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Delayed Optimisation for Robust and Linear Pose-Graph SLAM

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Abstract

This paper addresses a robust and efficient solution to eliminate false loop-closures in a pose-graph linear SLAM problem. Linear SLAM was recently demonstrated based on submap joining techniques in which a nonlinear coordinate transformation was performed separately out of the optimisation loop, resulting in a convex optimisation problem. This however introduces added complexity in dealing with any false loop-closures, which mostly stems from two factors: a) the limited local observations in submap-joining stages and b) the non block-diagonal nature of the information matrix of each submap. To address these problems, we propose a Robust Linear SLAM (RL-SLAM) by 1) developing a delayed optimisation for outlier candidates and 2) utilising a Schur complement to efficiently eliminate corrupted information block. Based on this new strategy, we prove that the spread of outlier information does not compromise the optimisation performance of inliers and can be fully filtered out from the corrupted information matrix. Experimental results based on public synthetic and real-world datasets in 2D and 3D environments show that this robust approach can cope with the incorrect loop-closures robustly and effectively.

1 Introduction

Autonomous navigation system, which is a critical component of autonomous vehicles, is commonly composed of GPS sensors integrated with dead-reckoning sensors, such as inertial measurement units (IMU) or wheel/laser/visual odometry. To overcome the inherent vulnerabilities of this integrated system in GPS-denied environments, map-aided navigation has been extensively investigated and applied for environments such as indoor, the urban canyon and underwater. Thus building accurate maps becomes an essential prerequisite for a large number of robotics applications. Simultaneous Localization and Mapping, well-known as SLAM [Smith and Cheeseman, 1986], was creatively proposed to circumvent the extra loads of map building. SLAM presents that an autonomous robot can incrementally map an unknown environment without any prior information, whilst simultaneously determining its location utilizing the generated map. No infrastructure or any prior knowledge is required, making it quite attractive for robots operating in partially or fully unknown environments [Guivant and Nebot, 2001],[Newman *et al.*, 2002], [Kim and Sukkarieh, 2007], [Ribas *et al.*, 2008].

There exist a large number of SLAM solutions which can be broadly categorised as a) filtering based and/or b) optimisation based approach. For the former, the SLAM problem is formulated in an augmented state space with the current robot pose and observed landmark positions as the state variables. Following the theoretical work on Bayesian filtering, extended Kalman filter (EKF) [Dissanayake *et al.*, 2001], compressed EKF (CEKF) [Guivant and Nebot, 2001], particle filter known as FastSLAM [Montemerlo, 2002] and sparse extended information filter (SEIF) [Thrun *et al.*, 2004] were successfully demonstrated. Due to their incremental nature, these approaches are generally acknowledged as on-line SLAM techniques. On the other hand, the latter, also called as graph-based SLAM, models the SLAM system in a graphical structure in which each robot pose or landmark position is represented as a node, whilest a constraint resulted from either odometry or scan matching is represented as an edge to connect two related nodes. Finding its solution is then equivalent to finding an optimal configuration of nodes by a nonlinear least square optimisation, thus called an off-line or batch processing technique.

Motivated by the progresses on the nonlinear optimisation techniques, and Linear SLAM (L-SLAM) which utilises submap joining approach by separating the nonlinear transformation out of the optimisation process [Zhao *et al.*, 2013], we present a robust implementation of Linear SLAM under outliers. In particular, the main contributions of this paper are:

- We introduce a delayed optimisation strategy for L-SLAM to effectively handle any potential loop-closure outliers in each map-joining stage. This is crucial in utilising map joining techniques as the outlier detection becomes very challenging due to the limited local information available.
- We realise efficient outlier-detection in the local submap joining process by using the Expectation-Maximization (EM) method. At the expectation step, all erroneous loop closures except the delayed constraints are iteratively down-weighted.
- We prove that the outlier information in the corrupted information matrix does not degrade the optimisation performance before the outlier-rejection. In particular, The corrupted information matrix block can be efficiently eliminated by using the Schur complement.

The remainder of the paper is organized as follows. Following an overview of related work in the next section, pose-graph based SLAM is described in Section 3. In Section 4, an insight into Linear SLAM is provided. Section 5 presents a robust strategy for solving the Linear SLAM problem, including the delayed optimisation, the detection of outliers and the recovery of information matrix, namely the exclusion of outliers. The performance of the proposed RL-SLAM is validated in Section 6 through publicly available synthetic and practical datasets in 2D and 3D environments. Finally, conclusions and future work are provided in Section 7.

2 Related Work

The optimisation-based approaches to solve the SLAM problem consider all measurements in a batch process. Lu and Milios [Lu and Milios, 1997] were the first in formulating the SLAM problem as a global graph optimisation based on maximum likelihood criterion. It is quite common that the optimisation process refers to thousands of nonlinear equations, resulting high computational complexity. Several approaches have been proposed to significantly improve the performance, in terms of efficiency and convergence, such as relaxationased [Howard *et al.*, 2001], square root smoothing and mapping ($\sqrt{SAM}$) [Dellaert and Kaess, 2006], incremental SAM (iSAM) [Kaess *et al.*, 2007], stochastic gradient descent-based (SGD) [Olson *et al.*, 2006] and so forth. Grisetti *et al.* [Grisetti *et al.*, 2010] provided a comprehensive introduction to the graph-based SLAM problem, in particular for the manifold-based approach to handle the non-Euclidean nature of angular orientation. More recently, Kümmerle *et al.* [Kümmerle *et al.*, 2011] presented a general framework of graph-based SLAM optimisation, called g2o, with open-source C++ codes to implement nonlinear minimization efficiently. Different from filtering-based approaches, these approaches aim to optimize the full trajectory. Thus they are also known as smoothing methods.

Local submap joining techniques have shown to be computationally efficient in large-scale map building. The key idea of this method is to partition the whole map into a sequence of submaps with limited, manageable size, and then to combine them to build up the original large map. Under the bounded size, the local computation or transformation commonly holds better performances. Tardós *et al.* [Tardós *et al.*, 2002] applied sonar sensor measurements to build the local maps and finished the joining with sequential operations. Pérez *et al.* [Paz *et al.*, 2008] proposed the Divide and Conquer joining method which efficiently reduced the computational complexity to be linear with the number of the features at each joining step. Taking into account the sparse property of the graphical structure, a sparse local submap joining filter (SLSJF) based on extended information filter (EIF) was proposed in [Huang *et al.*, 2008], where each local submap is processed as an integrated measurement, resulting in an exactly sparse information matrix and no information lost.

Efforts on the linear formulation of graph-based SLAM have been devoted to reducing or avoiding the nonlinear and nonconvex optimisation which is susceptible to local minima. One theoretical analysis work in [Huang *et al.*, 2010] showed that the nonlinearity of the SLAM problem is mainly introduced by orientations, while the map joining strategy can significantly reduce the linearization error in optimisation stage. With the independence assumption of the position and orientation measurements, Carlone [Carlone *et al.*, 2011] proposed a closed-form approximation to the 2D pose-graph based SLAM problem, so that the nonlinear optimisation problem is cast into a linear optimisation one. In exploring why the conventional nonlinear SLAM solutions commonly do not trap into local minima even with a poor initial guess, Huang *et al.* [Huang *et al.*, 2012] provided an insight into the number of local minima in SLAM systems. In addition, another crucial performance of the nonlinear solutions to SLAM is the convergence, which is affected by the measurement accuracy, the inter-nodal distance and the connectivity of the graph as discussed in [Carlone, 2013]. To address these issues, Zhao *et al.* [Zhao *et al.*, 2013] proposed a linear SLAM algorithm which firstly transfers two submaps into the same coordinate frame before the joining. Then the joining process can be implemented by a linear least square. In particular, this method is available for 2D/3D cases and feature/pose-based SLAM and has been successfully used for the monocular SLAM problem [Zhao *et al.*, 2014].

Considering the vulnerability of state-of-the-art nonlinear optimisation to outliers, different robust strategies have been investigated in the last few years. Olson and Agarwal [Olson and Agarwal, 2012] proposed the Max-Mixture method (MM) in which loop closures were modeled as a multi-model distribution with a null hypothesis, which holds a very large variance. By iteratively verifying and weighting these hypotheses, the null hypotheses are down-weighted and are accepted for outliers. Sünderhauf and Protzel [Sünderhauf and Protzel, 2012b][Sünderhauf and Protzel, 2012a] proposed switchable constraints-based method (SC) in which switch variables are introduced for each loop closures, so that the outliers are switched off in the optimizing process. Agarwal et al. [Agarwal *et al.*, 2013][Agarwal *et al.*, 2014] proposed Dynamic Covariance Scaling (DCS), a variant of switchable constraints. This method efficiently reduces the dimension of state vector by providing close-form solutions to scaling factors. The Realizing, Reversing, Recoving algorithm (RRR) in [Latif *et al.*, 2013] was proposed to detect and remove incorrect loop-closures by a series of chi-square tests between loop closures and optimisation results. A robust approach based on Expectation-Maximization (EM) in [Lee *et al.*, 2013] iteratively computed weights for each loop-closure constraints and optimizes the configuration of poses in expectation and maximization steps, respectively. Different form the first three approaches where the failed constraints are always maintained with low weights, the EM rejects all failed constraints as in the RRR. However, all aforementioned robust methods are proposed for the full SLAM problem, it still requires a further research on the robust strategy for the local map joining-based SLAM.

3 Pose-Graph SLAM

Compared with the filtering SLAM, the pose-graph SLAM constructs a graph with robot poses as its vertices and inter-pose constraints as edges as briefed in Introduction. The solution to a pose-graph SLAM problem can be seen as finding the pose trajectory which maximally satisfies those edge constraints. This is also equivalent to minimising a specified error function to obtain an optimal pose configuration.

Let $\mathcal{M} = (\mathcal{V}, \mathcal{E})$ denote the directed graph (or map) of SLAM, where each vertex $\boldsymbol{x}_i \in \mathcal{V}, (1 \leq i \leq |\mathcal{V}|)$, indicates a robot pose, and each edge $\boldsymbol{z}_{ij} \in \mathcal{E}$, represents a constraint from $\boldsymbol{x}_i$ to $\boldsymbol{x}_j$. In particular, if $j = i+1$, $\boldsymbol{z}_{ij}$ indicates a sequential motion constraint obtained from the odometry measurement, otherwise a loop-closure constraint. Let the total number of vertices and edges be n and m, that is $n = |\mathcal{V}|$ and $m = |\mathcal{E}|$, respectively. Then, the measurement error corresponding to $\boldsymbol{z}_{ij}$ is commonly defined as

$$\boldsymbol{e}_{ij} = f_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) - \boldsymbol{z}_{ij} \tag{1}$$

where $f_{ij}(\cdot,\cdot)$ is a nonlinear measurement function from $\boldsymbol{x}_i$ to $\boldsymbol{x}_j$, and $\boldsymbol{e}_{ij}$ is a measurement error vector with a zero mean and an information matrix $\mathcal{I}_{ij}$. Under Gaussian assumption, the pose-graph SLAM problem is to maximise the conditional likelihood function

$$p\left(\boldsymbol{X}|\boldsymbol{Z}\right) = C \cdot \exp\left(-\sum_{(i,j)} \boldsymbol{e}_{ij}^T \mathcal{I}_{ij} \boldsymbol{e}_{ij}\right), \tag{2}$$

where C is a normalisation constant, $\boldsymbol{X} = \{\boldsymbol{x}_i\}$ and $\boldsymbol{Z} = \{\boldsymbol{z}_{ij}\}$ are the stacked vertex and measurement vectors, respectively. This is equivalent to minimise the negative log-conditional likelihood as

$$\underset{\boldsymbol{X}}{\operatorname{argmin}}\left(-\ln p(\boldsymbol{X}|\boldsymbol{Z})\right) = \underset{\boldsymbol{X}}{\operatorname{argmin}} \sum_{(i,j)} \boldsymbol{e}_{ij}^T \mathcal{I}_{ij} \boldsymbol{e}_{ij}, \tag{3}$$

where the log constant term is dropped. A detail introduction can be found in [Grisetti *et al.*, 2010]. There exist a variety of popular approaches to find the minimum of Eq. (3), such as Gaussian-Newton and Levenberg-Marquardt (LM). In particular, g2o [Kümmerle *et al.*, 2011] is an open-source package providing efficient and accurate solutions. These approaches, however, all require an initial guess of the poses for linearsation and iteration. In case the initial value is poor, the optimisation results can easily be trapped in local minima.

4 Generalised Least Squares-based Linear SLAM

In L-SLAM the nonlinear coordinate transformation is performed out of the optimisation process, thus enabling convex optimisation methods such as least-squares optimisation. In this work, we will only consider a pose-graph SLAM (thus with no feature maps) and assume no odometry errors as in the previous work.

Adopting the definition of local submaps in [Zhao *et al.*, 2013], $\mathcal{M}_j^i$ represents a j^{th} local map anchored by a pose $\boldsymbol{x}_i$. That is the coordinate sytem is defined by the anchor pose with its position vector being the origin and the attitude (orientation) the coordinate axes.

Each submap is typically built along the sequential odometry measurements and thus the last pose in one submap can be another anchor pose in the following submap. For example, suppose i^{th} map $\mathcal{M}_i^a$ is anchored by a pose $\boldsymbol{x}_a$ and has a pose $\boldsymbol{x}_b$ as its last pose, which subsequently becomes the next anchor pose in the following j^{th} map $\mathcal{M}_j^b$, where $j = i+1$.

The key operation in L-SLAM is to perform the coordinate transformation before applying the map joining rather than within the joining. Consequently, the old anchor pose-a in $\mathcal{M}_i^a$ should be transformed into a new anchor pose-b so that the submap is in the same coordinate system of $\mathcal{M}_j^b$ as

$$\boldsymbol{X}_i^b = h(\boldsymbol{X}_i^a) \quad (4)$$
$$\mathcal{I}_i^b = (\boldsymbol{J^a} \boldsymbol{P}_i^a \boldsymbol{J}^{a,T})^{-1} = \boldsymbol{J}^b \mathcal{I}_i^a \boldsymbol{J}^a, \quad (5)$$

where $h(\cdot){:}R^n \mapsto R^n (n = |\mathcal{V}|$ is the nonlinear coordinate transformation on a pose manifold (see Eq. (9) in [Zhao *et al.*, 2013]), and $\boldsymbol{J}^a$ and $\boldsymbol{J}^b$ are the Jacobians of $h(\cdot)$ and $h^{-1}(\cdot)$, respectively. In addition, the new vertex set should be formed by replacing the pose b with a in $\mathcal{V}_p^a$ as pose b becomes now the new origin of the coordinates. Please note that $\boldsymbol{J}^b$ is the inverse of the $\boldsymbol{J}^a$ from the inverse theorem of the coordinate transformation.

Although the information matrix of each original local submap is commonly set as a (block) diagonal matrix, it becomes a non-(block) diagonal matrix from a sequence of nonlinear coordinate transformations in L-SLAM. This will be further discussed in Section 5.3.

Stacking two submaps, the combined map becomes a union of two submaps

$$\tilde{\mathcal{M}}_{ij}^b = \mathcal{M}_i^b \cup \mathcal{M}_j^b, \quad (6)$$

with $\tilde{\boldsymbol{X}}_{ij}^b = \boldsymbol{X}_i^b \cup \boldsymbol{X}_j^b$ and $\tilde{\mathcal{I}}_{ij}^b = \text{blkdiag}(\mathcal{I}_i^b, \mathcal{I}_j^b)$. Thus Eq. (3) can be rewritten for the estimated map $\mathcal{M}_{ij}^b$ in the *generalised linear least squares* framework

$$\underset{\hat{\boldsymbol{X}}}{\text{argmax}}\ p\ (\tilde{\boldsymbol{X}}_{ij}^b | \boldsymbol{X}_{ij}^b) = \underset{\hat{\boldsymbol{X}}}{\text{argmin}}\ \|\tilde{\boldsymbol{X}}_{ij}^b - \mathbf{H}\boldsymbol{X}_{ij}^b\|_{\tilde{\mathcal{I}}_{ij}^b}, \quad (7)$$

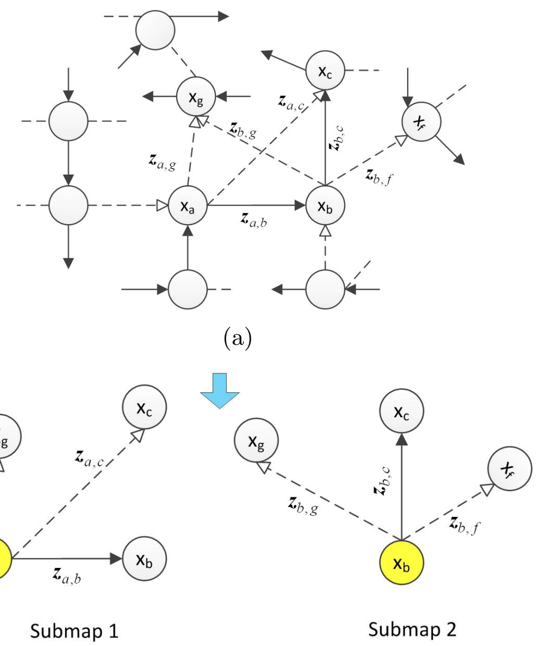
and the estimated $\hat{\boldsymbol{X}}_{ij}^b$ and $\hat{\mathcal{I}}_{ij}^b$ become

$$\hat{\boldsymbol{X}}_{ij}^b = \mathbf{H}^{+} \tilde{\boldsymbol{X}}_{ij}^b, \quad \hat{\mathcal{I}}_{ij}^b = \mathbf{H}^T \tilde{\mathcal{I}}_{ij}^b \mathbf{H}, \quad (8)$$

where $\mathbf{H}^+$ is a general Moore-Penrose inverse given by $(\mathbf{H}^T \tilde{\mathcal{I}}_{ij}^b \mathbf{H})^{-1} \mathbf{H}^T \tilde{\mathcal{I}}_{ij}^b$ and $\mathbf{H}$ is a coefficient matrix of dimension $(n_i{+}n_j){\times}n_{ij}$. If the i^{th} edge in $\tilde{\boldsymbol{X}}_{ij}^b$ corresponds to the j^{th} vertex in $\boldsymbol{X}_{ij}^b$, it has the block $\mathbf{H}_{i,j} = \mathbf{I}$, otherwise zeros.

5 Robust Linear SLAM

L-SLAM gains the advantages of linearity by performing the nonlinear transformation before the optimisation. It however, introduces added difficulties in handling the loop-closure outliers due to insufficient information and the correlated information matrix resulted from a series of nonlinear transformations. The aforementioned robust approaches in the SLAM problem mainly focus on the (block) diagonal covariance matrix, where the outlier-rejection could be facilely conducted by deactivating or eliminating incorrect constraints and its information matrix blocks. In contrast, the outlier information can spread out over the whole information matrix



(a) (b)

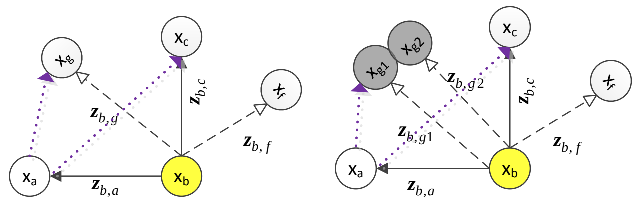
Figure 1: A pose-graph SLAM example used in this paper. Solid lines denote odometry measurements, whilst dashed lines represent loop-closure constraints. Each vertex $\boldsymbol{x}_i$ indicates a robot pose, and edges are marked as $\boldsymbol{z}_{ij}$.

in non block-diagonal cases as considered in this work. Thus a necessary operation is required to correctly recover the information matrix. Different from the nonlinear SLAM utilising a batch optimisation, L-SLAM performs a series of local optimizations on two submaps. Due to the locality and thus limited information available in each optimisation stage, it becomes challenging to screen out outliers reliably. To address this, we introduce a robust approach to L-SLAM with three stages: delayed optimisation, outliers detection and exclusion.

5.1 Delayed optimisation

Following the adjustment of the coordinate frame, the multi-map $\tilde{\mathcal{M}}_{ij}^b$ describes all available local information. In general, there exist multiple pose constraints from the new anchor pose. Without taking into account outliers, all constraints are *averagely* fused in a least-square sense in L-SLAM. This joining strategy can provide acceptable results if there are no loop-closure outliers.

To handle potential erroneous edges, *delayed optimisation* augments all pose candidates into the graph and postpones the merge to the next joining level, while combining unambiguous poses. Once odometry pose measurements arrive, which are assumed highly accurate in this work, the delayed loop-closures are declaired as inliers or outliers.



(a) (b)

Figure 2: The joined and delayed map from two submaps $\mathcal{M}_1^b$ and $\mathcal{M}_2^b$. The index $(1, 2, \cdots)$ is used to separate different edges to a same pose. The dotted lines represent the hidden correlation between two poses. Colour vertexes are the origins where submaps are projected to, while gray vertexes represent the delayed pose appearing in both two submaps.

Figure 1 illustrates an example of pose-graph SLAM which is used in this work to demonstrate the delayed optimisation. From this graph, two successive local submaps can be extracted as $\mathcal{M}_1^a = (\mathcal{V}_1^a, \mathcal{I}_1^a)$ and $\mathcal{M}_2^b = (\mathcal{V}_2^b, \mathcal{I}_2^b)$, where $\mathcal{V}_1^a = \{b, c, g\}$ and $\mathcal{V}_2^b = \{c, f, g\}$. Note that, the two local submaps can also be delayed submaps from the previous joining step and can include two or more edges between two poses. The pose $\boldsymbol{x}_b$ is the last odometry-pose in $\mathcal{M}_1^a$ and thus acting as a new anchor pose of $\mathcal{M}_2^b$.

As discussed in Section 4, $\mathcal{M}_1^a$, defined in the anchor pose $\boldsymbol{x}_a$, needs to be transformed with respect to the pose $\boldsymbol{x}_b$, giving $\mathcal{M}_1^b$ with $\mathcal{V}_1^b = \{a, c, g\}$. The stacked multmap $\tilde{\mathcal{M}}_{12}^b$ can then be obtained by including all poses, giving $\tilde{\mathcal{V}}_{12}^b = \{a, c, g, \mathcal{V}_2^b\}$. Please note that in the diagram $\boldsymbol{x}_c$ is an odometry-pose which is assumed outlier-free and $\boldsymbol{x}_g$ is potentially an outlier. In L-SLAM, these were equally combined giving $\mathcal{V}_{12}^b = \{a, c, g, f\}$ as illustrated in Fig. 2(a), whilst the delayed pose set is augmented with all candidates as $\mathcal{V}_{12}^b = \{a, c, f, g_1, g_2\}$ as shown in Fig. 2(b).

5.2 Outlier-Detection

In the robust nonlinear pose-graph SLAM with (block) diagonal information matrix, potential outlier constraints are gradually down-weighted in each nonlinear equation. Similarly, we introduce such weights in the submap joining process. Then the conditional likelihood (Eq.(2)) can be rewritten as a marginalisation of a joint density with respect to the weight variables

$$\begin{aligned} p\left(\boldsymbol{X}|\boldsymbol{Z}\right) &= \int_{\mathbf{W}} p\left(\boldsymbol{X}, \mathbf{W}|\boldsymbol{Z}\right) d\mathbf{W} \qquad (9) \\ \mathbf{W} &= \{w_{ij} | i = 1, \cdots, n; j = 2, \cdots, n; i < j\} \end{aligned}$$

where $\mathbf{W}$ is a diagonal weight-coefficient matrix. Lee et al. [Lee *et al.*, 2013] has shown that the classification EM algorithm is an efficient approach to find the maximum probability solution to $p(\boldsymbol{X}|\boldsymbol{Z})$, where the expectation and maximization steps are performed iteratively to calculate the weights and poses, respectively. In particular, pose constraints with low weights are regarded as outliers and filtered out. Thus this strategy is employed to detect outliers in this work.

Under the assumption of zero wheel-slips in odometry measurements, the weights of odometry and delayed constraints are all set to 1. In contrast, the weight w_{ij} for the constraint $\boldsymbol{z}_{ij}$ from a pose i to a pose j can be modelled as a Cauchy function

$$w_{ij} = \frac{C^2}{C^2 + \|f_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) - \boldsymbol{z}_{ij}\|^2_{\mathcal{I}_{ij}}}, \qquad (10)$$

where the weight will have 1 if the errors are zero and will approach to 0 as the matching errors increase. The half-weight point is controlled by the constant C. During each iteration, if the weight w_{ij} corresponding to the constraint $\boldsymbol{z}_{ij}$ is larger than the pre-specified threshold δ, this constraint is declared as an inlier loop closure, otherwise an incorrect loop closure with $w_{ij} = 0$.

5.3 Outlier-Exclusion

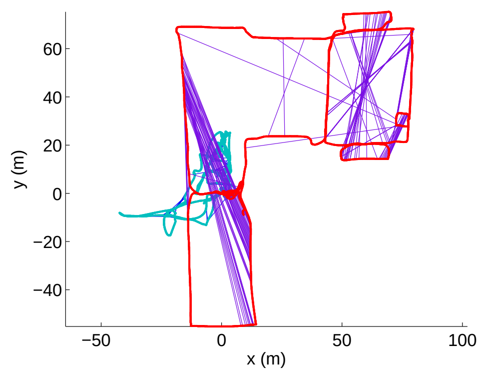
As mentioned in Section 5.1, the information of outliers are retained untill they are successfully resolved in the joining process, resulting in a corrupted information matrix. It can be shown that the outlier-rejection can be performed by cutting out the corresponding rows and columns in the covariance matrix. Thus a direct approach is to first compute its covariance matrix, eliminated the corrupted rows/columns, and then recover the information matrix. However, it is not efficient due to involvement of the full matrix inversion. To avoid the full matrix inverion while retaining the benefits of the information matrix, we propose to use Shur complement. Suppose that $\mathcal{I}_a$ is an inlier information block and $\mathcal{I}_b$ an outlier block

$$\mathcal{I} = \begin{bmatrix} \mathcal{I}_a & \mathcal{I}_{ab} \\ \mathcal{I}_{ba} & \mathcal{I}_b \end{bmatrix}. \qquad (11)$$

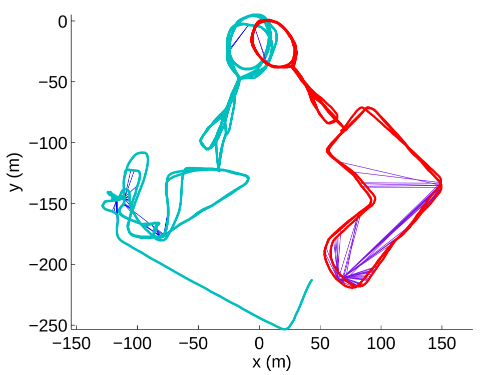
Unlike the covariance matrix case, the $\mathcal{I}_a$ is fully corrupted by the $\mathcal{I}_b$ which can be recovered by utilising the Shur complement,

$$\mathcal{I}_a = \mathcal{I}_a - \mathcal{I}_{ab}\mathcal{I}_b^{-1}\mathcal{I}_{ba}. \qquad (12)$$

The number of the non-zero entries in $\mathcal{I}_{ab}$ is no larger than $3m\log_2(n)$ in 2D cases or $6m\log_2(n)$ in 3D cases. Particularly, outliers are commonly distributed in different submaps and will be eliminated gradually in the joining process. Consequently, false constraints information can be fully filtered out by the Schur Complement with low computational complexity.

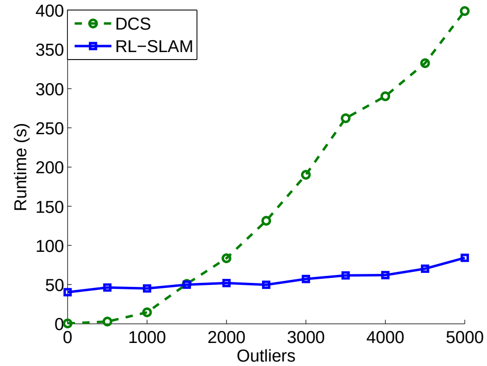


(a)

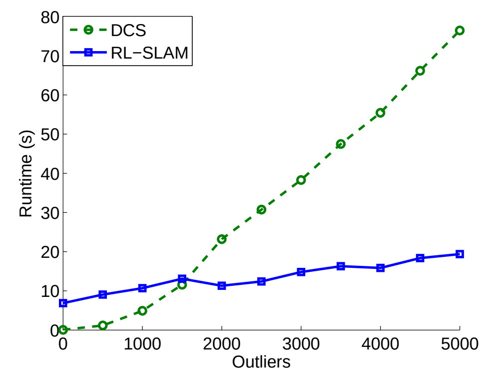


(b)

Figure 3: RL-SLAM results (red color) for two real datasets: Bicocca (a) and New College (b) with real outliers shown as cross-lines. L-SLAM results are shown in light cyan colour for comparison.



(a)



(b)

Figure 4: The runtime comparions of the RL-SLAM (in Matlab) and DCS/g2o (in C++) on the City10000 (a) and ManhtanOlson datasets (b) corrupted with up to 5000 outliers. Please note that the runtime of RL-SLAM is not much affected by the number of outliers contrary to the DCS, showing clear benefits if the number of outliers exceeds 1500.

6 Results and Evaluations

There are several publicly available benchmark datasets for the pose-graph SLAM. To validate the performance of the proposed approach against different numbers of outliers, we chose five synthetic datasets in 2D and 3D environments: City10000, ManhattanOlson3500, Sphere2500, Intel Research Lab, and Parking Garage datasets. Submaps generated from these datasets can be found in [Zhao *et al.*, 2013] as well as the open-source matlab codes to perform the submap joining. Due to the limited space, those results were not shown here. The robustness of the RL-SLAM are evaluated using the Bicocca and NewCollege datasets which contain real outliers as shown in Fig. 3. For comparision, we run the DCS in g2o with its suggested parameters $\Phi = 5$ for the Bicocca dataset and $\Phi = 1$ for the NewCollege dataset.

Figure 4 shows the computational time of both algorithms against 0 to 5000 outliers in the ManhtanOlson and City10000 datasets. It can be observed that the DCS/g2o is faster than our method with a small number of outliers. However, RL-SLAM outperforms when the number of outliers becomes large (over 1500 outliers), which is due to its submap strategy: that is outliers are distributed in submaps and they are successively eliminated in the submap joining processes. This also reveals the cause of fluctuations in the RL-SLAM runtime. The longer one loop-closure is delayed, the more the extra computation time is required.

7 Conclusions

Although the existing Linear SLAM is effective in solving the nonlinear pose-graph SLAM problem, it is vulnerable to loop-closure outliers. To mitigate this problem, Robust Linear SLAM (RL-SLAM) was proposed in this work by applying a delayed optimisation sterategy where any ambiguous loop-closure outliers are left to the next submap joining processes, and efficiently recovering the corrupted information matrix using Schur complement. Experimental results using publicly available datasets confirmed the feasibility and efficiency of the proposed method, recovering a large number of outliers effectively. The state-of-the-art solutions from the nonlinear SLAM showed still better pose accuracy compared to RL-SLAM, particurarly in 3D datasets. However RL-SLAM could outperforme nonlinear SLAM in computational time when the number of outliers increased. In future work, we will focus on 6DOF/3D scenarios to improved the accuracy and investigate the effets of odometry errors which were assumed outlier-free, and implement the methods in C++ framework for the comparision with DCS/g2o.

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