

# Decentralized Control of Mobile Robotic Sensors for a Smooth Sweep Coverage along an Arbitrary Boundary

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## Abstract

This paper addresses the decentralized control of mobile robotic sensors performing sweep coverage along an arbitrary boundary. Current sweep coverage control does not exhibit a smooth trajectory for a mobile robotic sensor when following an abrupt change in the direction of an arbitrary boundary. Simulation results of the recent sweep coverage problem reveal that the control needs to be more sensitive for some of the mobile robotic sensor(s) after the detection of an arbitrary boundary. The goal is to attain a smooth sweep coverage along an arbitrary boundary by a swarm of mobile robots which are driven under decentralised control strategy. We introduce normalised weighting factors as part of the mobile robotic sensors consensus to calculate nearest neighbour rules. Current simulation results demonstrate that the weighted average functions to calculate nearest neighbour rules give rise to a smooth sweeping trajectory for each mobile robot detecting an arbitrary boundary. The concept of these weighted average functions also make some of the mobile robotic sensor(s) more sensitive in following an abrupt change in the direction of an arbitrary boundary.

## 1 Introduction

Decentralized coordination of groups of mobile sensors is an active area of research in robotics<sup>1</sup>. In this paper, vision based sensors are considered, which provide the position of neighbouring robotic sensors together with adjacent obstacle boundaries. Possible applications include surveillance, reconnaissance, maintenance, inspection and training [Gage, 1992; 1995]. A swarm of mobile robotic sensors is basically inspired from flocks of birds and schools of fish, who or-

<sup>1</sup>The terms, "sensor" or "the mobile robotic sensor" or simply, "the mobile robot" will be used throughout this paper for mobile robotic sensor having an on-board computation, boundary detection and communication capability.

ganise themselves with mutual coordination [Reynolds, 1987; Savkin, 2004].

Gage has classified the coverage of the mobile robots into three basic patterns: Blanket Coverage, Barrier Coverage and Sweep Coverage [Gage, 1992]<sup>2</sup>. We can define these coverage patterns as follows [Gage, 1992]:

- Blanket Coverage occurs when the mobile robots form a static arrangement to cover a region such that any intruding objects has probability of being detected for maximum number of times.
- Barrier Coverage occurs when the mobile robots arrange themselves in the form of a static barrier, which minimizes the probability of undetected intruders passing through it. Fig. 1(ii) is a good example of a Barrier Coverage demonstrated by five mobile robotic sensors, each one being represented by a black arrow as its heading angle and a red circle as its sensing range. Fig. 1(ii) shows a virtual boundary moving mobile robot with a yellow arrow as its heading angle. The purpose of a virtual mobile robot is to share the information of the boundary with a neighbour mobile robot(s).
- If the formed static barrier starts moving such that every mobile robot inside it maintains an equal distance from its neighbour(s) and gains the same speed as that of its neighbours, then this sort of collective behaviour is known as the Sweep Coverage.

Many researchers ([Cheng and Savkin, 2009; 2009; 2010; 2010; 2012; 2013; Cheng *et al.*, 2011; Savkin *et al.*, 2012]) have developed decentralised control algorithms based on Gage's classification ([Gage, 1992]) for coverage problems.

Our main objective is to develop an improved decentralized control, which could smoothly drive the randomly placed mobile robots (shown in Fig. 1(i)), to form a sweep coverage and to also sweep smoothly across an arbitrary boundary (as shown in Fig. 1(ii)). Sweep coverage has potential applications for minesweepers [Acar *et al.*, 2003;

<sup>2</sup>The terms, "sweep coverage" or simply "sweeping of the mobile robots" will be used throughout this paper to address a sweep coverage problem.

Cassinis *et al.*, 1999], patrolling borders [Kumar *et al.*, 2007], environment monitoring of the deep ocean floor [Jeremi' c and Nehorai, 1998] and underwater oil exploration [Brhaug *et al.*, 2007]. While centralised systems can also be used to accomplish the aforementioned tasks, there is high complexity associated with each mobile robot's communication with a central system. A decentralised control strategy for the mobile robots to accomplish a sweep coverage is considered a cost effective solution.

The coverage path planning problem has been addressed by a number of researchers where the environment is assumed to be known [Choset, 2001; Kurabayashi *et al.*, 1996; Garcia *et al.*, 2004]. Other research has achieved path planning by introducing autonomous and cooperative behaviour within the algorithm [Min and Yin, 2010; Butler *et al.*, 2001].

An alternative to path planning uses simple average functions to provide a fully decentralised sweep coverage along an arbitrary boundary [Cheng *et al.*, 2011]. Our current research extends this idea to ensure a smooth and an efficient movement especially when there is an abrupt change required in the direction of the sweeping mobile robots. In order to make our sweep coverage smooth and sensitive to an abrupt change in the direction of an arbitrary boundary, we introduce weighted average functions for the calculation of nearest neighbour rules. Our approach broadens nearest neighbour rules specifically for the heading angle and velocity of the mobile robots sweeping across an arbitrary boundary. A fully decentralised control strategy with weighted average nearest neighbour rules has been adopted throughout the control design. Simulation results with weighted average functions are presented in order to compare the improvement on recent work [Cheng *et al.*, 2011].

## 2 Problem Statement

The sweep coverage problem ([Cheng *et al.*, 2011]) may be stated as follows:

Let  $s_i(kT)$  be a position and  $\theta_i(kT)$  be a heading angle of the mobile robotic sensor measured counter clockwise from the  $X$ -axis. Let a mobile robot  $i$  has a linear velocity  $v_i(kT)$ .

Let a unit vector function  $u(\gamma)$  be such that for any  $\gamma \in [-\pi/2, \pi/2)$  measured with respect to the  $X$ -axis. It follows that:

$$u(\gamma) = [\cos(\gamma) \quad \sin(\gamma)]^T \quad (1)$$

Let  $B$  be a boundary line with a direction  $\phi_b$ , and it has an associated scalar  $b_1$ . It can be mathematically defined as follows:

$$B := \{p \in \mathbb{R}^2 : u^T p = b_1\} \quad (2)$$

$$\phi_b := \gamma + \pi/2 \quad (3)$$

Let  $B_1$  and  $B_2$  be two sets defined as follows:

$$B_1 := \{p \in \mathbb{R}^2 : u^T p > b_1\} \quad (4)$$

$$B_2 := \{p \in \mathbb{R}^2 : u^T p < b_1\} \quad (5)$$

Next, we define a below mentioned moving line  $L_0(kT)$  with points  $p_i$  (as shown in Fig. 1(ii)) for  $i = 1, 2, \dots, n$ :

$$L_0(kT) := \{p \in \mathbb{R}^2 : p^T u = F_0 + kT v_0\} \quad (6)$$

for  $k = 0, 1, 2, \dots$ , where  $F_0$  is a scalar and  $v_0$  is the desired sweeping speed. The desired points denoted by  $p_i$  (as shown in Fig. 1(ii)) on  $L_0(kT)$  can be mathematically described as follows:

$$p_i(kT) := \begin{cases} p_b(kT) + (d \times i)u & \text{if } \mathcal{P} \subset B_1 \\ p_b(kT) - (d \times i)u & \text{if } \mathcal{P} \subset B_2 \end{cases} \quad (7)$$

for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$ , where  $p_b(kT) := L_0(kT) \cap B$  and  $\mathcal{P} \in \mathbb{R}^2$  is a bounded set with Lebesgue measure for initial headings of the mobile robotic sensors.

The discrete-time position,  $s_i(kT) \in \mathbb{R}^2$ , of the mobile robotic sensor with control inputs, velocity  $v_i$  and heading angle  $\theta_i$ , can be written as follows:

$$s_i((k+1)T) = s_i(kT) + T v_i(kT) u(\theta_i(kT)) \quad (8)$$

for  $i = 1, 2, \dots, n$ , and  $k = 0, 1, 2, \dots$ .

There are some physical constraints associated with the mobile robot, that is,  $|v_i(t)| \leq v_{\max}$  for  $i = 1, 2, \dots, n$  and  $t \geq 0$ . The initial heading of each mobile robotic sensor meets the condition:  $\theta_i(0) \in [0, \pi)$  for  $i = 1, 2, \dots, n$ .

Mathematically, a control law is said to be a sweep coverage with sweeping speed  $v_0$  along boundary  $B$  for  $n$  mobile robotic sensors maintaining mutual distance  $d$  if for almost all initial positions of the mobile robotic sensors, there exists permutation  $c_1, c_2, \dots, c_n$  of set  $1, 2, \dots, n$  such that the following condition is satisfied:

$$\lim_{k \rightarrow \infty} \|s_{c_i}(kT) - p_i(kT)\| = 0, i = 1, 2, 3, \dots, n. \quad (9)$$

Hence, the main objective is to develop a decentralized control law to sweep the mobile robots (meeting assumptions) smoothly along an arbitrary boundary  $B$ . The control action should produce a smooth trajectory for each mobile robot especially facing a sudden change in the direction of an arbitrary boundary. The control action should adjust a weighted average movement (not just an equally averaged movement) for each mobile robotic sensor after the detection of an arbitrary boundary, which is an improvement on recent research [Cheng *et al.*, 2011].

## 2.1 Assumptions

The following assumptions [Cheng *et al.*, 2011] are to be made during control development:

**Assumption 2.1** Define a disk of radius  $R_c > 0$  for  $t \in [kT, (k+1)T)$  and  $k = 0, 1, 2, \dots$  as follows:

$$C_{i,R_c}(kT) := \{p \in \mathbb{R}^2 : \|p - s_i(kT)\| \leq R_c\} \quad (10)$$

where  $\|\cdot\|$  denotes the Euclidean norm. So, any mobile robotic sensor with a disk of radius  $R_c$  has the capability to communicate with its neighbour(s) within this range, as long as the condition,  $R_c < v_{max}T/\sqrt{2}$ , remains valid for it.

**Assumption 2.2** A mobile robotic sensor can detect any operation specific target within a disk of radius  $R_s > 0$  for  $t \in [kT, (k+1)T)$  and  $k = 0, 1, 2, \dots$  defined as follows:

$$S_{i,R_s}(kT) := \{p \in \mathbb{R}^2 : \|p - s_i(kT)\| \leq R_s\} \quad (11)$$

where  $\|\cdot\|$  denotes the Euclidean norm.

Here, we also assume that any mobile robotic sensor  $i$  can detect and find the slope of an arbitrary boundary  $B$  within a range  $R_b$  such that the condition,  $R_b/R_c > \sqrt{2}$ , holds.

**Assumption 2.3** There are  $n$  number of the mobile robots satisfying the below mentioned condition to form a sensor barrier of length  $D$  from boundary  $B$ .

$$(n+1)R_s > D \quad (12)$$

for  $R_s \in (0, R_c)$ .

**Assumption 2.4** The physical parameters (position, heading angle and the desired sweeping speed) of the mobile robotic sensor have the constraints as follows:

The initial randomly placed mobile robotic sensors come in a bounded set  $\mathcal{P} \in \mathbb{R}^2$  with Lebesgue measure. Initially, each randomly placed mobile robotic sensor satisfies  $\theta_i(0) \in [0, \pi)$  for  $i = 1, 2, \dots, n$ . The desired sweeping speed satisfies the condition as follows:

$$0 < |v_0| \leq 1/T \min\{(v_{max}T - R_c\sqrt{2}), (R_b - R_c\sqrt{2})\} \quad (13)$$

**Assumption 2.5** There exists an infinite sequence of contiguous, non-empty, bounded, time intervals  $[k_i, k_{i+1}]$  for  $i = 0, 1, 2, \dots, n$  and  $k_0 = 0$ , such that for all  $[k_i, k_{i+1}]$  the graph from the union of the collection  $G(kT) \in \mathcal{P}$  for  $kT \in [k_i, k_{i+1}]$  is connected.

## 3 Nearest Neighbour Rules with Weighted Average Functions

We mathematically amend the variables for the nearest neighbour rules of recent sweep coverage problem [Cheng *et al.*, 2011]. We define the weighted average functions (which replaces simple average functions as used by [Cheng and Savkin, 2009; 2009; 2010; 2010; 2012; 2013; Cheng *et al.*, 2011; Savkin *et al.*, 2012; Savkin, 2004; Savkin and Teimoori, 2010]) for the decentralised coordinated control.

We consider the very first sensor detecting the slope of a boundary switches its heading angle by considering a weighting factor. Similarly, each mobile robot inside the rest of the barrier also gets a weighted motivation to adjust its heading angle. Formally, we first define a weighted average variable for the heading angle update of the mobile robot:

$$\Theta_i(kT) := w_i\phi_i(kT) + \sum_{j \in \mathcal{F}_i(kT)} w_j\phi_j(kT) \quad (14)$$

for  $i = 1, 2, 3, \dots, n$ , where  $\mathcal{F}_i(kT)$  denotes the set of neighbors of the mobile robotic sensor  $i$  at a particular time  $kT$ . We have defined a non-negative weighting factor,  $w_i$ , for the mobile robotic sensor itself and a non-negative weighting factor,  $w_j$ , for its neighbour(s) at a particular time ( $kT$ ) such that the following condition is satisfied.

$$w_i + \sum_{j \in \mathcal{F}_i(kT)} w_j = 1 \quad (15)$$

The above equation means that we have used normalised weighting factors for a mobile robot to decide its heading angle. The mobile robotic sensor decides these weighting factors by considering the highest value for the neighbour mobile robot nearest to the boundary, whereas, the lowest value is given to the neighbour mobile robot farthest from the boundary. Hence, this approach makes the mobile robots nearer to the boundary to be more sensitive in adjusting individual heading angle, and the adjustment of individual heading angle becomes less sensitive for the mobile robots sweeping farther from the boundary. For example, if a mobile robotic sensor (say  $i$ ) nearest to the boundary has two neighbours ( $|\mathcal{F}_i(kT)| = 2$ ), then  $w_i = 2/6$ ,  $w_j = 1/6$  for the neighbour farthest from the boundary and  $w_j = 3/6$  for the neighbour (virtual mobile robotic sensor) based on the boundary. Similarly, if this mobile robotic sensor has just one neighbour, the weighting factors will be  $w_i = 1/3$  for the mobile robot itself and  $w_j = 2/3$  for its neighbour (virtual mobile robotic sensor). However, if any of the mobile robotic sensor has not detected the boundary, then the control law [Cheng *et al.*, 2011] does not need any modification for the sweep coverage problem, that is:

$$w_i = w_j \text{ for } B \cap R_b \neq \emptyset. \quad (16)$$

The above condition means an equal weighting factor (that is, for one neighbour:  $w_i = w_j = 1/2$  or for two neighbours:

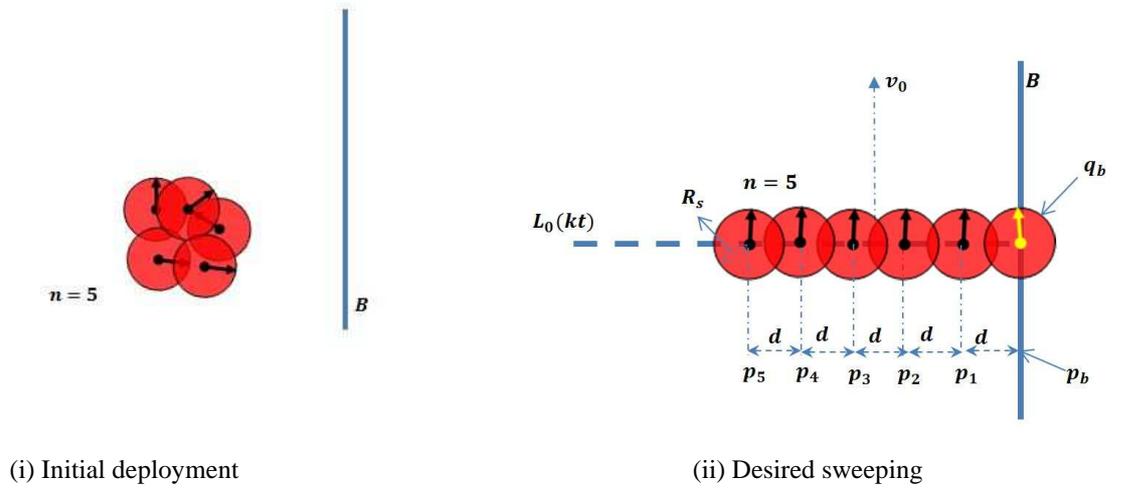


Figure 1: Objective

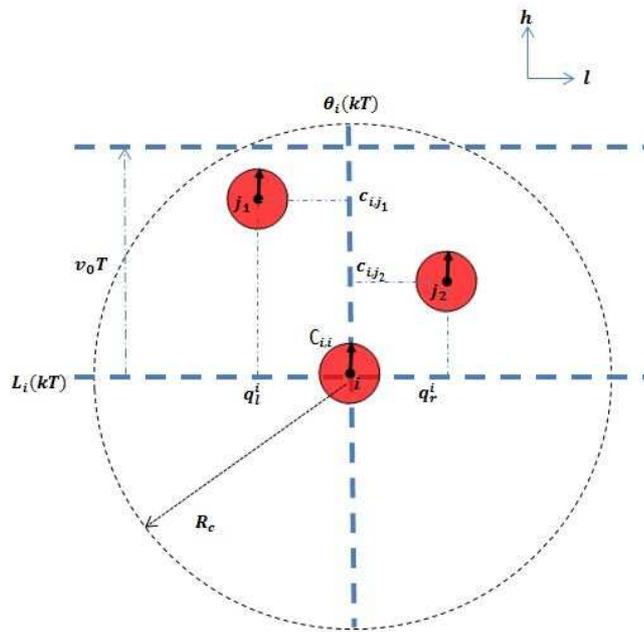


Figure 2: Movement with projection of mobile robots

$w_i = w_{j1} = w_{j2} = 1/3$ ) is given to a mobile robot itself and to its neighbours in the absence of boundary  $B$ . If two or more than two mobile robots detect the boundary at the same time, then the weighting factors with different value can be decided in the desired direction of sweeping.

The heading angle of the mobile robot is our one control input, which is being calculated by normalised weighted average function in a decentralised way.

We define the coordination variable  $\phi_i(kT)$  for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$  to be updated using the following rule:

$$\phi_i((k+1)T) = \Theta_i(kT) \quad (17)$$

By using  $\Theta_i(kT)$ , we define a scalar at time  $kT$  ( $k = 0, 1, 2, \dots$ ) for individual sensor  $i$  itself:

$$c_{i,i}(kT) = s_i^T(kT)u(\Theta_i(kT)) \quad (18)$$

Similarly, we define a scalar for each neighbor  $j$  of mobile robot  $i$ :

$$c_{i,j}(kT) = s_j^T(kT)u(\Theta_i(kT)) \quad (19)$$

Similar to the heading angle of the mobile robot, there is a requirement that its velocity component along the direction of the boundary should also be calculated in a weighted average fashion. So, we again define a non-negative weighting factor  $m_i$  for the mobile robotic sensor itself and a non-negative weighting factor  $m_j$  for its neighbour(s),  $\mathcal{F}_i(kT)$ , such that the following condition is satisfied:

$$m_i + \sum_{j \in \mathcal{F}_i(kT)} m_j = 1 \quad (20)$$

For example, if mobile robotic sensor  $i$  has two neighbours (i.e.,  $|\mathcal{F}_i(kT)| = 2$ ) and it is closest to the boundary, then  $m_i = 2/6$ ,  $m_j = 3/6$  for the neighbour farthest from the boundary and  $m_j = 1/6$  for the neighbour nearest to the boundary (which could be the virtual boundary based mobile robotic sensor). Similarly, if a mobile robotic sensor has just one neighbour, then the weighting factors will be  $m_i = 2/3$  for itself and  $m_j = 1/3$  for its neighbour.

However, it should be noted that:

$$m_i = m_j \text{ for } B \cap R_b \neq \emptyset. \quad (21)$$

The above condition again states that an equal weight (that is, for one neighbour:  $m_i = m_j = 1/2$  or for two neighbours:  $m_i = m_{j1} = m_{j2} = 1/3$ ) is given to mobile robot  $i$  itself and to its neighbours, unless boundary  $B$  is out of boundary detection range  $R_b$  of the mobile robot(s). As mentioned earlier, if two or more than two mobile robots detect boundary  $B$  at the same time, then the weighting factors can be decided in the desired direction of sweeping.

Now, we can amend the scalar variable used in recent control ([Cheng *et al.*, 2011]) to calculate velocity component along the direction of the mobile robot. So, we define this

scalar variable as the weighted average function (not a simple average function) at a particular time ( $kT$ ):

$$\mathcal{C}_i(kT) := m_i c_{i,i}(kT) + \sum_{j \in \mathcal{F}_i(kT)} m_j c_{i,j}(kT) \quad (22)$$

Thus, the update rule for  $c_{i,i}(kT)$  is as follows:

$$c_{i,i}(k+1)T = \mathcal{C}_i(kT) \quad (23)$$

### 3.1 Control Action

By utilising the above mentioned nearest neighbour rules with weighted average functions, we can write the control action of recent research [Cheng *et al.*, 2011] as follows:

The line to be followed by the mobile robot  $i$  can be mathematically defined with the help of  $\Theta_i(kT)$ :

$$L_i(kT) = \{(x, y) \in \mathbb{R}^2 : x \cos(\Theta_i(kT)) + y \sin(\Theta_i(kT)) = \mathcal{C}_i(kT)\} \quad (24)$$

for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$

Let the projection of mobile robot  $i$  and the projection of its neighboring mobile robot  $j$  on  $L_i(kT)$  at time  $kT$  be  $q_i^i(kT)$  and  $q_j^i(kT)$ , respectively:

$$q_j^i(kT) = [\sin(\Theta_i(kT)) - \cos(\Theta_i(kT))]s_j^T(kT) \quad (25)$$

for  $k = 0, 1, 2, \dots$

Similarly, we define  $q_l^i(kT)$  and  $q_r^i(kT)$  be the projections of the neighbouring mobile robots to the left and to the right of mobile robot  $i$  such that the following condition is satisfied:

$$q_l^i(kT) < q_i^i(kT) < q_r^i(kT). \quad (26)$$

In order to keep the mobile robots away from the boundary, we introduce a variable  $q_b^i$ .

First, a variable  $b_i$  is defined:

$$b_i(kT) = L_i(kT) \cap B \quad (27)$$

Then it follows that:

$$q_b^i = [\sin(\Theta_i(kT)) - \cos(\Theta_i(kT))]b_i(kT) \quad (28)$$

Thus we can limit the mobile robots as follows:

$$\begin{aligned} q_r^i(kT) &= q_b^i, \text{ if } q_b^i \leq q_i^i(kT) \text{ and } 0 \notin \mathcal{F}_i(kT) \\ q_l^i(kT) &= q_b^i, \text{ if } q_b^i \geq q_i^i(kT) \text{ and } 0 \notin \mathcal{F}_i(kT) \end{aligned} \quad (29)$$

The coordinates of the mobile robot are updated as follows:

$$\mathcal{Q}_i(kT) := \begin{cases} (q_l^i(kT) + q_r^i(kT))/2 & \text{if } l \text{ and } r \text{ exist} \\ (q_l^i(kT) + q_i^i(kT) + d)/2 & \text{if only } l \text{ exists} \\ (q_r^i(kT) + q_i^i(kT) - d)/2 & \text{if only } r \text{ exists} \\ q_i^i(kT) & \text{if } l \text{ and } r \text{ do not exist} \end{cases} \quad (30)$$

The projection of mobile robotic sensor  $i$  itself on  $L_i(kT)$  is updated as follows:

$$q_i^i(k+1)T = \mathcal{Q}_i(kT) \quad (31)$$

for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$

We can define the velocity components of mobile robot  $i$  along  $L_i(kT)$  and  $\Theta_i(kT)$  as follows:

$$\begin{aligned} \bar{v}_i(kT) &:= (\mathcal{Q}_i(kT) - q_i^i(kT))/T \\ \hat{v}_i(kT) &:= (\mathcal{C}_i(kT) - c_{i,i}(kT) + v_0T)/T \end{aligned} \quad (32)$$

for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$

Hence, a set of control inputs can be written as follows:

$$\begin{aligned} v_i(kT) &= \sqrt{\bar{v}_i(kT)^2 + \hat{v}_i(kT)^2} \\ \theta_i(kT) &= \begin{cases} \Theta_i(kT) + \beta_i(kT) - \pi/2, & \text{if } \hat{v}_i(kT) \geq 0 \\ \Theta_i(kT) - \beta_i(kT) - \pi/2, & \text{if } \hat{v}_i(kT) < 0 \end{cases} \end{aligned} \quad (33)$$

where  $\beta_i(kT) := \cos^{-1}(\bar{v}_i(kT)/v_i(kT))$ , for  $i = 1, 2, \dots, n$  and  $k = 0, 1, 2, \dots$

### 3.2 Theorem

If  $n$  autonomous mobile robots governed by dynamics (8) meeting the assumptions are supposed to sweep across line  $B$  with angle  $\phi_{q_b}(kT)$ , then control law (33) is a decentralised control with sweeping speed  $v_0$  along  $(\phi_{q_b}(kT))$ .

**Proof outline:** As we have only changed the weighted average functions to calculate the nearest neighbour rules, our control action is similar to [Cheng *et al.*, 2011]. So, the step-by-step mathematical arguments have been provided in [Cheng *et al.*, 2011] to achieve the following condition.

$$\lim_{k \rightarrow \infty} \|s_{c_i}(kT) - p_i(kT)\| = 0, i = 1, 2, 3, \dots, n, \quad (34)$$

## 4 Simulation Results

We obtain Figs. 3(i-iv) and Figs. 4(i-iv) for  $n = 6$ ,  $R_c = 2$ ,  $R_b = 2$ ,  $R_s = 2$ ,  $v_0 = 0.05$  and  $\phi_b = \pi/2$ . We represent the moving mobile robot with a blue circle and its heading angle with a red arrow. We divide the simulations to make a clear comparison of the results obtained from recent research [Cheng *et al.*, 2011] with that of the current control law using weighted average functions.

Figs. 5(i-iv) and Figs. 6(i-iv) also show stage-wise simulations for a worst case scenario in order to make a full comparison of recent research [Cheng *et al.*, 2011] with the current work.

## 5 Brief Conclusions and Future Research

We can compare simulation results obtained from recent research [Cheng *et al.*, 2011], Fig. 3(iii, iv) with that of current research, Fig. 4(iii, iv). After detection of the boundary

shown in Fig. 4(iii, iv), the mobile robot nearest to the boundary is making a smooth trajectory, and consequently neighbouring mobile robot is more sensitive in adjusting its velocity and position. Note that, there is an agreement between Fig. 3(ii) and Fig. 4(ii), because our revised nearest neighbour rules give equal weight (i.e.  $w_i = w_j$  and  $m_i = m_j$ ) during calculation of the velocity and the heading angle in the absence of the detected boundary. So, our broadened nearest neighbour rules with weighted average functions can behave well in both manners, that is, an equal normalised weighting value is considered without the detection of a boundary (which has already been demonstrated by recent research [Cheng *et al.*, 2011]) and a normalised but an unequal weighting value for the calculation of control inputs (velocity and heading angle) is given to each mobile robot after the detection of a boundary. Remember that, recent sweep coverage algorithm [Cheng *et al.*, 2011] always gives a normalised equal weighting value to each mobile robot for the calculation of its own control inputs.

The advantage of weighted average functions can be further noticed, when there is a worst boundary angle in the path of slightly inclined (less than 2 degree) sweeping mobile robots. In Fig. 5(iii), we can see the upper mobile robots are unable to make a smooth turn and these mobile robots had to proceed backwards in order to adjust the respective trajectory, but clearly in Fig. 6(iii) all the mobile robots can make a smooth turn in order to adjust individual trajectory against a worst angle boundary coming ahead. We can also see Fig. 5(iv) is not showing a smooth individual trajectory for each mobile robot against a worst angle boundary, whereas Fig. 6(iv) demonstrates a smooth and an efficient trajectory for each mobile robot against the same boundary.

Future work could incorporate the tools for state estimation and control in case of missing data with limited communication [Savkin and Peterson, 1997; Matveev and Savkin, 2004; Savkin, 2006] among the mobile robotic sensors.

## Acknowledgment

This work was supported in part by the Australian Research Council (ARC) with an allocated grant (DP130103898). The main objective of this grant is to conceptually develop new design rules in the area of robust control of mobile networked systems.

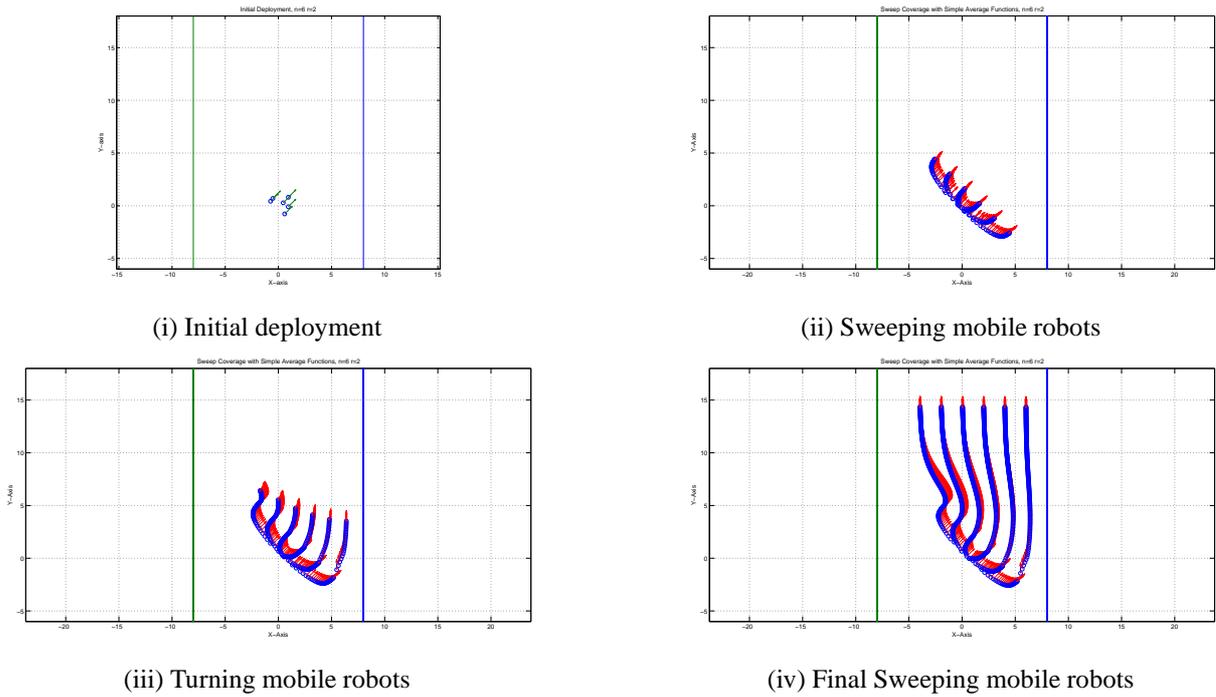


Figure 3: Sweep Coverage with Simple Average Functions

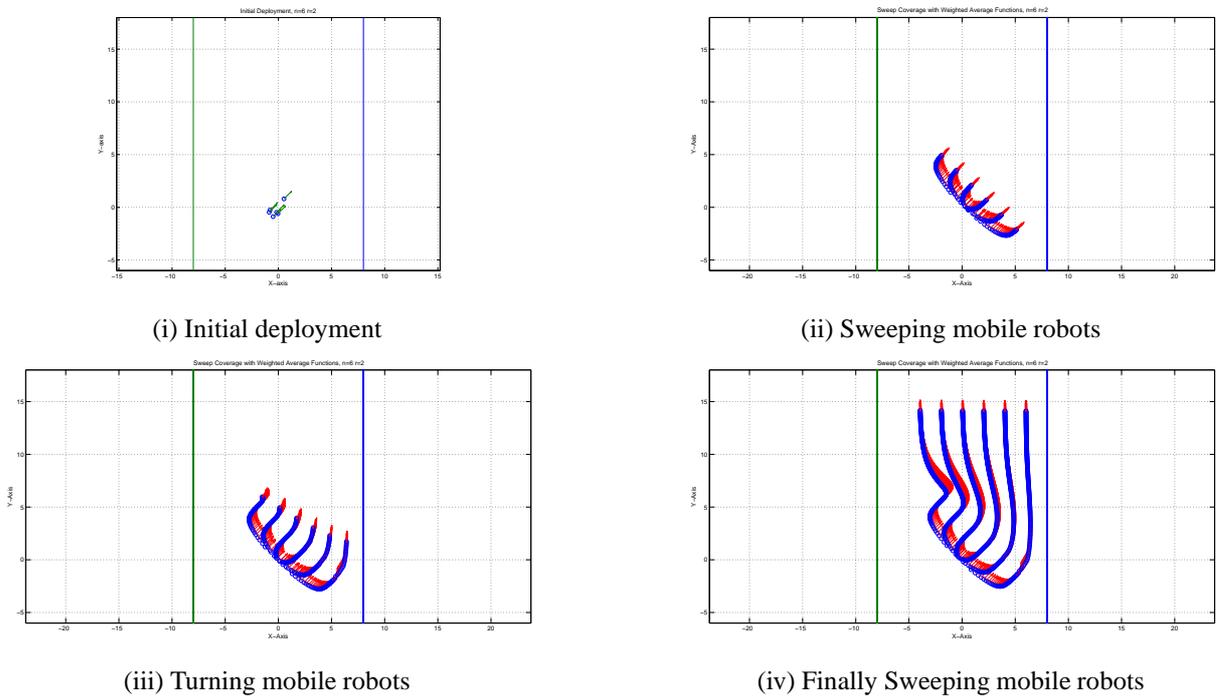


Figure 4: Sweep Coverage with Weighted Average Functions

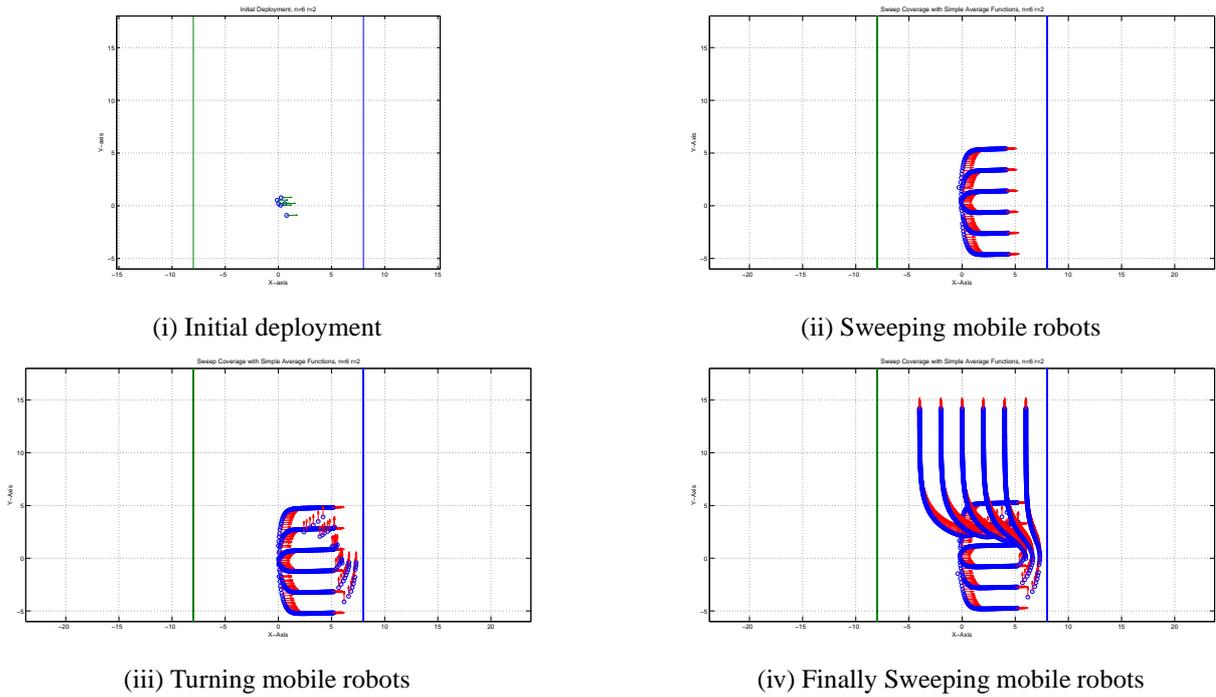


Figure 5: Sweep Coverage with Simple Average Functions for a Worst Boundary Angle

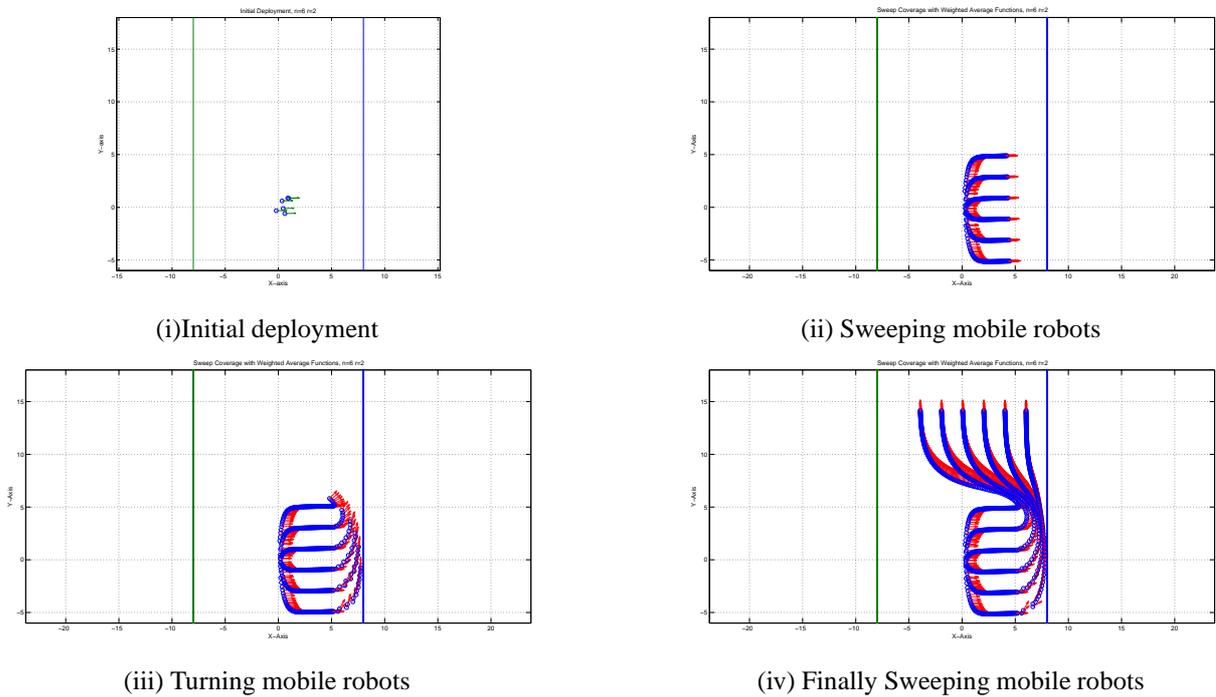


Figure 6: Sweep Coverage with Weighted Average Functions for a Worst Boundary Angle

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