models do not consider the essential standard constraints on the angular and linear velocities. Indeed, all actual vehicles such as Unmanned ground vehicles (UGVs) and unmanned aerial vehicles (UAVs) have standard hard constraints on their angular and linear velocities, [Low *et al.*, 2007]. For example, with no constraints on the angular velocity, it may become a large value which results a small turning radius for the robots that is impossible to obtain with actual robots. Therefore, the linear system approaches considered without constraints on the control inputs are not applicable. In [Savkin and Teimoori, 2010b], an algorithm of flocking for a group of wheeled robots described by the unicycle model with hard constraints on angular and linear velocities was proposed, however, the much more difficult problem of formation building was not considered. We consider a more difficult problem of formation building where a nonlinear model with hard constraints on the angular and linear velocities describing the robots.

Communication between the robots is another issue that was considered in many of the papers in this

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area. However, many such publications consider leader-follower in the team therefore, they must consider quite restrictive classes of robot communication graphs; see e.g [Consolini *et al.*, 2012], [Defoort *et al.*, 2008]. In some other papers, robots communication graph is assumed to be minimally rigid [Krick *et al.*, 2009], [Wang and Tian, 2012] or time-invariant and connected [Mehrjerdi *et al.*, 2011] which is also quite restrictive.

The existence of obstacles in the environment is the other problem has been considered in many recent researches on mobile robots control methods; see e.g. [Hoy *et al.*, 2015], [Teimoori and Savkin, 2010a], [Matveev *et al.*, 2011], [Matveev *et al.*, 2012], [Savkin and Wang, 2014], [Savkin and Wang, 2013] and the references therein. The problem of existence of obstacles in the environment is not considered in most aforementioned references for formation building methods [Savkin *et al.*, 2013]. However, there are some papers that assume obstacles in the environment and include some obstacle avoidance methods in their proposed algorithms; see e.g. [Liang and Lee, 2006], [De La Cruz and Carelli, 2008] , [Rezaee and Abdollahi, 2014] and the references therein. In [Liang and Lee, 2006], it was assumed that the shape of the obstacle is convex and known to the robots. Obstacle avoidance strategy in [De La Cruz and Carelli, 2008] is based on the concept of impedance with fictitious forces. The technique for obstacle avoidance of mobile robots in [Rezaee and Abdollahi, 2014] is based on a rotational potential field. In this paper, we use a new obstacle avoidance method by which the robots maintain a given distance to the obstacles.

We propose a distributed motion coordination control algorithm so that the robots collectively move in a desired geometric pattern from any initial position while avoiding the obstacles on their routes. In the proposed method, the robots have no information on the shape and position of the obstacles and only use range sensors to obtain the information. We use standard kinematic equations for the robots with hard constraints on the linear and angular velocities. There is no leaders in the team and the robots apply a distributed control algorithm based on the local information they obtain from their nearest neighbours. We take the advantage of using the consensus variables approach that is a known rule in multi-agent systems. Also an obstacle avoidance technique based on the information from the range sensors is used. We consider quite general class of robot communication graphs which are not assumed to be time-invariant or always connected. A mathematically rigorous proof of the proposed control algorithm is given also the effectiveness of the algorithm is illustrated via computer simulations.

The reminder of the paper is organized as follows. Section 2 describes the networked multi-robot system under consideration. Section 3 presents our formation building algorithm and its mathematical analysis. In Section 4 the obstacle avoidance is described. Computer simulation results of the proposed algorithm are given in Section 5. Finally, brief conclusions are given in Section 6.

2 Multi-Robot System

We consider a system consisting of n autonomous mobile robots labeled 1 through n moving in a plane. The kinematic equations of motion for robots are given by

$$\begin{aligned} \dot{x}_i(t) =&\ v_i(t)\cos(\theta_i(t)) \\ \dot{y}_i(t) =&\ v_i(t) sin(\theta_i(t)) \\ \dot{\theta}_i(t) =&\ \omega_i(t) \end{aligned} \tag{1}$$

for all $i = 1, 2, \ldots, n$, where $(x_i(t),\ y_i(t))$ are the Cartesian coordinates of the robot i at time t and $\theta_i(t)$ is its orientation with respect to the x-axis measured in the counterclockwise direction. Also, $v_i(t)$, the speed of the robot, and $\omega_i(t)$, its angular velocity, are the control inputs. Furthermore, we need the following practical constraints:

$$-\omega^{\max} \le \omega_i(t) \le \omega^{\max} \qquad \forall t \ge 0 \tag{2}$$

$$V^m \le v_i(t) \le V^M \qquad \forall t \ge 0 \tag{3}$$

for all $i = 1, 2, \ldots, n$. Here $\omega^{\max} > 0$ and $0 < V^m < V^M$ are given constants.

Moreover, let $z_i(t)$ be the vector of the robots' coordinates and $V_i(t)$ as the robots' velocity vector defined by

$$z_i(t) := \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix},\ V_i(t) := \begin{pmatrix} v_i(t)\cos(\theta_i(t)) \\ v_i(t)\sin(\theta_i(t)) \end{pmatrix} \tag{4}$$

for all $i = 1, 2, \ldots, n$.

We assume that the robots share their information via a wireless communication at discrete time instants $k = 0, 1, 2, \ldots$. Due to limited communication range of the robots, we assume r_c as the communication range for all the mobile robots, meaning that a robot can only receive information from the robots which are located not farther than r_c.

Definition 2.1. Robot j is the neighbour of robot i at time k if and only if it is located on the disk of radius r_c with the center of robot i's position. Also let $\mathcal{N}_i(k)$ be the set of all neighbours of the robot i at time k and $|\mathcal{N}_i(k)|$ be the number of elements in $\mathcal{N}_i(k)$.

The relationship among the robots can be defined by an undirected graph $\mathcal{G}(k)$. We assume that any robot of the multi-robot team is a node of the graph $\mathcal{G}(k)$ at time

k, i.e., i in $V_{\mathcal{G}} = \{1, 2, \dots, n\}$, the node set of $\mathcal{G}(k)$, is related to robot i. In addition, robot i is the neighbour of robot j at time k if and only if there is an edge between the nodes i and j of graph $\mathcal{G}(k)$ where $i \neq j$. Therefore, the problem of communication among the team of robots equals the problem of the connectivity of the related graph. Note that, robot i doesn't need to be the neighbour of robot j to get the information from. The information can be transferred through the other robots which connect these robots in the related graph. We will also need the following assumption.

Assumption 2.1. There exists an infinite sequence of contiguous, non-empty, bounded time-intervals $[k_j, k_{j+1})$, $j = 0, 1, 2, \dots$, starting at $k_0 = 0$, such that across each $[k_j,\ k_{j+1})$, the union of the collection $\{\mathcal{G}(k) : k \in [k_j,\ k_{j+1})\}$ is a connected graph.

To achieve the common heading and speed of formation, we use the consensus variables $\tilde{\theta}_i(k)$ and $\tilde{v}_i(k)$, respectively. Also, we need a common origin of coordinates of the formation for the multi-robot system, therefore $\tilde{x}_i(k)$ and $\tilde{y}_i(k)$ are used as the consensus variables for coordinates of the robots. In the other words, the robots start with different initial values of consensus variables $\tilde{x}_i(0), \tilde{y}_i(0), \tilde{\theta}_i(0)$ and $\tilde{v}_i(0)$, and each robot calculates these consensus variables at any time k such that eventually the consensus variables converge to some consensus values which define a common speed and orientation in a common coordinate system.

Assumption 2.2. The initial values of the consensus variables $\tilde{\theta}_i$ satisfy $\tilde{\theta}_i(0) \in [0,\ \pi)$ for all $i = 1, 2, \dots, n$.

Assumption 2.3. The information on other robots that is available to the robot i at time k is the coordinates $(x_j(k),\ y_j(k))$ and the consensus variables $\tilde{\theta}_j(k), \tilde{x}_j(k), \tilde{y}_j(k)$ and $\tilde{v}_j(k)$ for all $j \in \mathcal{N}_i(k)$.

In practice, the coordinates of neighbouring robots can be obtained using Kalman state estimation via limited capacity communication channels [Malyavej and Savkin, 2005].

3 Formation Building

We propose the following rules for updating the consensus variables $\tilde{\theta}_i(k), \tilde{x}_i(k), \tilde{y}_i(k)$ and $\tilde{v}_i(k)$:

$$
\begin{aligned}
\tilde{\theta}_i(k+1) &= \frac{\tilde{\theta}_i(k) + \sum_{j \in \mathcal{N}_i(k)} \tilde{\theta}_j(k)}{1 + |\mathcal{N}_i(k)|} \\
\tilde{x}_i(k+1) &= \frac{x_i(k) + \tilde{x}_i(k) + \sum_{j \in \mathcal{N}_i(k)} (x_j(k) + \tilde{x}_j(k))}{1 + |\mathcal{N}_i(k)|} - x_i(k+1) \\
\tilde{y}_i(k+1) &= \frac{y_i(k) + \tilde{y}_i(k) + \sum_{j \in \mathcal{N}_i(k)} (y_j(k) + \tilde{y}_j(k))}{1 + |\mathcal{N}_i(k)|} - y_i(k+1) \\
\tilde{v}_i(k+1) &= \frac{\tilde{v}_i(k) + \sum_{j \in \mathcal{N}_i(k)} \tilde{v}_j(k)}{1 + |\mathcal{N}_i(k)|}
\end{aligned}
\tag{5}
$$

Based on rule (5), the mobile robots use the consensus variables to achieve a consensus on the heading, speed and origin of coordinate system of the formation.

Lemma 3.1. Suppose that Assumptions 2.1 and 2.2 hold and the consensus variables are updated according to the decentralized control rule (5). Then there exist constants $\tilde{\theta}_0, \tilde{X}_0, \tilde{Y}_0$ and $\tilde{v}_0$ such that

$$
\begin{aligned}
\lim_{k \to \infty} \tilde{\theta}_i(k) &= \tilde{\theta}_0 \\
\lim_{k \to \infty} \tilde{v}_i(k) &= \tilde{v}_0 \\
\lim_{k \to \infty} (x_i(k) + \tilde{x}_i(k)) &= \tilde{X}_0 \\
\lim_{k \to \infty} (y_i(k) + \tilde{y}_i(k)) &= \tilde{Y}_0
\end{aligned}
\tag{6}
$$

for all $i = 1, 2, \dots, n$. Furthermore, the convergence in (6) is exponentially fast.

The statement of Lemma 3.1 immediately follows from the main result of [Jadbabaie *et al.*, 2003]. Note that the constants $\tilde{\theta}_0, \tilde{X}_0, \tilde{Y}_0$ and $\tilde{v}_0$ are the same for all robots.

Definition 3.1. A navigation law is said to be globally stabilizing with initial conditions $(x_i(0),\ y_i(0),\ \theta_i(0))$, $i = 1, 2, \dots, n$ and the given values of configuration $\mathcal{C} = \{X_1,\ X_2,\ \dots,\ X_n,\ Y_1,\ Y_2,\ \dots,\ Y_n\}$, if there exists a Cartesian coordinate system and $\tilde{v}_0$ such that the solution of the closed-loop system (1) with these initial conditions and the proposed navigation law in this Cartesian coordinate system satisfies:

$$
\begin{aligned}
\lim_{t \to \infty} (x_i(t) - x_j(t)) &= X_i - X_j \\
\lim_{t \to \infty} (y_i(t) - y_j(t)) &= Y_i - Y_j
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\lim_{t \to \infty} \theta_i(t) &= 0 \\
\lim_{t \to \infty} v_i(t) &= \tilde{v}_0
\end{aligned}
\tag{8}
$$

for all $1 \leq i \neq j \leq n$. where $X_1, X_2, \dots, X_n, Y_1, Y_2, \dots,$ Y_n are given constants.

Rule (7) means that as $t \to \infty$ all the robots will finally move in the same direction along the x-axis with the same speed. Furthermore, rule (8) indicates that a geometric configuration of the robots given by $\mathcal{C}$ will be obtained. For instance, if we have four robots and $\mathcal{C} = \{0, 0, 2, 2, 0, 1, 0, 1\}$, then the geometric formation of the robots will be a rectangle of sides 1 and 2.

Since we use the discrete time consensus variables $\tilde{\theta}_i(k), \tilde{x}_i(k), \tilde{y}_i(k)$ and $\tilde{v}_i(k)$ updated according to (5), we need to define the corresponding piecewise constant continuous time variables as

$$\begin{aligned}\tilde{\theta}_i(t) &:= \tilde{\theta}_i(k)\ \forall t \in (k,\ k+1)\\ \tilde{x}_i(t) &:= \tilde{x}_i(k)\ \forall t \in (k,\ k+1)\\ \tilde{y}_i(t) &:= \tilde{y}_i(k)\ \forall t \in (k,\ k+1)\\ \tilde{v}_i(t) &:= \tilde{v}_i(k)\ \forall t \in (k,\ k+1).\end{aligned} \quad (9)$$

For any time t and any robot i, we consider a Cartesian coordinate system with the x-axis in the direction $\tilde{\theta}_i(t)$ (according to the definition (9), $\tilde{\theta}_i(t)$ is piecewise constant). In other words, in this coordinate system $\tilde{\theta}_i(t) = 0$ and $x_i(t)$, $y_i(t)$ are now coordinates of the robot i in this system. Notice that we now formulate our decentralized control law for each robot in its own coordinate system. Since according to Lemma 3.1, $\tilde{\theta}_i(k)$ converges to the same value for all i, all these robots' coordinate systems converge to the same coordinate system in which (7) holds.

Assumption 3.1. Let $c > 0$ be any constant such that

$$c > \frac{2V^M}{\omega^{\max}}. \quad (10)$$

We assume that the constant c and also the configuration $\mathcal{C}$ are known to all the robots.

Introduce the functions $h_i(t)$ as

$$h(t) := (x_i(t) + \tilde{x}_i(t)) + X_i + t\tilde{v}_i(t) \quad (11)$$

for all $i = 1,2,\ldots,n$. Also introduce two-dimensional vector $g_i(t)$ as

$$g_i(t) := \begin{pmatrix} g_i^x(t) \\ g_i^y(t) \end{pmatrix} \quad (12)$$

where

$$\begin{aligned} g_i^x(t) &:= \begin{cases} h_i(t) + c \text{ if } x_i(t) \le h_i(t) \\ x_i(t) + c \text{ if } x_i(t) > h_i(t) \end{cases} \\ g_i^y(t) &:= (y_i(t) + \tilde{y}_i(t)) + Y_i \end{aligned} \quad (13)$$

and two-dimensional vector $d_i(t)$ as

$$d_i(t) := g_i(t) - z_i(t) \quad (14)$$

for all $i = 1,2,\ldots,n$, where $z_i(t)$ is defined by (4).

Now we introduce the following decentralized control law:

$$\begin{aligned} v_i(t) &= \begin{cases} V^M & \text{if } x_i(t) \le h_i(t) \\ V^m & \text{if } x_i(t) > h_i(t) \end{cases} \\ \omega_i(t) &= \omega^{max} sign(\psi_i(t)) \end{aligned} \quad (15)$$

for all $i = 1, 2, \ldots, n$ where $\psi_i(t)$ is the angle between $V_i(t)$ and $d_i(t)$ measured from $V_i(t)$ in the counter-clockwise direction, i.e.,

$$\psi_i(t) = \angle(V_i(t), d_i(t)) \quad (16)$$

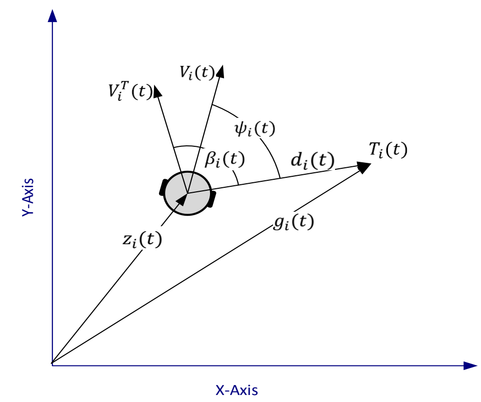


Figure 1: Vectors geometry

(see Fig.1) and $sign(\cdot)$ is defined by

$$sign(\alpha) := \begin{cases} -1 & \text{if } \alpha < 0 \\ 0 & \text{if } \alpha = 0 \\ 1 & if\ \alpha > 0 \end{cases} \quad (17)$$

We also need the following assumption.

Assumption 3.2. The initial robots' speeds satisfy

$$V^m < v_i(0) < V^M$$

for all $i = 1, 2, \ldots, n$.

Notice that Assumption 3.2 is just slightly stronger than the requirement (3) for $t = 0$ where non-strict inequalities are required.

The proposed algorithm is based on robots' headings and coordinates which, of course, depend on initial conditions. Therefore, the proposed law depends on initial conditions on robots' headings and coordinates. The connectivity of the multi-robot formation is maintained due to Assumption 2.1 which is a standard assumption in numerous papers on multi- agent systems; see e.g. [Jadbabaie *et al.*, 2003], [Savkin and Teimoori, 2010b] and references therein.

Now we are in a position to present the main result of this section.

Theorem 3.1. Consider the autonomous robots described by the equations (1) and the constraints (2), (3). Let $C = \{X_1,\ X_2,\ \ldots,\ X_n,\ Y_1,\ Y_2,\ \ldots,\ Y_n\}$ be a given configuration. Suppose that Assumptions 2.1, 2.2, 3.1 and 3.2 hold. Then, the decentralized control law (5), (15) is globally stabilizing with any initial conditions and the configuration C.

Proof of Theorem 3.1: Let $1 \le i \le n$. We consider a fictitious target T_i moving on the plane with coordinates $g_i(t)$ defined by (13). Furthermore, introduce another

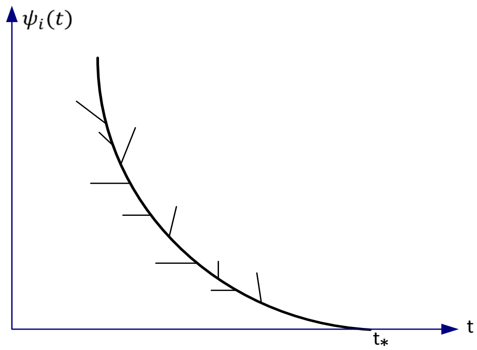


Figure 2: Sliding mode solution

fictitious target $\tilde{T}_i$ moving on the plane with coordinates $\tilde{g}_i(t)$ defined by

$$\tilde{g}_i(t) \ := \begin{pmatrix} \tilde{g}_i^x(t) \\ \tilde{g}_i^y(t) \end{pmatrix} \tag{18}$$

where

$$\tilde{g}_i^x(t) := \begin{cases} X_0 + X_i + t\tilde{v}_0 + c & \text{if } x_i(t) \leq \tilde{X}_0 + X_i + t\tilde{v}_0 \\ x_i(t) + c & \text{if } \ x_i(t) > \tilde{X}_0 + X_i + t\tilde{v}_0 \end{cases}$$
$$\tilde{g}_i^y(t) := \tilde{Y}_0 + Y_i \tag{19}$$

It immediately follows from Lemma 3.1, that

$$\lim_{t\to\infty} (\tilde{g}_i(t) - g_i(t)) = 0 \tag{20}$$

and this convergence is exponentially fast. Let $\psi_i(t)$ be the angle between the velocity vector $V_i(t)$ of the robot i and the line-of-sight between the robot and $\tilde{T}_i$, $\beta_i(t)$ be the angle between the velocity vector $V_i^T(t)$ of $\tilde{T}_i$ and the line-of-sight from the robot i to $\tilde{T}_i$; see Fig.1.

It is well-known (see e.g. [Teimoori and Savkin, 2010b]) that the following equation holds:

$$\dot{\psi}_i(t) = \frac{\|V_i(t)\| \sin \psi_i(t)}{\|\tilde{d}_i(t)\|} - \omega_i(t) - \frac{\|V_i^T(t)\| \sin \beta_i(t)}{\|\tilde{d}_i(t)\|} \tag{21}$$

where $\tilde{d}_i(t)$ is defined as

$$\tilde{d}_i(t) \ := \tilde{g}_i(t) - z_i(t), \tag{22}$$

$z_i(t)$ is defined by (4), and $\|\cdot\|$ denotes the standard Euclidean vector norm. It obviously follows from (19),(22) that

$$\|\tilde{d}_i(t)\| \geq c \quad \forall t \geq 0. \tag{23}$$

Furthermore, (19) implies that

$$V_i^T(t) := \begin{pmatrix} (V_i^{Ty}(t) \\ V_i^{Tx}(t) \end{pmatrix};$$

$$V_i^{Tx}(t) = \begin{cases} \tilde{v}_0 \text{ if } \ x_i(t) \leq X_0 + X_i + t\tilde{v}_0 \\ v_i(t) \text{ if } \ x_i(t) > X_0 + X_i + t\tilde{v}_0 \end{cases}$$
$$V_i^{Ty}(t) = 0. \tag{24}$$

It follows from (24) and (3) that

$$\|V_i^T(t)\| \leq V^M \tag{25}$$

Now, we consider the control law (15) with d_i replaced by $\tilde{d}_i$. The inequality (25) together with (10), (21) and (23) implies that under this control law, there exists a constant $\epsilon > 0$ such that

$$\begin{aligned} \dot{\psi}_i(t) < -\epsilon \ \ if \ \ \psi_i(t) > 0 \\ \dot{\psi}_i(t) > \epsilon \ \ if \ \ \psi_i(t) < 0. \end{aligned} \tag{26}$$

Therefore, there exists a time $t_* > 0$ such that

$$\psi_i(t) = 0 \quad \forall t \geq t_*. \tag{27}$$

Notice that the closed-loop system (1), (15) is a system of differential equations with discontinuous right-hand sides, the equation $\psi_i = 0$ defines a switching surface of this system, a solution satisfying (27) is a sliding mode (see e.g. [Utkin, 2013]). The inequalities (26) guarantee that this sliding mode solution of the closed-loop system looks as it is shown in Fig.2 and satisfies

$$\dot{\psi}_i(t) = 0 \quad \forall i \ \ \forall t \geq t_*. \tag{28}$$

From this and (21), we obtain that

$$\omega_i(t) = -\frac{\|V_i^T(t)\| \sin \beta_i(t)}{||\tilde{d}_i(t)\|} \tag{29}$$

for all sliding mode solutions. Therefore, for any initial condition, the sliding mode solution is unique and well-defined. Furthermore, (29), (25), (10) and (23) imply that the constraint (2) holds for any sliding mode solution satisfying (27).

Furthermore, the condition (27) means that the velocity vector $V_i(t)$ is parallel to the vector $\tilde{d}_i(t)$ for all $t \geq t_*$. Hence, for all $t \geq t_*$, we have that the the robot's velocity vector is always pointed at $\tilde{g}_i(t)$. Since $\tilde{g}_i^y(t) = \tilde{Y}_0 + Y_i$, we obtain that $y_i(t) \to \tilde{Y}_0 + Y_i$. The second of the conditions (7) immediately follows from this. Furthermore, Assumption 3.2 implies that $V^m \leq \tilde{v}_0 \leq V^M$. This, the fact that the velocity vector $V_i(t)$ is parallel to the vector $\tilde{d}_i(t)$ for all $t \geq t_*$ and the control law (15) with d_i replaced by $\tilde{d}_i$ imply that

$$\tilde{d}_i(t) = \begin{pmatrix} c \\ 0 \end{pmatrix}$$

for all i and all large enough t. The first of the conditions (7) immediately follows from this. We proved the statement of the theorem for the control law (15) with d_i replaced by $\tilde{d}_i$. This and the exponential convergence (20) together with the inequality (23) imply that the same statement holds for the original control law (15). This completes the proof of Theorem 3.1. □

Remark 3.1. It is clear from the proof of Theorem 3.1 that the main idea of the control law (15) can be explained as follows. Each robot i is guided towards a fictitious target T_i that is always located ahead of the desired robot's position relative to its neighbours. The reason why we guide the robot towards a fictitious target but not the desired relative robot's position itself is clear from (29). If we guided the robot towards the desired relative position, we would have $\|d_i(t)\| \to 0$, therefore, $\omega_i(t) \to \infty$ and the constraint (2) would be violated. Notice that our method for guidance towards a fictitious target T_i is a pure pursuit type guidance law (see e.g. [Savkin and Teimoori, 2010a]).

4 Obstacle Avoidance

Navigation of a group of mobile robots for formation in the existence of obstacles is more challenging. The map of the environment, information about the obstacles including their shapes, positions and geometric distribution are not known to the robots a priori. To detect an obstacle, the robots must be equipped with a range sensor like sonar or laser. The robots can detect an obstacle when it lies within their range then they can obtain range and angle to the obstacle. The algorithm of obstacle avoidance uses this information and calculates an appropriate route to avoid collision with the obstacle.

We apply an algorithm for obstacle avoidance that uses angles and distances provided by range sensors. We assume that the range sensors are located on the robot's perimeter, in the forepart with 180° filed of view, i.e., ±90° with respect to robot's heading. Also, we assume that maximum range of range sensors is r_s. As shown in Fig.3, a robot moving toward an obstacle detects the obstacle as soon as the obstacle is placed in the sensing range of the robot. Then the robot changes its route to turn the obstacle preserving a distance to it. Suppose R_t be the turning radius of the robot and d be the distance to the obstacle when the robot's heading is parallel to the obstacle surface. Since

$$R_t^{max} = \frac{V^M}{\omega^{min}}$$

then

$$d_{min} = r_s - R_t^{max}$$

thus we need following assumption.

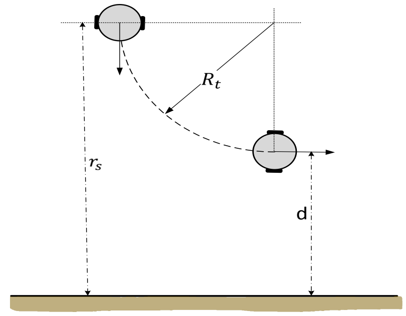


Figure 3: Detecting an obstacle

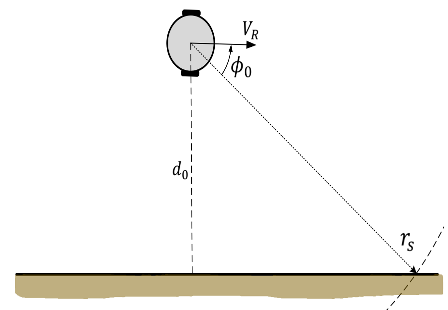


Figure 4: Moving with a constant distance to the obstacle

Assumption 4.1. $d_{min} > d_0$ where d_0 is a given constant.

Assume a robot is moving along the circumference of the obstacle; see Fig.4. Also, suppose that the curvature radius of the obstacle is big enough such that the surface of the obstacle is assumed flat. As shown in Fig. (4), if the robot picks the farthest detectable point on the obstacle surface using its range sensor as a reference point, there exists an angle between the robot's heading and range sensor's ray is termed as avoiding angle. To have a constant distance to the obstacle, we need a constant avoiding angle ϕ_0 satisfying $d_0 = r_s \sin \phi_0$.

Now consider the robot encounters a curved obstacle (see Fig.5). Therefore, the robot must follow a trajectory preserving the given distance of d_0 to the obstacle surface. For instance, as shown in Fig.5, the robot's distance to the obstacle is d_0 but the range sensor detects that the avoiding angle ϕ is greater than ϕ_0 and their difference is $\Delta\phi = |\phi - \phi_0|$. Thus the robot must turn in order to remove this gap; by turning equal to $\Delta\phi$ to the right in this case.

Fig.6 shows the details of obstacle avoidance approach when the obstacle is convex. As shown in Fig.6, the robot is moving along the surface of the obstacle with avoiding distance of d_0. As previously mentioned and shown in Fig.4, there is an angle of ϕ_0 between the robot's heading and range sensor's ray for a flat surface. However, in order to keep moving with the avoid-

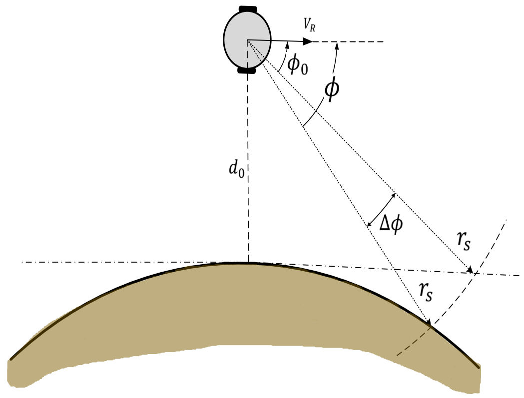


Figure 5: Detecting the curvature of an obstacle

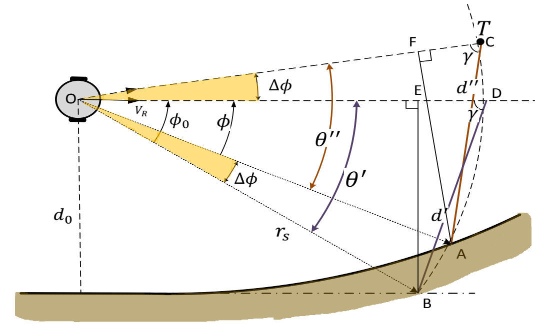


Figure 7: A concave obstacle

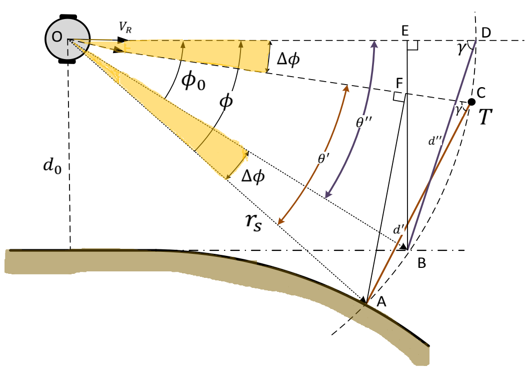


Figure 6: A convex obstacle

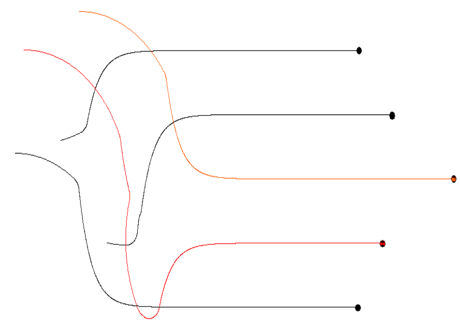


Figure 8: Robots form the desired pattern without any obstacles

ing distance of d_0, the robot must turn by $\Delta\phi$ toward the obstacle. Assume a fictitious target T, a point with distance of r_s to the robot and angle of $\Delta\phi$ respect to the robot's heading toward the obstacle (see Fig.6). As depicted in Fig.6, angles θ' and θ'' are equal thus the line segments d' and d'' will be equal. In addition, since $\widehat{AB} = \widehat{CD}$ thus $\measuredangle ODB = \measuredangle OCD = \gamma$ which satisfies that triangles BED and AFC are equal. Therefore, line segment AF, which is the distance to the obstacle at F, will be equal to BE= d_0. It means that, if point C is selected as the fictitious target, the distance to the obstacle can be maintained to a given constant.

Fig.7 shows the case that the obstacle is concave. This case is similar to the convex case except that the fictitious target is away from the obstacle therefore the robot must turn by $\Delta\phi$ away from the obstacle.

As a result, we propose the following control law that enables robots to avoid collision by calculating a smooth path around the obstacles.

$$\begin{aligned} v_i(t) &= V^M \\ \omega_i(t) &= \omega^{max} sign(\psi_i(t)) \end{aligned} \tag{30}$$

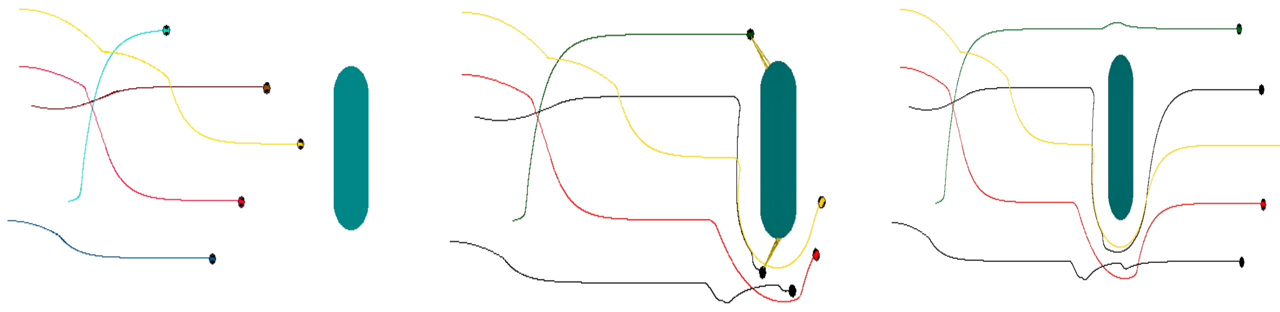
for all $i = 1, 2, \ldots, n$, where

$$\psi_i(t) = \begin{cases} 1 & \text{If } \phi < \phi_0 \\ 0 & \text{If } \phi = \phi_0 \\ -1 & \text{If } \phi > \phi_0 \end{cases} \tag{31}$$

Now we are in a position to present the main result of this paper.

Theorem 4.1. Consider the autonomous mobile robots described by the equations (1) and the constraints (2), (3). Let $\mathcal{C} = \{X_1, X_2, \ldots, X_n, Y_1, Y_2, \ldots, Y_n\}$ be a given configuration. Suppose that Assumptions 2.1, 2.2, 3.1 and 3.2 hold, and c is a constant satisfying (10). Then, the distributed control law (5), (15), (30) is globally stabilizing with any initial conditions and the configuration $\mathcal{C}$.

Proof of Theorem 4.1: proof of Theorem 4.1 is completely similar to the proof of Theorem 3.1. Both control laws, (15) for formation building and (30) for obstacle avoidance are the same. The main difference is that, the fictitious target T in this case is variable between (12) and what is defined in this Section. In other words, whenever a robot encounters an obstacle, the fictitious target switches from (12) to a point with distance of r_s to



(a) (b) (c)

Figure 9: Robots pass the obstacle and build the desired form

the robot and angle of $\Delta\phi$ respect to the robot's heading toward the obstacle (point C in Fig.6 and 7).

5 Simulation Results

In this section, computer simulation results are presented. We consider a team consisting of five robots randomly located on the plane with different headings. The goal is to build a formation as well as avoiding the obstacles that maybe obstruct robots' movement. The robots are to build the edge ('>') by applying the proposed algorithm. First, we assume that there is no any obstacle, therefore only the formation building rule of the proposed algorithm is used. As depicted in Fig.8, the robots build the desired formation ('>').

Second, we assume the same problem but this time with an obstacle. The simulation results of applying the purposed algorithm are displayed in Fig.9. As shown in Fig.9(a), the robots build the desired formation before they encounter the obstacle, and move such that the formation configuration holds. When the robots detect an obstacle on their direction, they avoid the obstacle by turning around. Fig.9(b) shows the snapshot of the this phase. Passing the obstacle, robots restart the formation building phase and as Fig.9(c) shows, the desired formation is built again. Note that as shown in Fig.9, it is not necessary for the robots to pass the obstacle all together and then start the formation building, e.g., while one robot is still in the obstacle avoidance phase, the other robots that have passed the obstacle start the formation building phase again.

To confirm that the proposed algorithm is effective even with any number of obstacles with different shapes and sizes, more simulations are fulfilled. Fig.10 shows the results of these simulations in which more obstacles with different shapes and sizes are used. The results confirm that the proposed algorithm is effective even with any number of obstacles with different shapes and sizes.

It should also be pointed out that the proposed obstacle avoidance algorithm prevents the collision between robots too as a robot considers another robot in its sensing range as an obstacle.

6 Conclusions

The problem of formation building with obstacle avoidance for a team of mobile robots have been considered. The algorithm of global formation building have been combined with a local obstacle avoidance algorithm. We have proposed a distributed motion coordination control algorithm so that the robots collectively move in a desired geometric pattern from any initial position while avoiding the obstacles on their way. We have considered unicycles with standard kinematic equations and hard constraints on the their linear and angular velocities for the type of the robots. A consensus variable rule have been used for the formation building phase that is based on the local information, also a technique based on the information from the range sensors have been used for the the obstacle avoidance phase. A mathematically rigorous proof of the proposed control algorithm has been given and the effectiveness of the algorithm has been confirmed via computer simulations. The future work will be modifying the proposed algorithm so that the formation holds while passing the obstacles.

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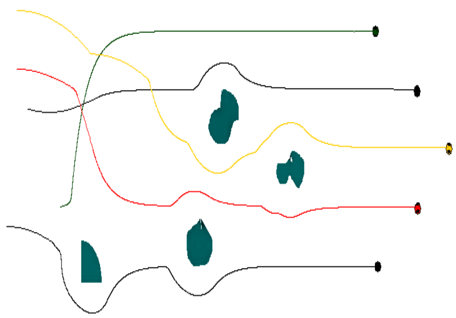
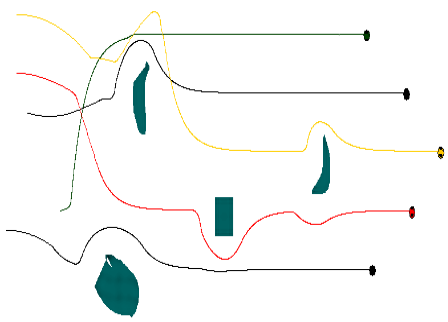


(a)



(b)

Figure 10: Robots form the desired pattern and avoid obstacles

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