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A Novel Unknown-Input Estimator for Disturbance Estimation and Compensation

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Abstract

A novel unknown-input estimator (UIE) for estimating and rejecting disturbances is proposed in this paper. Effective treatment of unknown disturbing inputs is of vital importance to maintaining satisfactory performance of control systems. Advanced methods that estimate these disturbances and cancel them accordingly outperform traditional approaches. However, limitations remain, including requirements for some knowledge on unknown inputs, derivatives of measured outputs, inversion of plant dynamics, constrained state observer design, parameter optimisation (global optimum not guaranteed), or complicated structures. The proposed UIE is exempted from the aforementioned limitations. It consists of an estimation gain matrix, a state observer, and a low-pass-filter-characterised subsystem. Comparison via simulation is drawn between the new UIE and a benchmark disturbance observer on a multi-input multi-output system. The proposed UIE is shown to be more effective in estimating and compensating unknown inputs.

1 Introduction

In reality, a control system is always subject to unknown inputs in the form of internal uncertainties and exogenous loads that cannot be measured or inconvenient to measure. If these disturbances are not properly treated, poor performance of the control system may result, especially in applications where high-precision set-point tracking or regulation of controlled variables (e.g. position, speed, or torque, etc) is required [Schuhmann *et al.*, 2012; Erenturk, 2013]. To attenuate the negative impact from unknown inputs, various methods can be used. A common practice is to maximise the inherited robustness of a controller by careful parameter design and tuning [Åström and Hägglund, 2006]. Further robustness improvement can be achieved via advanced techniques such as nonlinear control [Chen *et al.*, 2009], H_∞ robust control [Dietz and Scherer, 2010], and adaptive intelligent control [Mohammadzaheri and Chen, 2010a; Mohammadzaheri and Chen, 2010b]. A more attractive approach distinguished from the aforementioned comes with the concept of unknown-input estimation, which brings disturbance rejection to a higher level, as demonstrated by various industrial applications [She *et al.*, 2011; Mitsantisuk *et al.*, 2012; Erenturk, 2013; Chandar and Talole, 2014]. This technique provides the controller with better tolerance to uncertainties by constructing counteractive control efforts to cancel the effects from unknown inputs according to the estimations.

With an extended state observer, a class of partially unknown inputs modelled in the form of constant-coefficient differential equations are estimated together with the states of a linear time-invariant plant [Johnson, 1971; Ohishi *et al.*, 1987]. The well-known internal model principle works in a similar way but the plant states are not estimated [Francis and Wonham, 1975; Hara *et al.*, 1988]. A prerequisite for all these methods is that characteristics of the exogenous inputs such as the frequency of each sinusoidal component must be known. Although such information can be obtained via field or experimental data, there is possibility that the data collected may not cover all situations. In this case, disturbances cannot be sufficiently well modelled and unsatisfactory estimation performance may result. Moreover, it is sometimes difficult or inconvenient to collect relevant data for disturbance modelling.

When only the order of the exogenous inputs is known and these disturbances perturb the system through channels other than those of the control inputs, a generalised extended state observer can be used [Li *et al.*, 2012; Godbole *et al.*, 2013]. For nonlinear systems, nonlinear disturbance observers are also available [Yang *et al.*, 2012; Wei and Chen, 2012]. However, higher-order observers are required when dealing with complex disturbing exogenous inputs, and this increases the observer dimension as a result.

Instead of using a model based on a priori assumptions or empirical knowledge, estimating completely unknown inputs is possible by combining an unknown-input-decoupled observer (UIDO) with an additional disturbance-estimation function [Hou and Müller, 1992]. The former is used to observe plant states while the latter is for approximating the unknown inputs. Without decoupling unknown inputs, an ordinary state observer [Luenberger, 1964] can still work properly under the situation that the unknown inputs are cancelled by corresponding control inputs. Similarly in this case, an estimation function is still needed besides the state observer to predict unknown inputs. Among these studies, some methods require the derivatives of measured outputs [Liu and Peng, 2002; Chandar and Talole, 2014], which make the estimation sensitive to noises or modelling errors; Although some approaches do not differentiate the measured outputs [Corless and Tu, 1998; Liu and Peng, 2000; Xiong and Saif, 2003; She *et al.*, 2008; Liu *et al.*, 2013], some other limitations are imposed. Specifically, some rank and boundedness conditions apply to unknown inputs in the work of Corless and Tu [1998], Liu and Peng [2000], and Xiong and Saif [2003]; The computation becomes another problem using the method of Liu and Peng [2000] when the number of unknown inputs increases; The complexity of parameter selection rises due to the less restrictive conditions on unknown inputs [Xiong and Saif, 2003; Kim and Rew, 2013]; The state observer gain is constrained by the estimation scheme for unknown inputs, and in the mean time limited design freedom is given to the unknown-input estimation [She *et al.*, 2008; Liu *et al.*, 2013].

As a counterpart of design in the state space, the disturbance observer (DOB) synthesised in the frequency domain relies on a customised low-pass filter and the inversion of plant dynamics [Ohnishi, 1987; Umeno and Hori, 1991; Umeno *et al.*, 1993; Kempf and Kobayashi, 1999; Choi *et al.*, 2003]. The custom low-pass filter design, which influences the stability of the plant, can be intricate when an optimal set of filter coefficients are desired through an optimization procedure. The plant inversion also causes some difficulties in employing DOBs in multi-input multi-output (MIMO) systems since an inverse cannot always be found. The generalisation of the DOB to MIMO cases bypasses the plant inversion and adopts parameterised controller design through an optimisation process [Shahruz, 2009; Du *et al.*, 2010]. Nevertheless, the global optimum is not guaranteed in these methods.

The work in this paper therefore aims to develop an alternative approach for unknown-input estimation exempted from the aforementioned limitations without sacrificing its performance. Instead of discussing unknown-input estimation as a subsystem or an add-on of an existing regulator or tracking controller, the study presented in this paper solely focuses on the new unknown-input estimation scheme and treats it as a standalone mechanism. The outcomes of this study not only contribute to the theory framework of unknown-input estimation but also deliver potentials of providing existing controllers in literature with enhanced tolerance to disturbances for further improvement on regulation or trajectory tracking performance.

To avoid confusion and for unity, the scheme for unknown-input estimation is termed as 'unknown-input estimator' (UIE) in this paper. Subsequent sections are arranged as follows. A novel UIE is derived and analysed in Section 2; Simulations are presented in Section 3 in the form of comparisons between the proposed UIE and a benchmark DOB on an MIMO system. Conclusions are drawn in Section 4.

2 Unknown-Input Estimator (UIE)

2.1 Problem Statement and Assumptions

The general plant model to be considered is given by

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{B}_d\boldsymbol{d}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases}, \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$, $\boldsymbol{y}(t) \in \mathbb{R}^{n_y}$, and $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ are vectors of system states, measured outputs, and control inputs, with n_x, n_y, and n_u elements, respectively; $\boldsymbol{A} \in \mathbb{R}^{n_x \times n_x}$ represents system internal dynamics; $\boldsymbol{B} \in \mathbb{R}^{n_x \times n_u}$ and $\boldsymbol{B}_d \in \mathbb{R}^{n_x \times n_d}$ denote the distribution of control and unknown inputs, respectively; $\boldsymbol{C} \in \mathbb{R}^{n_y \times n_x}$ describes output dynamics; $\boldsymbol{d}(t) \in \mathbb{R}^{n_d}$ contains n_d unknown inputs that the system is subjected to, including uncertain, nonlinear, time-varying, and state-dependent terms.

The following two assumptions are made:

Assumption 1: $(\boldsymbol{A}, \boldsymbol{B})$ is controllable.

Assumption 2: $(\boldsymbol{C}, \boldsymbol{A})$ is observable.

The first assumption is not directly associated with the UIE to be discussed but made to confine the plant of interest to the controllable system category in industry. The second assumption is necessary for state estimation which is one of the essential components of the UIE proposed in this paper.

Note that $\boldsymbol{B}_d$ can be either known or unknown, with no rank condition required, and there is no restriction on the number of unknown inputs $\boldsymbol{d}(t)$. This means, there can be more unknown inputs than the control inputs and the measured outputs. When both $\boldsymbol{B}_d$ and n_d are unknown, it is difficult or even impossible to precisely estimate the components of $\boldsymbol{d}(t)$.

Given the fact that the estimation of unknown inputs in most servo systems are for disturbance rejection to maintain prescribed performance, it is thus possible to assume the existence of equivalent unknown inputs entering the plant through the control input channels $\boldsymbol{B}$. This is a practical technique commonly adopted in DOBs, and it has been shown by She et al. [2008] that such equivalent quantities exist for the unknown inputs $\boldsymbol{d}(t)$ in a system as in Eq. (1) under *Assumptions 1* and *2*.

Denote the equivalent unknown inputs by $\boldsymbol{d}_e(t)$, then Eq. (1) can be expressed as

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}[\boldsymbol{u}(t) + \boldsymbol{d}_e(t)] \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases}. \tag{2}$$

Now we can use Eq. (2) as an equivalent of Eq. (1), and the control problem at this point is to estimate and feed back $\boldsymbol{d}_e(t)$ to cancel $\boldsymbol{d}(t)$.

2.2 UIE Structure

In order to cancel the effects from unknown inputs $\boldsymbol{d}(t)$, or equivalently, $\boldsymbol{d}_e(t)$, the control input $\boldsymbol{u}(t)$ is defined as

$$\boldsymbol{u}(t) = -\hat{\boldsymbol{d}}_e(t). \tag{3}$$

Now we construct $\hat{\boldsymbol{d}}_e(t)$ in the following form:

$$\hat{\boldsymbol{d}}_e(t) = \hat{\boldsymbol{d}}_{ev}(t) + \boldsymbol{K}_d\left[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)\right], \tag{4}$$

where $\hat{\boldsymbol{d}}_{ev}(t) \in \mathbb{R}^{n_d}$ is an adaptive auxiliary variable for estimating the equivalent unknown inputs, $\boldsymbol{K}_d \in \mathbb{R}^{n_d \times n_y}$ is an estimation gain, and $\hat{\boldsymbol{y}}(t)$ is an estimate of the plant outputs.

The auxiliary variable $\hat{\boldsymbol{d}}_{ev}(t)$ is obtained and updated in real time via a subsystem $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$ with $\hat{\boldsymbol{d}}_e(t)$ as its input:

$$\begin{cases} \dot{\boldsymbol{x}}_f(t) = \boldsymbol{A}_f \boldsymbol{x}_f(t) + \boldsymbol{B}_f \hat{\boldsymbol{d}}_e(t) \\ \hat{\boldsymbol{d}}_{ev}(t) = \boldsymbol{C}_f \boldsymbol{x}_f(t) \end{cases}, \tag{5}$$

where $\boldsymbol{A}_f \in \mathbb{R}^{n_f \times n_f}$, $\boldsymbol{B}_f \in \mathbb{R}^{n_f \times n_d}$, and $\boldsymbol{C}_f \in \mathbb{R}^{n_d \times n_f}$ are the internal, input, and output dynamics of the subsystem, respectively, with n_f denoting the number of the subsystem states.

Note that Eq. (5) only gives a general input-output form of the subsystem. In addition, $\boldsymbol{x}_f(t)$ do not have any direct physical implication in this general form, and n_f depends on the detailed structure of the subsystem. It is because the design of $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$ is flexible and varies among different applications. Proper selection of the structure and parameters for the subsystem is addressed in detail in Sections 2.3 and 2.4 for ease of discussion in consideration of its direct impact on system closed-loop characteristics.

In order to obtain the estimated plant outputs $\hat{\boldsymbol{y}}(t)$, a state observer that takes $\boldsymbol{d}_e(t)$ into account is needed and constructed as:

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}[\boldsymbol{u}(t) + \hat{\boldsymbol{d}}_e(t)] + \boldsymbol{L}[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)] \\ \hat{\boldsymbol{y}}(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \end{cases}, \tag{6}$$

where $\boldsymbol{L} \in \mathbb{R}^{n_x \times n_y}$ is the estimation gain for the state observer, and $\hat{\boldsymbol{x}}(t)$ is the estimate of plant states.

In summary, the subsystem $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$, the gain $\boldsymbol{K}_d$, and the state observer are the three major components to design for the proposed UIE.

2.3 Closed-loop Analysis

Since we are able to use system (2) as an equivalent of Eq. (1), the mechanism of estimating and suppressing unknown inputs can be revealed by the relation between the equivalent unknown inputs $\boldsymbol{d}_e(t)$ and the system outputs $\boldsymbol{y}(t)$ in Eq. (2).

From Eqs (4), (5), and (6), we have in frequency domain:

$$\hat{\boldsymbol{y}}(s) = \boldsymbol{C}(s\boldsymbol{I}_{nx} - \boldsymbol{A} + \boldsymbol{LC})^{-1}\boldsymbol{L}\boldsymbol{y}(s) = \boldsymbol{G}_{yy}(s)\boldsymbol{y}(s), \tag{7}$$

$$\hat{\boldsymbol{d}}_{ev}(s) = \boldsymbol{C}_f(s\boldsymbol{I}_{nf} - \boldsymbol{A}_f)^{-1}\boldsymbol{B}_f\hat{\boldsymbol{d}}_e(s) = \boldsymbol{G}_{dd}(s)\hat{\boldsymbol{d}}_e(s), \tag{8}$$

$$\begin{aligned} \hat{\boldsymbol{d}}_e(s) &= \left[\boldsymbol{I}_{nu} - \boldsymbol{G}_{dd}(s)\right]^{-1}\boldsymbol{K}_d\left[\boldsymbol{y}(s) - \hat{\boldsymbol{y}}(s)\right] \\ &= \boldsymbol{G}_{yd}(s)\left[\boldsymbol{y}(s) - \hat{\boldsymbol{y}}(s)\right] \end{aligned}, \tag{9}$$

and thus,

$$\begin{aligned} \boldsymbol{y}(s) &= \left\{\boldsymbol{I}_{ny} + \boldsymbol{P}(s)\boldsymbol{G}_{yd}(s)\left[\boldsymbol{I}_{ny} - \boldsymbol{G}_{yy}(s)\right]\right\}^{-1}\boldsymbol{P}(s)\boldsymbol{d}_e(s) \\ &= \boldsymbol{G}(s)\boldsymbol{d}_e(s) \end{aligned}, \tag{10}$$

where $\boldsymbol{P}(s)$ is the transfer function matrix of the MIMO plant from $\boldsymbol{u}(s)$ to $\boldsymbol{y}(s)$; $\boldsymbol{G}_{yy}(s)$, $\boldsymbol{G}_{dd}(s)$, $\boldsymbol{G}_{yd}(s)$, and $\boldsymbol{G}(s)$ are the transfer function matrices of corresponding input/output pairs; $\boldsymbol{I}_{nx}$, $\boldsymbol{I}_{nf}$, $\boldsymbol{I}_{nu}$, and $\boldsymbol{I}_{ny}$ are identity matrices with a dimension of $n_x \times n_x$, $n_f \times n_f$, $n_u \times n_u$, and $n_y \times n_y$, respectively.

Note that Eq. (10) is in an MIMO form. For ease of discussion, the i^{th} row and j^{th} column element in $\boldsymbol{G}(s)$ and $\boldsymbol{G}_{dd}(s)$ is denoted by $G^{ij}(s)$ and $G^{ij}_{dd}(s)$, respectively. As shown in Eq. (10), for a given $\boldsymbol{P}(s)$, the effects from unknown inputs can be effectively suppressed by minimising $\| G^{ij}(s) \|_\infty$ $(i = 1, \cdots, n_y;\ j = 1, \cdots, n_u)$, and this can be achieved by a proper design of $\boldsymbol{G}_{yd}(s)$. In other words, if a $\boldsymbol{G}_{yd}(s)$ exists so that $\| G^{ij}(s) \|_\infty$ is sufficiently small, then estimates of the equivalent unknown inputs can be obtained with adequate accuracy. Feeding these estimates back into the system can effectively counteract the actual unknown inputs, alleviating disturbances. According to Eq. (9), this requires appropriate design of $\boldsymbol{G}_{dd}(s)$ and $\boldsymbol{K}_d$.

Specifically, for any positive scalars $\varepsilon_1 \ll 1$, $\varepsilon_2 \ll 1$, and $\varepsilon_3 \ll 1$, with a properly determined $\boldsymbol{K}_d \neq 0$ assumed, if

$$1 - \left|G^{ii}_{dd}(s)\right| \le \varepsilon_1; \quad (i = 1, \cdots, n_u), \tag{11}$$

and
$$\sum_{j=0}^{n_u}\left|G^{ij}_{dd}(s)\right| - \left|G^{ii}_{dd}(s)\right| \le \varepsilon_2; \quad (i, j = 1, \cdots, n_u), \tag{12}$$

then from Eqs. (9) and (10),

$$\| G^{ij}(s) \|_\infty \le \varepsilon_3; \quad (i = 1, \cdots, n_y;\ j = 1, \cdots, n_u),$$

and as a result, $\boldsymbol{y}(t) \approx 0$ in Eq. (10) for $t \to \infty$.

To avoid confusion, define $\Delta\boldsymbol{y}(s)$ as the error between the disturbed and undisturbed outputs. That is, $\boldsymbol{y}(t) \approx 0$ in Eq. (10) means $\Delta\boldsymbol{y}(s) = \boldsymbol{P}_n(s)[\boldsymbol{d}_e(s) - \hat{\boldsymbol{d}}_e(s)] \approx 0$, which leads to $\hat{\boldsymbol{d}}_e(s) \approx \boldsymbol{d}_e(s)$. According to Eq. (5), it is also true that $\hat{\boldsymbol{d}}_{ev}(t) \approx \boldsymbol{d}_e(t)$ at steady state.

With a properly designed $\boldsymbol{G}_{dd}(s)$ satisfying Eqs. (11) and (12), the speed of estimation can be further fine tuned via adjustment of $\boldsymbol{K}_d$, as indicated by Eq. (9).

2.4 UIE Design

Selection of State Observer Gain $\boldsymbol{L}$

According to Eqs. (7) and (10) as well as the state estimation principle, there is limited contribution from $\boldsymbol{G}_{yy}(s)$ in minimising $\| G^{ij}(s) \|_\infty$ for a given $\boldsymbol{G}_{yd}(s)$. Therefore, the state observer gain $\boldsymbol{L}$ can be designed normally as the one for an ordinary observer.

When a system is subject to unknown inputs that are non-Gaussian and vary dynamically, an ordinary state observer assuming disturbances as white noises can have considerable estimation error. However, with the presence of the proposed UIE, these non-Gaussian and time-varying unknown inputs can be mostly cancelled, and the Kalman filter technique can be employed to compute $\boldsymbol{L}$.

Remark 1: The dynamics of state estimation is given by

$$\dot{\boldsymbol{e}}_x(t) = (\boldsymbol{A} - \boldsymbol{LC})\boldsymbol{e}_x(t) + \boldsymbol{B}\boldsymbol{\xi}(t), \tag{13}$$

where $\boldsymbol{\xi}(t) = \boldsymbol{d}_e(t) - \hat{\boldsymbol{d}}_e(t)$. As shown in Eq. (13), state estimation of adequate accuracy is ensured by the presence of the UIE, with estimation errors subjected only to the residual equivalent unknown inputs $\boldsymbol{\xi}(t) \approx 0$ as $t \to \infty$.

Selection of $\boldsymbol{G}_{dd}(s)$ or $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$

As discussed in Section 2.3, the design of $\boldsymbol{G}_{dd}(s)$ should satisfy the conditions as in Eqs. (11) and (12) for effective unknown-input estimation and cancellation. However, it is known that $\| G^{ij}(s) \|_\infty$ $(i = 1, \cdots, n_y;\, j = 1, \cdots, n_d)$ cannot be minimised ideally throughout $\omega \in [0, \infty)$. For this reason, an assumption is made that the natural frequency of the unknown inputs to be suppressed are below ω_c. This holds true for a majority of servo systems as most disturbances are from low-frequency sources. Therefore, it is viable to introduce $\boldsymbol{G}_{dd}(s)$ in the form of a low-past-filter-characterised subsystem so that $\max\limits_{\omega \in \Omega} |\boldsymbol{G}(j\omega)|$ is minimised for effective estimation and suppression of unknown inputs of frequency within $\Omega \triangleq (0, \omega_c)$ instead. Noises of higher frequency are filtered.

For simplicity, decoupling of multiple unknown inputs is preferred, and thus $\boldsymbol{G}_{dd}(s)$ can be constructed directly as a diagonal matrix:

$$\boldsymbol{G}_{dd}(j\omega) = \operatorname{diag}\left\{ G_{dd}^{11}(j\omega_{11}), \cdots, G_{dd}^{ii}(j\omega_{ii}) \right\}, \tag{14}$$

where $\omega_{ii} \in \boldsymbol{\Omega}_{ii}$, with $\boldsymbol{\Omega}_{ii} \subseteq \boldsymbol{\Omega}$ and $i = 1, \cdots, n_u$.

In this paper, a 1st-order filter is used for illustration purpose given its least phase lag and simplest structure:

$$G_{dd}^{ii}(s) = \frac{1}{\dfrac{1}{\omega_{cii}} s + 1} = \frac{1}{\tau_{ii} s + 1}, \tag{15}$$

where ω_{cii} is the cut-off frequency of the i^{th} diagonal element and τ_{ii} is the corresponding time constant.

The state-space representation $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$ can be obtained through an appropriate transform from $\boldsymbol{G}_{dd}(s)$.

Remark 2: It is worth emphasis that the design of the subsystem $\boldsymbol{G}_{dd}(s)$ or $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$ is not limited to the low-pass-filter-characterised structure discussed herein, but flexible and can vary for situations other than the industrial servo systems commonly seen. The most suitable configuration of the subsystem depends on the individual applications under consideration, and the design can simply follow the guidelines provided in this paper.

Selection of $\boldsymbol{K}_d$

From Eqs. (9) and (10), $\boldsymbol{K}_d$ of a higher matrix norm contributes to smaller $|G^{ij}(j\omega)|$ for frequencies within $(0, \omega_c)$. However, it is impractical to have infinitely large $\|\boldsymbol{K}_d\|$, which can amplify noises and reduce damping. For a balanced design of $\boldsymbol{K}_d$, the linear-quadratic-regulator (LQR) computation can be performed on the reconstructed plant dynamics

$$\begin{cases} \dot{\boldsymbol{e}}_x(t) = (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\boldsymbol{e}_x(t) + \boldsymbol{B}\boldsymbol{e}_{dv}(t) \\ \boldsymbol{e}_y(t) = \boldsymbol{C}\boldsymbol{e}_x(t) \end{cases}, \tag{16}$$

where

$$\boldsymbol{e}_x(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t),$$

$$\boldsymbol{e}_y(t) = \hat{\boldsymbol{y}}(t) - \boldsymbol{y}(t),$$

and

$$\boldsymbol{e}_{dv}(t) = -\boldsymbol{K}_d \boldsymbol{C}\boldsymbol{e}_x(t).$$

According to duality, this is to select $\boldsymbol{K}_v = (\boldsymbol{B}\boldsymbol{K}_d)^{\mathrm{T}}$ that minimises the performance index

$$J_d = \int_0^\infty \left\{ \boldsymbol{e}_x^{\mathrm{T}}(t)\boldsymbol{Q}_d\boldsymbol{e}_x(t) + \boldsymbol{e}_{dv}^{\mathrm{T}}(t)\boldsymbol{R}_d\boldsymbol{e}_{dv}(t) \right\} dt \tag{17}$$

with symmetric positive-definite weighting $\boldsymbol{Q}_d$ and $\boldsymbol{R}_d$ of respective size $n_x \times n_x$ and $n_u \times n_u$.

As a result,

$$\boldsymbol{K}_d = \boldsymbol{B}^{\dagger}\boldsymbol{K}_v^{\mathrm{T}}, \tag{18}$$

where $\boldsymbol{B}^{\dagger}$ is the Moore-Penrose pseudo inverse of $\boldsymbol{B}$.

Remark 3: Unlike the estimator of She et al. [2008], some relative independence in the state observer design is retained in our approach although dynamics of the state observer are associated with the UIE. The introduction of an additional parameter $\boldsymbol{K}_d$ in our method gives a third degree of freedom in the UIE design, and hence the performance of the UIE is not merely dependent on the state observer gain and the low-pass-filter-characterised subsystem. As a result, less constraint is imposed on the state observer design. This feature not only offers more flexibility in UIE tuning, but also plays an important role when the proposed UIE is integrated with an existing controller designed in state space such as a standard linear-quadratic-Gaussian (LQG) tracking controller. To be specific, such flexibility allows independent and separate design of the proposed UIE as an add-on to an existing state-space controller for enhanced robustness. These two components can share the same state observer without imposing additional constraints on the state observer design (The integration of the proposed UIE into an existing controller is not the focus of the current study).

3 Simulations

In this section, the performance of the proposed new UIE is evaluated via simulations. For comparison, a DOB specifically generalised for MIMO systems [Shahruz, 2009] is selected as a benchmark. This DOB takes the form of a conventional DOB, with inherited reliability. Moreover, the H_∞ optimisation technique is applied in design, which gives additional credits to such a DOB for its claimed superior performance.

A mass-spring-damper system of two degrees of freedom as in Figure 3 with two control inputs $\boldsymbol{u} = [u_1, u_2]^{\mathrm{T}}$ and two measured outputs $\boldsymbol{q} = [q_1, q_2]^{\mathrm{T}}$ is considered [Shahruz, 2009]. Exogenous unknown inputs are not shown in the figure, because no restrictions are imposed on the number and distribution of these disturbances.

The corresponding equation of motion is given by

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{R}\dot{\boldsymbol{q}}(t) + \boldsymbol{S}\boldsymbol{q}(t) = \tilde{\boldsymbol{B}}\boldsymbol{u}(t) + \tilde{\boldsymbol{B}}_d\boldsymbol{u}_d(t), \tag{19}$$

where the mass, damping, and stiffness matrices are

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \boldsymbol{R} = \begin{bmatrix} 0.4 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}, \text{and } \boldsymbol{S} = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix},$$

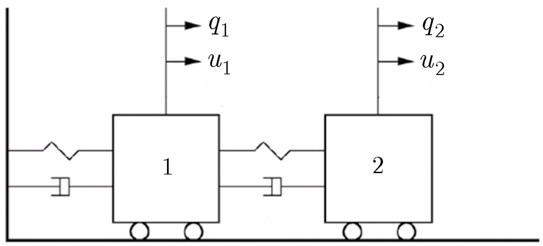

Figure 1: A mass-spring-damper system of two degrees of freedom with two control inputs (image adapted from [Shahruz, 2009])

and the control input distribution matrix $\tilde{\boldsymbol{B}}$ is fixed as

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

while the distribution of unknown inputs $\tilde{\boldsymbol{B}}_d$ is not determined herein as different assumptions are to be made in simulations that follow.

Let $\boldsymbol{x} = [q_1, q_2, \dot{q}_1, \dot{q}_2]^{\mathrm{T}}$. Then a state-space representation for Eq. (19) is obtained as in Eq. (1). Throughout simulations, the proposed UIE takes the following design:

$$\boldsymbol{K}_d = \begin{bmatrix} 1950 & 217.8 \\ 217.8 & 2603.3 \end{bmatrix},$$

$$\boldsymbol{L} = \begin{bmatrix} 316.8982 & 0.0792 \\ 0.0792 & 317.1327 \\ 212.2322 & 25.1145 \\ 25.1145 & 286.5825 \end{bmatrix},$$

and

$$\boldsymbol{A}_f = \begin{bmatrix} -100 & 0 \\ 0 & -100 \end{bmatrix}, \boldsymbol{B}_f = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \text{and} \, \boldsymbol{C}_f = \begin{bmatrix} 12.5 & 0 \\ 0 & 12.5 \end{bmatrix}$$

for subsystem $(\boldsymbol{A}_f, \boldsymbol{B}_f, \boldsymbol{C}_f)$, equivalent to a two-channel low-pass filter of 1st order as in Eqs. (14) and (15) with cut-off frequency $\omega_{c11} = \omega_{c22} = 100$ rad/s.

Note that the DOB of Shahruz [2009] is synthesised in frequency domain, with corresponding closed-loop transfer function given by

$$\boldsymbol{y}(s) = [\boldsymbol{P}(s) - \boldsymbol{P}(s)\boldsymbol{K}(s)\boldsymbol{P}(s)]\boldsymbol{d}_e(s), \qquad (20)$$

and the parameter $\boldsymbol{K}(s)$ is optimised as

$$\boldsymbol{K}(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix},$$

where

$$K_{11}(s) = \frac{9999.6474 \times (s^2 + 0.3932228 \times s + 3.0179556)}{s^2 + 210.0506 \times s + 9946.5299},$$

$$K_{12}(s) = \frac{-1007.9167 \times (s + 9.9221867)}{s^2 + 198.2221 \times s + 9988.6870},$$

$$K_{21}(s) = \frac{-1007.9167 \times (s + 9.9221867)}{s^2 + 198.2221 \times s + 9988.6870}, \text{ and}$$

$$K_{22}(s) = \frac{9993.3611 \times (s^2 + 0.1021055 \times s + 0.9928496)}{s^2 + 195.0442 \times s + 9943.9212}.$$

The control effort provided by the DOB is the same as in Eq. (3), but with $\hat{\boldsymbol{d}}_e$ obtained via

$$\hat{\boldsymbol{d}}_e(s) = \boldsymbol{K}(s)[\boldsymbol{P}_a(s)\boldsymbol{d}(s) - \boldsymbol{P}(s)\boldsymbol{u}(s)],$$

where $\boldsymbol{P}_a(s)$ denotes the actual dynamics of the plant.

In the following simulations, two scenarios are considered.

Case I:

Exogenous unknown inputs are to be estimated under the condition that $\tilde{\boldsymbol{B}} = \tilde{\boldsymbol{B}}_d$. This allows direct evaluation on the performance of unknown-input estimation by comparing the disturbances with corresponding control efforts according to Eq. (3).

The disturbances are simulated as:

$$d_1(t) = \sin(2t),$$

and $$d_2(t) = a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t),$$

where $$\begin{cases} a_1 = 2\sqrt{t}, \\ \omega_1 = 0.1\cos(\pi t/5), \\ a_2 = 2\cos(\pi t/5), \\ \omega_2 = 0.25t + 1. \end{cases}$$

It is found in simulation that both the UIE and DOB can effectively estimate the simulated unknown inputs with high accuracy, and the associated estimation errors can be barely identified directly from plots with $\boldsymbol{d}$ and $\hat{\boldsymbol{d}}_e$, as in Figures 2 and 3.

The estimation performance can be better revealed by the corresponding estimation errors $\boldsymbol{e}_d = \boldsymbol{d} - \hat{\boldsymbol{d}}_e = \boldsymbol{d} + \boldsymbol{u}$. As shown in Figures 4 and 5, both the UIE and DOB produce very small estimation errors. It is also notable that the UIE has much smaller estimation errors than the DOB.

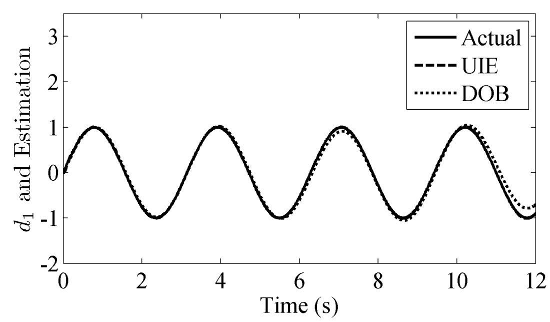

Figure 2: Disturbance d_1 and estimations

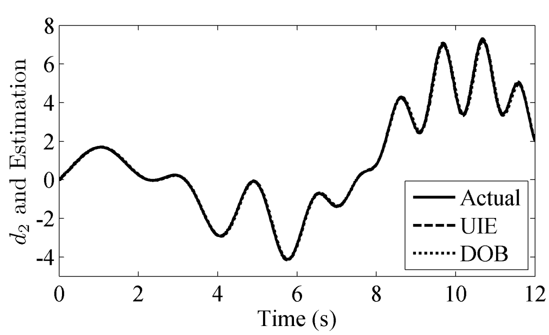

Figure 3: Disturbance d_2 and estimations

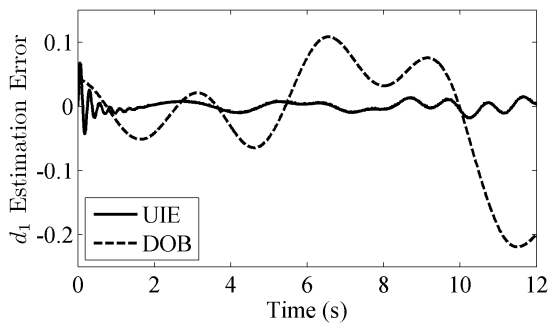

Figure 4: Estimation error of disturbance d_1

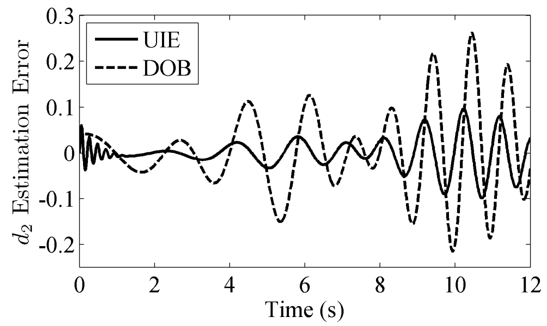

Figure 5: Estimation error of disturbance d_2

Case II:

As mentioned in Section 2, it is expected that exogenous unknown inputs can also be effectively cancelled under the situation of $\tilde{\boldsymbol{B}} \neq \tilde{\boldsymbol{B}}_d$ and $n_d > n_u$. In this case, the performance of estimation and compensation can be evaluated by observing the system outputs. That is, $\boldsymbol{y}(t) \approx 0$ when $t \to \infty$ indicates satisfactory results.

Specifically,

$$\tilde{\boldsymbol{B}}_d = \begin{bmatrix} 1 & 0.5 & 0.75 \\ 0.5 & 1 & 0.75 \end{bmatrix} \text{ and } \begin{cases} d_1(t) = \sin(2t) \\ d_2(t) = 0.5\sin(3t) \\ d_3(t) = 2\sin(t) \end{cases}.$$

Without the UIE and DOB, the outputs $\boldsymbol{y}$ of the system oscillate into large amplitude under disturbances (see Figure 6). The oscillation can be effectively suppressed, however, by the UIE or DOB, and notably, the UIE outperforms the DOB, as shown in Figures 7 and 8. It is worth emphasis that the H_∞ optimisation method is performed for the DOB design, which can be verified by the satisfactory disturbance suppression results of the DOB. In comparison, the much better disturbance compensation achieved by the UIE demonstrates a notable advantage of the proposed UIE over the optimised DOB.

4 Conclusion

In this paper, a novel UIE capable of effective estimation and compensation of completely unknown inputs for MIMO systems is proposed. The new UIE is exempted from the various limitations of existing unknown-input estimation

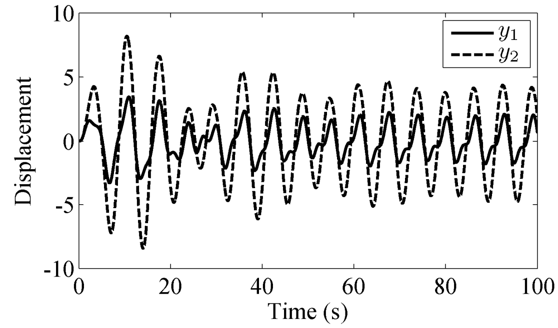

Figure 6: Open-loop system outputs under disturbances

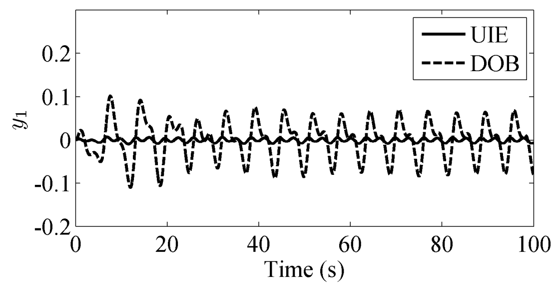

Figure 7: Closed-loop system outputs under disturbances

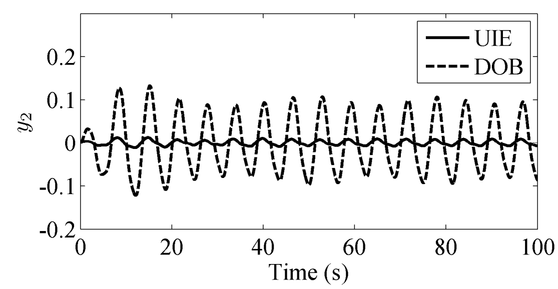

Figure 8: Closed-loop system outputs under disturbances

schemes while superior performance is maintained. Comparison through simulation is drawn between the proposed UIE and an optimised benchmark DOB on an MIMO mass-spring-damper system, and it is shown that the new UIE outperforms the DOB. Further analysis on characteristics of the UIE and its integration into existing state-space servo controllers will be addressed in the future.

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