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A Decentralized Control Algorithm for Target Search by a Multi-Robot Team

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Abstract

This paper introduces a novel decentralized random algorithm to search of an unknown area by a multi-robot system. The proposed algorithm applies a triangular grid pattern for the search procedure that guarantees complete search of the whole area. The algorithm is based only on information about the nearest neighbours of each robot. The monitoring region is of an arbitrary shape and not known to the robots a priori. A mathematically rigorous proof of convergence with probability 1 of the proposed algorithm is given and effectiveness of the algorithm is also illustrated via computer simulation.

1 Introduction

Multi-robot systems have recently become one of the most attractive fields of studies for researchers of robotics. Advances in microelectronics, sensors, mobile communications as well as progress in algorithms and networks are the main reasons which have brought researchers to pay attention to multi-agent systems. There are multiple potential applications for multi-agent systems to areas such as search and rescue [Guarnieri *et al.*, 2009], patrolling and building surveillance [Almeida *et al.*, 2004], autonomous sensor networks [Cheng and Savkin, 2011a], [Cheng and Savkin, 2013], home and office assistance, air and underwater pollution monitoring [Ferri *et al.*, 2010], fire detection [Marjovi *et al.*, 2009] and formation systems [Savkin *et al.*, 2013], [Cheng and Savkin, 2011b].

Although, many of these applications can be achieved by single-robot systems, using multi-robot systems has many advantages that cannot be obtained by using a single robot. Compared to single-robot systems, multi-robot systems are more reliable, robust, scalable and flexible [Zhu *et al.*, 2011]. Search and coverage are two close is sues which have been considered in many recent studies. Gage defined three types of multi agent coverage [Gage, 1995], which are blanket coverage [Ghosh and Das, 2008], barrier coverage [Cheng and Savkin, 2012] and sweep coverage [Cheng *et al.*, 2011]. Various algorithms have been suggested to fulfil these tasks, many of them are heuristic methods while there are several methods which have been mathematically proved [Cheng and Savkin, 2009], [Savkin and Javed, 2011]. Dynamic coverage which is very close to the search problem, is a new issue in this area [Liu *et al.*, 2013]. Due to the extent of the area to be searched, it may not be applicable to use a large number of agents to cover the whole area. Instead, employing a dynamic coverage with a few agents can achieve the same task so that the probability of missing the goal can be minimized.

The other subject matter of multi-robot systems is how the robots communicate with each other to share their information while doing their task. While the algorithms based on centralized methods seem to be easier than decentralized ones, there are various matters which restrict their application. The main problem is the lack of the global information for the robots. Therefore, using decentralized algorithms is essential and more applicable in most applications.

The objective of this paper is to propose a method for decentralized control of a multi-robot system with the task to search an unknown region based on a specific pattern. The suggested algorithm is based on ideas from [Savkin *et al.*, 2012]. The proposed algorithm uses a triangular grid pattern i.e. robots certainly visit the vertices of a triangular grid during the search procedure. To accomplish this, a two-stage algorithm is used. In the first stage, robots apply an algorithm according to consensus variables to deploy themselves on the vertices of a common triangular grid which covers the region. In the second stage, they begin to search the area by mov-

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ing between vertices of the triangular grid. They take advantage of a random-based algorithm which ensures complete search of the whole area in the second phase.

Suppose the map of the area is unknown to the robots, It means that, robots eventually complete their maps while searching the area. Furthermore, a robot also shares its map with the others, but only with the robots which are close enough to it. In other words, because of limited communication ranges between robots, It is considered a neighbourhood for any robot that depends on its communication range. Therefore, a robot only communicates with its neighbours which can be altered within search time.

Indeed, the proposed search algorithm uses the advantages of the triangular grid coverage. It is obvious that any triangular grid coverage of any region is a complete blanket coverage. The main advantage of deploying robots in a triangular grid pattern is that it is asymptotically optimal in terms of minimum number of robots required for the complete coverage of an arbitrary bounded set [Kershner, 1939]. Therefore, using the vertices of this triangular grid coverage guarantees complete search of the region.

The triangular grid pattern has been studied in some papers, however, most of them were heuristic algorithm which were only verified by simulation studies without any theoretically proof analysis. Moreover, they often consider the boundaries of the monitoring region have to be known by all the robots a priori. That is a severe constraint in practice. Unlike many other algorithms proposed in this area, this algorithm is theoretically verified. In particular, a mathematically rigorous proof is given for convergence of the algorithm with probability 1 for any initial positions of the mobile robots.

The reminder of the paper is organized as follows. In section 2, the problem is defined and some assumptions are assumed to clarify the subject and the goal. Section 3 presents a novel algorithm to search a region with obstacles, based on a specific pattern. To illustrate the algorithm, computer simulations are presented in section 4. Finally, a brief conclusion is given in section 5.

2 Problem statement

To define the problem, it is helpful takeing advantages of the method of definition in [Savkin and Javed, 2011]. Consider a bounded planar area $\mathcal{R}$, a limited number of obstacles $\mathcal{O}_1$,$\mathcal{O}_2$,... $\mathcal{O}_m$ (see Fig.1). Suppose a multi-robot system which consists of some autonomous mobile robots. The goal is to search the whole area by the multi-robot system in a specific method. The method is a decentralized algorithm that drives the robots to the vertices of a triangular grid consisting of equilateral triangles while avoiding the obstacles. The sides of the triangles r is known to all the robots a priori.

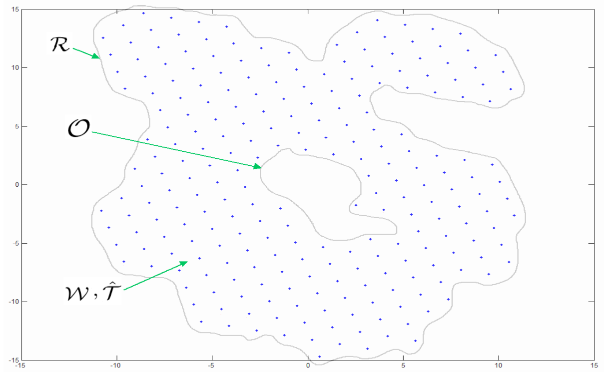


Figure 1: A triangular grid (dotted area) covers the area to be searched

Assume each robot has a communication range $r_c > 0$ and a sensing range $r_s > 0$ where $r_c \geq \sqrt{3}r_s$. This means that each robot can communicate with its surrounding neighbours in the range of r_c at the discrete time instances k, $k = 0, 1, 2, \ldots$, in order to share its information of motion with the other robots. In other words, robot i can gather information of its neighbours that are in a disk of radius r_c defined by $D_{i,r_c}(k) := \{p \in \mathbb{R}^2 : \|p - p_i(k)\| \leq r_c\}$, where $p_i(\cdot) \in \mathbb{R}^2$ denotes the Cartesian coordinates of the robot i and $\|\cdot\|$ denotes the Euclidean norm.

In addition, robot i can gather information of the area $\mathcal{R}$ that is in a disk of radius r_s defined by $D_{i,r_s}(k) := \{p \in \mathbb{R}^2 : \|p - p_i(k)\| \leq r_s\}$. The other assumption is that, the map of the area which is to be searched by the robots is unknown to the robots. Robots distinguish the borders of the area $\mathcal{R}$ and the obstacles $\mathcal{O}_1$,$\mathcal{O}_2$,... $\mathcal{O}_m$ while searching the area and eventually make their maps. The robots share their maps with each other at the discrete time instances k, $k = 0, 1, 2, \ldots$. The multi-robot system under consideration is an example of a networked control system in which information about the environment is obtained via limited communication with each robot's neighbours and robust/extended Kalman state estimation; see e.g. [Matveev and Savkin, 2003], [Matveev and Savkin, 2004], [Savkin, 2006], [Savkin and Petersen, 1998], [Savkin and Petersen, 1997], [Pathirana *et al.*, 2005].

Assumption 2.1: The area $\mathcal{R}$ is bounded, connected and Lebesgue measurable set.

Assumption 2.2: The obstacles $\mathcal{O}_n$ are separated, closed, bounded, and linearly connected sets for any $n \geq 0$.

Definition 2.1: Let $\mathcal{O} := \cup \mathcal{O}_n$ for all $n \geq 0$, then introduce $\mathcal{W} := \{ p \in \mathcal{R} : p \notin \mathcal{O}\}$

Definition 2.2: Consider all possible triangular grids cutting the plane into equilateral triangles with the sides of r. Let $\mathcal{T}$ be the infinite set of all vertices of this grid.

The set $\hat{\mathcal{T}} := \mathcal{T} \cap \mathcal{W}$ is called a triangular grid set in $\mathcal{W}$ (see Fig. 1).

Since the main goal is to search the area $\mathcal{W}$ by the robots in such a way that they certainly go through the vertices of a triangular grid in $\hat{\mathcal{T}}$, in the first stage, all robots should move to some vertices of a common triangular grid. After that, they start searching the area based on the algorithm of the second stage. Since robots may be distributed in the area such that for instance, a robot cannot communicate with all other robots, it is essential to apply decentralized algorithms both in the first and the second stages. Therefore, the algorithms are based on the information which robots share with their neighbours. For the first stage, an algorithm is applied that uses consensus variables to place the robots to the vertices of a triangular grid.

Definition 2.3: Let the neighbours of the robot r_i at discrete time k as $\mathcal{N}_i(k) = \{r_j : p_j \in D_{i,r_c}(k), j \in \{1, 2, \ldots, n\}, j \neq i\}$ where p_j denotes the Cartesian coordinates of the robot j. Also $|\mathcal{N}_i(k)|$ represents the number of elements in $\mathcal{N}_i(k)$.

To describe relationships between robots, using graph theory notation is suitable. Consider a undirected graph $G(k)$ which defines the relationships between robots at time $k \geq 0$. Consider node set of $G(k)$ as $V_G = \{1, 2, \ldots, n\}$ where i in V_G corresponds to robot i. In addition, there is an edge between the nodes i and j of graph $G(k)$ where $i \neq j$, if and only if robot i and robot j are neighbours at time k. To ensure that the connectivity of the graph is guaranteed, following assumption is accepted (see [Jadbabaie *et al.*, 2003] for more details).

Assumption 2.3: There exists an infinite sequence of contiguous, non-empty, bounded, time-intervals $[k_j, k_{j+1})$, $j = 0, 1, 2, \ldots$, starting at $k_0 = 0$, such that across each $[k_j, k_{j+1})$, the union of the collection $\{G(k) : k \in [k_j, k_{j+1})\}$ is a connected graph.

The aim is to develop a decentralized search algorithm for a multi-robot system that will result in the search of an area while robots pass the vertices of a equilateral triangular grid set.

3 Decentralized Search Algorithm

For any angle θ, suppose the vectors $n_1(\theta) := (\cos(\theta), \sin(\theta))$, $n_2(\theta) := (\cos(\theta + \frac{\pi}{3}), \sin(\theta + \frac{\pi}{3}))$. By adding $n_1(\theta) - n_2(\theta)$ to these two vectors, there are three vectors which determine the headings of three lines of a triangular grid. Any point q together with an angle θ uniquely define a triangular grid that q is a vertex of it and θ determines the angle of the grid. Accordingly, for any θ and q, $\hat{\mathcal{T}}[q, \theta]$ can be defined as a unique triangular covering set of $\mathcal{R}$. In addition, $\theta_i(k)$ and $q_i(k)$ are utilized as the consensus variables for each mobile robot i. It means that, the 2-dimensional consensus variable $q_i(k)$ and the scalar consensus variable $\theta_i(k)$ characterize the coordinates of a vertex and the direction of one of the three lines of a triangular grid, respectively. In other words, at any time k, robot i has the estimates $\theta_i(k)$ and $q_i(k)$ of the consensus grid parameters. The robots begin with different values of $\theta_i(0)$ and $q_i(0)$, and finally converge to some consensus values θ_0 and q_0 which define a common triangular grid for all robots.

Assumption 3.1: The initial values of the consensus variables θ_i satisfy $\theta_i(0) \in [0, \pi)$ for all $i = 1, 2, \ldots, n$.

Consider θ be an angle, q and p be points on the plane. Then $C[q, \theta](p)$ denotes the vertices of the triangular covering set $\hat{\mathcal{T}}[q, \theta]$ nearest to p (if there are more than one vertex in $\hat{\mathcal{T}}[q, \theta]$, any of them can be taken). To update the consensus variables $\theta_i(k)$, $q_i(k)$ and the robots' coordinates $p_i(k)$ the following rules are proposed:

$$\theta_i(k+1) = \frac{\theta_i(k) + \Sigma_{j \in \mathcal{N}_i(k)} \theta_j(k)}{1 + |\mathcal{N}_i(k)|};$$

$$q_i(k+1) = \frac{q_i(k) + \Sigma_{j \in \mathcal{N}_i(k)} q_j(k)}{1 + |\mathcal{N}_i(k)|} \qquad (1)$$

$$p_i(k+1) = C[q_i(k), \theta_i(k)] p_i(k) \qquad (2)$$

The algorithm (1), (2) can be explained as follows. The mobile robots use the variables θ_j to achieve the consensus on the triangular grid heading θ and the variables q_i to achieve the consensus on the triangular grid phase shift. Eventually, the robots converge to the vertices of a triangular grid set in $\mathcal{W}$.

Remark 3.1: The robots initially do not have a common coordinate system, otherwise the consensus building problem would be trivial. Therefore, each robot has consensus variables $\theta_j(k)$ and $q_i(k)$ in its own coordinate system. However, each robot knows the bearing and the distance to each of its neighbouring robots. Using this information, at any time instance k, the robot i sends to a neighbouring robot j the consensus variables $\theta_j(k)$ and $q_i(k)$ re-calculated in the coordinate system with the line (p_i, p_j) as the x axis p_j as the origin, and the angle $\theta_j(k)$ is measured from this axis in counter clockwise direction. Using this information, each robot can re-calculate the sums (1) at each time step in its own coordinate system.

Theorem 3.1: Suppose that Assumptions 2.1, 2.2, 2.3 and 3.1 hold and the mobile robots move according to the decentralized control law (1), (2). Then there exists a triangular grid set $\hat{\mathcal{T}}$ such that for any $i = 1, 2, \ldots, n$, there exists a $\tau \in \hat{\mathcal{T}}$ such that $\lim_{k \to \infty} p_j(k) = \tau$.

Proof: Assumption 2.2 and the update law (1) guarantee that there exist a θ_0 and q_0 such that

$$\theta_j(k) \to \theta_0, \; q_j(k) \to q_0 \; \forall i = 1, 2, \ldots, n \qquad (3)$$

(see [Jadbabaie *et al.*, 2003]). Furthermore, the update law (2) guarantees that $p_j(k+1) \in \hat{\mathcal{T}}[q_j(k), \theta(k)]$.

Therefore, this and (3) guarantee that $\lim_{k\to\infty} p_j(k) = \tau$ where $\tau \in \hat{\mathcal{T}}[q_0,\ \theta_0]$. This completes the proof of Theorem 3.1.

The algorithm (1), (2) locates all the robots to vertices of a triangular grid set. The next stage is to transfer the robots through other vertices of the triangular grid set. Its consequence is that, after a finite time all vertices of the triangular grid will be occupied at least one time by one of the robots. This guarantees that the whole area $\mathcal{W}$ is searched by the multi-robot system because the triangular grid $\hat{\mathcal{T}}$ covers the whole area $\mathcal{W}$. To develop the second stage of the algorithm, a random-based algorithm is used.

Assumption 3.2: The area $\hat{\mathcal{W}}$ where is to be searched in a triangular pattern, is unknown to the robots, therefore any robot gradually makes its own map during the search procedure and shares it with its neighbours. In other words, the map of the region related to each robot will gradually be completed during the search procedure.

Definition 3.1: Let introduce $\hat{\mathcal{T}}(k)$ as the set of all elements of $\hat{\mathcal{T}}$ that has been detected by robots at time k. Also Let $|\hat{\mathcal{T}}(k)|$ be the number of members of $\hat{\mathcal{T}}(k)$.

Definition 3.2: Let $V_\tau(k)$ be a Boolean variable which defines the state of vertex $\tau \in \hat{\mathcal{T}}(k)$ at time k. If the vertex τ have been already visited by any of the robots, then $V_\tau(k) = 1$, otherwise $V_\tau(k) = 0$

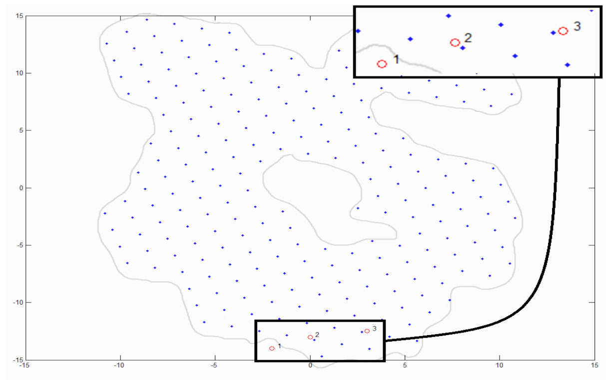
Assumption 3.3: The triangular grid set $\hat{\mathcal{T}}(k)$; $k = 0, 1, ...$ is connected: if $\tau \in \hat{\mathcal{T}}(k)$, then at least one of the six neighbours of τ also belongs to $\hat{\mathcal{T}}(k)$.

Let $\aleph(p_i(k))$ be a set containing all the six closest vertices to $p_i(k)$ on the triangular grid $\hat{\mathcal{T}}$ and $\hat{\aleph}(p_i(k))$ be all the elements of $\aleph(p_i(k))$ which have not been already visited by any of the robots. Also consider $|\aleph(p_i(k))|$ and $|\hat{\aleph}(p_i(k))|$ as the numbers of elements in $\aleph(p_i(k))$ and $\hat{\aleph}(p_i(k))$, respectively. It is obvious that $1 \le |\aleph(p_i(k))| \le 6$ and $0 \le |\hat{\aleph}(p_i(k))| \le 6$. In addition, assume ν be a randomly opted element of $\aleph(p_i(k))$ and $\hat{\nu}$ be a randomly opted element of $\hat{\aleph}(p_i(k))$. Now the random algorithm is proposed as follows:

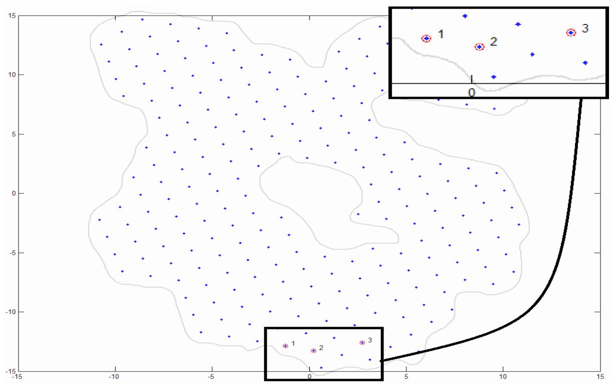
$$p_i(k+1) = \begin{cases} \hat{\nu} & \text{if } |\hat{\aleph}(p_i(k))| \neq 0 \quad \textit{with prob.} \quad \frac{1}{|\hat{\aleph}(p_i(k))|} \\ \nu & \text{if } |\hat{\aleph}(p_i(k))| = 0 \quad \textit{with prob.} \quad \frac{1}{|\aleph(p_i(k))|}. \end{cases} \tag{4}$$

The algorithm (4) ensures complete search of the whole area but it is not contain any condition to stop the search procedure. This case is enough for some applications such as continuously patrolling or surveying a region. On the other hand, it is not adequate for the case that robots should break the search procedure, for instance, when the robots' duty is finding some predetermined objects or putting sings on some vertices of the triangular grid. The algorithm (4) is enhanced by developing the following algorithm in order to bring the procedure to an end when needed.

$$p_i(k+1) = p_i(k) \quad if \quad \forall\tau \in \hat{\mathcal{T}}; \quad |\hat{\aleph}(p_\tau(k))| = 0 \tag{5}$$



(a) Initial locations of robots.

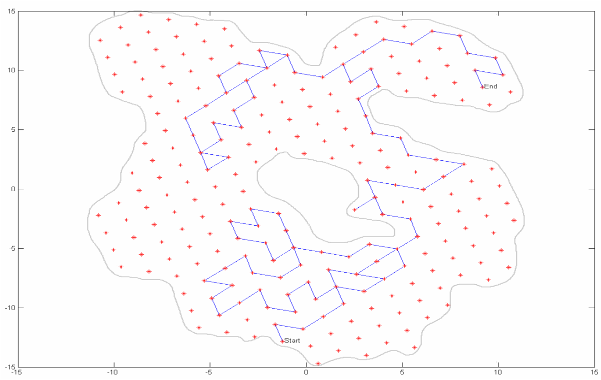


(b) After applying algorithm (2),(3).

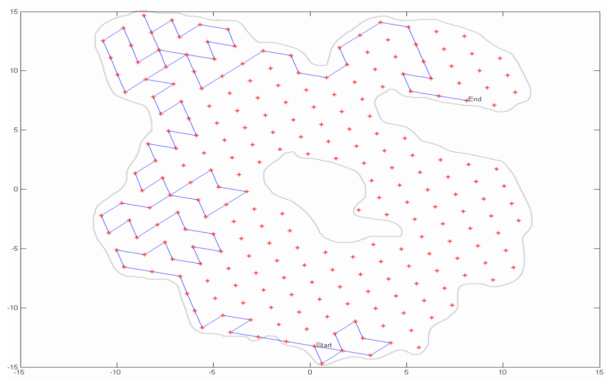
Figure 2: Robots' locations.

Theorem 3.2: Suppose that all assumptions hold and the mobile robots move according to the decentralized control law (4), (5). Then for any number of robots, with probability 1 there exists a time $k_0 \ge 0$ such that $V_\tau(k_0) = 1; \quad \forall\tau \in \hat{\mathcal{T}}$.

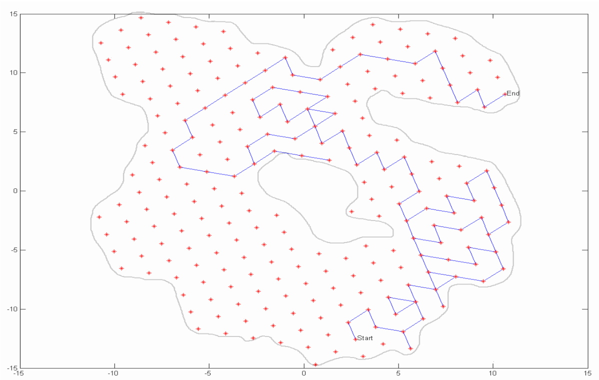
Proof: The algorithm (4), (5) defines an absorbing Markov chain which contains many transient states and a number of absorbing states that are impossible to leave. Transient states are all the vertices of the triangular grid $\hat{\mathcal{T}}$ which are occupied by robots during the search procedure. On the other hand, absorbing states are the vertices at where the robots stop. Using the algorithm, a robot goes to the vertices where may have not been visited previously, unless all robot's neighbours are already visited. Therefore, the number of transient states will eventually decrease. This continues until the quantity of



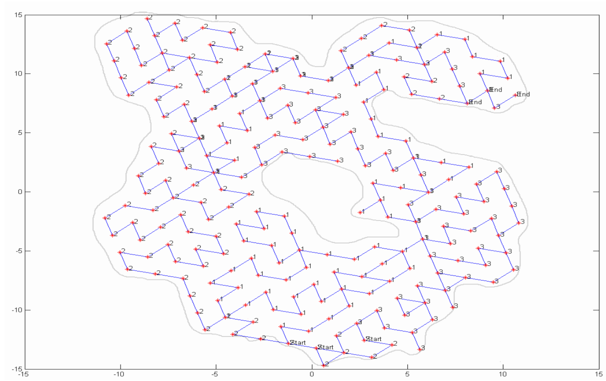
(a) Robot 1's trajectory



(b) Robot 2's trajectory



(c) Robot 3's trajectory



(d) Robots' trajectories

Figure 3: Robots' trajectories after applying algorithm (4),(5).

robots is equal to the quantity of unvisited vertices which are absorbing states. It is also clear that these absorbing states can be reached from any initial state, with a non-zero probability. This implies that with probability 1, one of the absorbing states will be reached. This completes the proof of Theorem 3.2.

4 Simulation Results

To verify the suggested algorithm, computer simulations are fulfilled. A region $\mathcal{W}$ is considered to be search by a few robots (see Fig1). It is supposed a multi-robot system with three robots which are randomly located in the region $\mathcal{W}$ with random initial values of angles. Fig.2(a) shows the the initial positions of the robots at $k = 0$. The goal is to search the whole area $\mathcal{R}$ by the robots, using proposed algorithm and considering triangular grid.

As mentioned before, a two-stage algorithm is used to achieve the goal. First, algorithm (1),(2) is applied which uses consensus variables in order to drive the robots to the vertices of a common triangular grid. Fig.2(b) displays the positions of the robots after applying this stage of the algorithm. As depicted in Fig.2(b), in this case, robots are located at desired place at $k = 6$. The second stage of the algorithm, consisting random algorithm (4) and its complement algorithm (5), is applied whenever the first stage is completed. Fig.3 demonstrates the effect of applying algorithm (4),(5) on robots. As seen in Fig.3(a), the robot number 1 goes through the vertices of common triangular grid based on the proposed algorithm until whole area is detected by robots. Fig.3(b) and Fig.3(c) display the paths of the robots 2 and 3, respectively. Fig.3(d) shows paths of all the robots together. It is obvious that the area $\mathcal{W}$ is completely searched by the robots such that each vertex of the covering triangular grid is occupied at least one time by the robots.

The procedure of search can be done by multi-robot system with any number of robots. It is clear that, more robots causes less time to search. But, more number of robots definitely increases the cost of the procedure. The question is that, how many number of robots should be used in the search procedure to have the best time and simultaneously the less cost. It seems that, it maybe somewhat depends on the shape of the region and the obstacles and the side of the triangles. For instance, simulation result for this case is depicted in Fig.4. As the graph shows, the time needed for the search procedure

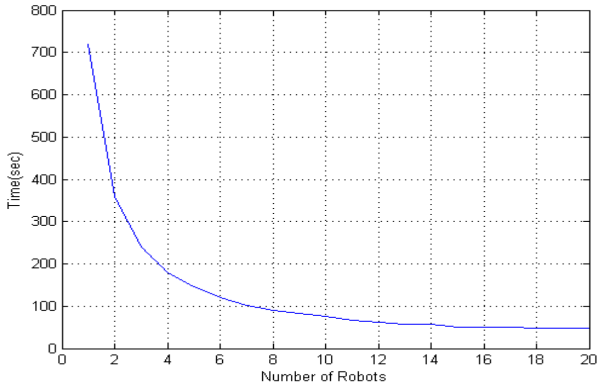


Figure 4: Duration of Search Vs. Number of Robots

is approximately decreases by increasing the number of robots. Moreover, It is noticeable that the decreasing of the search duration almost discontinues after increasing the number of the robots to a specific value, about 8 to 12 robots in this example. Therefore, optimal or semi-optimal number of robots is essential to be considered in applying this algorithm to avoid ineffectual increase of the number of the robots.

5 Conclusions

In this paper, a decentralized control algorithm was developed to drive a multi-robot system to search an unknown area. A triangular grid pattern used so that robots moved through the vertices of the grid during the search procedure. A mathematically proof of convergence of the proposed algorithm was demonstrated. Furthermore, Computer simulation results were presented to demonstrate the effectiveness and applicability of the algorithm. The next step can be considering the dynamics equations of motion for the robots and also making the algorithm into experiments. Therefore, in future research, we will consider more realistic scenarios with robotic sensors motion described by nonholonomic kinematic models with the problem of collision avoidance following methods of [Teimoori and Savkin, 2010], [Matveev *et al.*, 2011], [Savkin and Wang, 2013], [Matveev *et al.*, 2012], [Savkin and Hoy, 2013].

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