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Wheel Slip Identification and its Use in the Robust Control of Articulated Off-road Vehicles

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Abstract

In this paper, sliding mode controllers (SMC) are designed for a steerable tractor-trailer system, which is operating in the presence of longitudinal and lateral slips at front and rear wheels. Such vehicles intended for autonomous operations usually traverse at low speeds along straight lines. As very low accelerations are involved kinematic models seem sufficiently accurate for developing controllers. However undulating, sloping and very uncertain terrain, besides rough environment, introduce significant disturbances which makes the control design challenging. To take these effects into account, lateral and longitudinal sliding velocities are incorporated into otherwise ideal kinematic model, and an error model is developed. The slip velocities are estimated by an on-line recursive least squares (RLS) algorithm. The sliding mode controllers are updated adaptively and continuously based on the estimated slip velocities. Since there are two sets of steered wheels, two separate SMCs are developed; one for the tractor and the other for the trailer. For the tractor, an adaptive sliding mode controller with disturbance observer (aDOB-SMC) is proposed to cope with uncertainties and residual errors in the slip estimations. The proposed aDOB-SMC is able to guarantee precision path tracking as well as robustness for a typical tractor-steerable trailer pair. We prove that the combination of the on-line estimations and SMCs provide adaptability, stability and accuracy in the presence of significant slips. The simulation results are compared with two other controllers in the literature.

Keywords: Robust controllers, disturbance estimation, agricultural machines, agricultural robotics, back stepping, sliding mode control.

1 Introduction

The use of fully autonomous outdoor vehicles, especially in the agricultural industry has a great potential to bring about a number of advantages, especially to broad acre farming. Most of these advantages can only be harnessed with very high precision path tracking. Such accuracy will enable the use of the same autonomous system right throughout the cropping season for all tasks with no need for crop localisation as the crop has been planted at known locations with sufficiently high accuracy thanks to high precision path tracking. This paper presents methodologies that can ensure such accuracy. Undoubtedly, such systems can find many other applications in many other industries.

The task of path tracking by an autonomous vehicle starts with modelling. In general, two types of models are used in developing controllers for complex systems; the dynamic models and kinematic models. Complex dynamic models can be developed using known parameters or through parametric system identification. Although a dynamic model is more accurate than a kinematic model in principle, generally they suffer from parametric uncertainties and modelling uncertainties. A dynamic model is most useful in scenarios where high accelerations are involved. At other times, kinematic models are a reasonable approximation [Werner *et al.*, 2012]. In addition, the kinematic models are very manageable in developing controllers.

The system described in this paper consists of a tractor and a steerable trailer, which is towed by the tractor. This farm vehicle meets the criteria for it to be represented with a kinematic model due to slow changing speeds and the paths to be followed are mostly straight lines or paths with very low curvatures. Although there are recent studies about controlling a towed implement by a tractor [Huynh *et al.*, 2012; Werner *et al.*, 2012], the modelling and control have been done with no-slippage or small amount of constant slips. They also do not carry out slip estimation and as such the controlled system is very sensitive to the changes in

slip velocities' magnitude and direction especially along high curvature paths.

For our application, the kinematic model of a tractor-steerable trailer can easily be developed. To represent the wheel slippage in the model, slip velocities are incorporated. In this work slip identification is carried out only to determine the longitudinal and lateral slip velocities that affect the tractor. Since, the nature of slip depends to the physical characteristics of the interacting dynamics; slips are dependent on the tyres of the vehicle, condition of the terrain/ground and the planed path curvature. Hence, it is impossible to rely on slip estimation using off-line data as in many farming scenarios it is impractical to run the farm vehicle for data gathering beforehand with an acceptable degree of repeatability.

However, it is reasonable to assume the ground and tyre conditions to remain rather stationary in a typical farm. The slip velocities remain constant for a large amount of time especially on straight paths. Thus it is reasonable to expect the convergence of the estimation during real field operations.

It will be shown how the kinematic model of the tractor with slip velocities can be obtained. As such there is no need to consider non-parametric estimation methods. Moreover, the non-parametric methods such as transient and correlation analysis are not robust and precise in the presence of noise [Soderstrom and Stoica, 1989]. In this paper recursive least squares (RLS) [Haykin, 2002] algorithm is used for on-line parametric system identification.

In general system identification of nonlinear systems is not an easy task [Kwak *et al.*, May; Lacy and Bernstein, Dec; Kara and Eker, June]; *a priori* information about the order of the system and *a priori* knowledge of the model structure are essential [Ziaei and Sepehri, 2000]. For the tractor, the kinematic model is a nonlinear function; however the model structure and the order of the system are known. Moreover, the tractor's kinematic model is linear in the unknown parameters, i.e. the slip velocities. Hence, RLS method can be applied.

We do not consider estimation on the trailer's kinematic model. For the trailer we devise a controller based on the upper bound of the slip velocities that represent the practical slip values. It will be shown that the robustness of the controller provides stability and accuracy for the complete closed-loop system.

Although, the slip velocities are estimated by RLS method, the residual error and measurement noises dictate the choice of robust control approach. In this work estimation is used to provide adaptability to the control law, on one hand, and to provide robustness on the other hand.

In the context of nonlinear control, in works of Hoo *et al.* [Hoo and Kantor, 1985] and Kravaris *et al.* [Kravaris and Chung, 1986], feedback linearisation is used. However, feedback linearisation cannot provide sufficient robustness to our system. In this work two Lyapunov-based approaches, back stepping and sliding mode control (SMC) designs are considered. Both approaches are comprehensively used for control design of non-holonomic mobile vehicles. Back stepping technique has been used in works of Huynh *et al.* [Huynh *et al.*, 2012] and Fang Hao [Hao, 2004]. Although back stepping provides a good step by step design techniques, it is mathematically more complicated than SMC approaches. Moreover, back stepping is highly dependent to the bounded values of mismatched disturbances. Hence, sliding mode controllers (SMC) are nominated for both tractor and trailer to provide robustness. The concept of SMC is old and well known in nonlinear control design. They are simple and practical and are widely used in many industrial applications [Utkin, 1977; Hung *et al.*, 1993].

In this paper, an adaptive sliding mode controller with disturbance observer (aDOB-SMC) is developed, which is based on the work of Yang *et al.* [Yang *et al.*, 2011], for the tractor kinematic model. The aDOB-SMC uses the estimated values in the control law to provide adaptability as well as robustness. In addressing mismatched uncertainties, as reviewed in [Yang *et al.*, 2011], the sliding mode controller with disturbance observer is advantageous in comparison to LMI-based, adaptive and Riccati approaches on one hand, [Choi, 2007; Chang, 2009; Choi, 1998; Park *et al.*, 2007; Wen and Cheng, 2008], and integral sliding mode controllers (I-SMC) [Sam *et al.*, 2004; Hu, 2007; Park *et al.*, 2007] on the other hand. The first group requires a bounded H_2 norm that cannot be met in our system. And, for the second approach, I-SMC, the integration has adverse influence on the closed-loop system's performance causing phenomena such as large overshoots and long settling times. Our proposed aDOB-SMC is compared to the tractor's SMC presented in [Hao, 2004] as the SMC for perturbed chained system as well as the back stepping technique presented in [Huynh *et al.*, 2012].

For the trailer an adaptive sliding mode controller (a-SMC) is developed based on the bounded slip values for the trailer and estimated slips interacting with the tractor. Adaptability of the a-SMC is based on the estimated parameters in the controller law.

In the next sections, first the kinematic model and offset error model of the tractor-steerable trailer are presented; then system identification, control law and stability analysis are presented for both the tractor and the trailer. Afterwards, simulation results that prove accuracy and reliability of the proposed estimation and control strategies in path-following are presented. Finally a discussion and a conclusion are provided.

2 System Description

In this section, the kinematic model and the offset error model of the tractor-trailer system is presented. They are derived based on the assumption that the longitudinal and lateral slips at all four wheels as disturbances. Then the error model is introduced, which consists of the offset values in orientation and position of the tractor with the desired path as the reference.

2.1 Kinematic Model

Kinematic model of the tractor-steerable trailer is based on the approach presented in [Huynh *et al.*, 2010]. The vehicle model is depicted in Fig. 1.

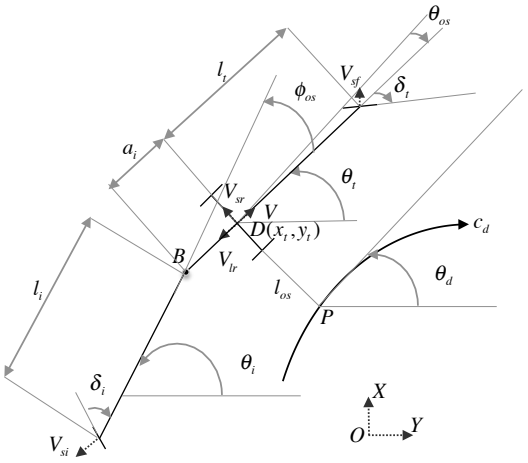

Figure 1: The error model variables for the tractor-steerable trailer with respect to the desired path.

As shown in Fig. 1, the tractor's front wheels and the trailer's wheels are steerable; the steering angles are δ_t and δ_i respectively. The steering is limited to the range $[-42, 42]$ degrees for the tractor and $[-22, 22]$ degrees for the trailer to match the steering angles of the real life vehicle that will be used to implement this controller in the future. Propulsion is effected through the rear wheels of the tractor. A constant speed of V is applied along the longitudinal axes of the tractor. The distance between the front axle and the rear axle of the tractor is $l_t = 1.7$ m; the distance between hitch point and the rear axle of the tractor is $a_i = 0.4$ m, and the distance between the rear axle of the trailer to the hitch point is $l_i = 2$ m. In the kinematic model presented in [Huynh *et al.*, 2010], position, x_t, y_t, and orientation, θ_t , of the body coordinate frame of the tractor is placed at the centre of the rear axle (point D in Fig. 1), and position, x_i, y_i, and orientation, θ_i of the trailer's body coordinate frame are placed at the centre of rear axle of the trailer. The angle between the tractor's longitudinal axis and the trailer's longitudinal axis is the hitch angle and is denoted by Φ_{os}. The tractor-steerable trailer's kinematic equations are expressed as:

$$\begin{aligned}
\dot{x}_t &= (V - V_{lr})\cos\theta_t - V_{sr}\sin\theta_t,\\
\dot{y}_t &= (V - V_{lr})\sin\theta_t + V_{sr}\cos\theta_t,\\
\dot{\theta}_t &= \frac{V - V_{lr}}{l_t}\tan(\delta_t + \beta_f) + \frac{V_{sr}}{l_t},\\
\dot{x}_i &= \dot{x}_t + a_i\dot{\theta}_t\sin\theta_t + \dot{\theta}_i l_i \sin\theta_i,\\
\dot{y}_i &= \dot{y}_t - a_i\dot{\theta}_t\cos\theta_t - \dot{\theta}_i l_i \cos\theta_i,\\
\dot{\theta}_i &= \frac{1}{G}(W_1 + W_2 + W_3 + W_4),
\end{aligned} \tag{1}$$

where, V_{lr} is the longitudinal slip velocity at rear wheels at point D; it is a positive quantity if vector $\mathbf{V_{lr}}$ is in opposite direction to vector $\mathbf{V}$. V_{sr} is side slip at the rear wheels at D. The vector $\mathbf{V_{sr}}$ points to the left of the tractor when it has a positive sign. The quantity β_f is a result of the disturbance at the front wheel. It is related to slip velocities through, $\tan\beta_f = \frac{V_{sf}}{V_f}$; when $V_f = \frac{V - V_{lr} + V_{sf}\sin\delta_t}{\cos\delta_t}$ and at the rear wheels, $V_r = V - V_{lr}$. And, for the trailer's angular velocity, $\dot{\theta}_i$, parameters are defined as: $G = l_i\cos(\Phi_{os})$, $W_1 = -(V - V_{lr})\sin(\delta_i + \Phi_{os})\sin(\Phi_{os})$, $W_2 = V_{sr}\sin(\Phi_{os})\cos(\delta_i + \Phi_{os})$, $W_3 = -a_i\dot{\theta}_t\sin(\Phi_{os})\cos(\delta_i + \Phi_{os})$ and $W_4 = V_{si}$.

2.2 Error Model

The two offset quantities are shown in Fig. 1 as l_{os} and θ_{os}, and are described below. As a result of the introduction of the two error quantities, the offset error dynamics has only two state variables. This suggests a single-input-single-output (SISO) system. This error model is well suited to apply a sliding mode controller with a disturbance observer. The controller is proposed in the next section.

As shown in Fig 1, l_{os} is the lateral position offset and θ_{os} is the orientation offset of the tractor with respect to the desired path with curvature c_d. The offset l_{os} is perpendicular to the desired path at P where θ_d is the desired orientation. The offset angle θ_{os} is defined as, $\theta_{os} = \theta_t - \theta_d$. The length of l_{os} is the distance from D to P. The sign of l_{os} is defined based on the position of D with respect to the desired path. It is negative if it is on the right hand side of the path, and positive if it is on the left hand side.

The tractor's error model is then defined as [Huynh *et al.*, 2010]:

$$\begin{aligned}
\dot{l}_{os} &= -\sigma|V - V_{lr}|\sin\theta_{os} - \sigma\zeta V_{sr}\cos\theta_{os},\\
\dot{\theta}_{os} &= \frac{V - V_{lr}}{l_t}\tan(\delta_t + \beta_f) + \frac{V_{sr}}{l_t}\\
&\quad - \sigma|V - V_{lr}|\frac{c_d\cos\theta_{os}}{1 + c_d l_{os}} + \sigma\zeta V_{sr}\frac{c_d\sin\theta_{os}}{1 + c_d l_{os}},
\end{aligned} \tag{2}$$

where σ is considered -1 if the prescribed path curvature c_d, is counter-clockwise, and it is 1 when the desired path's curvature is clockwise. The tractor is always assumed to be moving in the forward direction and as such ζ is always 1 and is dropped for brevity.

Similarly the steerable trailer's error model is developed based on the difference between the current angle between the tractor and the trailer, which is called hitch angle Φ_{os}; and Φ_d is the desired value for the hitch angle at B along the prescribed path. The trailer's error model is defined as [Huynh *et al.*, 2010]:

$$\begin{aligned}\dot{\Phi}_{os} = \frac{1}{l_i \cos\delta_i}\Big[&-(V - V_{lr})\sin(\delta_i + \Phi_{os} + \Phi_d) \\ &+ V_{sr}\cos(\delta_i + \Phi_{os} + \Phi_d) \\ &-\Big(\frac{V - V_{lr}}{l_t}\tan(\delta_t + \beta_f) + \frac{V_{sr}}{l_t}\Big) \\ &\times\Big(a_i\cos(\delta_i + \Phi_{os} + \Phi_d) + l_i\cos\delta_i\Big) - V_{si}\Big].\end{aligned} \tag{3}$$

3 SYSTEM IDENTIFICATION & CONTROL METHODOLOGY

In this section, the system identification and the control strategy are explained. The control loop is divided into three sections: 1. On-line estimation of slip velocities; 2. Tractor's path control; 3. Trailer's orientation control. The estimated slip velocities are fed in to the controllers. The continual estimation of the slip velocities while the system traverses the ground and their use in the controller provide controller adaptation.

3.1 Recursive System Identification

We consider a system description as a nonlinear function, F, which is linear in the unknown parameters as $F_i : y = \sum_{j=1}^{n} b_j^i f_j^i(\mathbf{x})$, where n is the number of nonlinear functions, $f_j^i(\mathbf{x})$, in the equation. And we have the set of input samples $\{\mathbf{x}(k-1), \mathbf{x}(k), \cdots, \mathbf{x}(N)\}$, and the corresponding outputs $\{y(1), y(2), \cdots, y(k)\}$, where $\mathbf{x}$ is a $1 \times m$ vector, m depends to the number of variables in $f_j^i(\mathbf{x})$, N is number of the available samples and $k \in \mathbb{N}$ is the samples' time stamp. Afterward, the unknown parameters $\mathbf{B} = [b_1, \cdots, b_n]^T$ are estimated as $\widehat{\mathbf{B}} = [\widehat{b}_1, \cdots, \widehat{b}_n]^T$ based on recursive estimation algorithm (RLS) where $0 < \lambda < 1$ is defined as the forgetting factor. Implementation of the recursive estimation is shown in Algorithm 1.

Based on the vehicle's kinematic model (1) two nonlinear equations are selected to be used in the RLS algorithm: 1. Longitudinal velocity of the tractor $\dot{x}_t$; 2. Angular velocity of the tractor $\dot{\theta}_t$. Hence on-line estimation is performed for the following nonlinear functions:

$$\begin{aligned}F_1 &: \dot{x}_t = b_1^1\cos\theta_t + b_2^1\sin\theta_t \\ F_2 &: \dot{\theta}_t = b_1^2 + b_2^2\tan\delta_t + b_3^2\,\frac{1}{\cos\delta_t},\end{aligned} \tag{4}$$

Algorithm 1 Slip Velocity Estimation by Recursive Least Squares

Inputs: $\mathbf{x}(k)$, $y(k)$
Step 1: Initialisation:
The initialisation of the estimated parameters can be done based on the available off-line data or prior-knowledge:
1) $\widehat{\mathbf{B}}_{k-1} = [\widehat{b}_1^0, \cdots, \widehat{b}_n^0]^T$.
2) Initialisation of the update rule matrix $\mathbf{P}_{k-1} \in \mathbb{R}^{n\times n}$: $\mathbf{P}_{k-1} = \mathbf{P}^0$, e.g. $\mathbf{P}^0 = \mathbf{1}_{\mathbf{n}\times\mathbf{n}}$.
3) Setting a forgetting factor. $0 < \lambda < 1$.

repeat
Step 2: Computing the matrix of current input $\mathbf{H}_k$ at the current time stamp, k from the measurements $\mathbf{x}_k$:
- $\mathbf{H}_k = [f_1(\mathbf{x}_k), \cdots, f_n(\mathbf{x}_k)]$

Step 3: Computing the vector of current gain factor $\mathbf{k}_k \in \mathbb{R}^{n\times 1}$:
- $\mathbf{k}_k = \frac{1}{\lambda}\left(\mathbf{P}_{k-1}\mathbf{H}_k^T(\lambda + \mathbf{H}_k\mathbf{P}_{k-1}\mathbf{H}_k^T)^{-1}\right)$.

Step 4: Updating vector of estimated parameters:
- $\widehat{\mathbf{B}}_k = \widehat{\mathbf{B}}_{k-1} + (y_k - \mathbf{H}_k\widehat{\mathbf{B}}_{k-1})\mathbf{k}_k$.

Step 5: Computing the current update rule matrix:
- $\mathbf{P}_k = \frac{1}{\lambda}(\mathbf{P}_{k-1} - \mathbf{k}_k\mathbf{H}_k\mathbf{P}_{k-1})$.

Step 6: Replacement of previous estimation vector and update rule matrix:
- $\mathbf{B}_{k-1} = \mathbf{B}_k, \mathbf{P}_{k-1} = \mathbf{P}_k$.

until Vehicle Stops ($V = 0$)

where the angular velocity is rearranged as: $\dot{\theta}_t = \frac{V_{sr}}{l_t} + \frac{V - V_{lr}}{l_t}\tan\delta_t + \frac{V_{sf}}{l_t}\frac{1}{\cos\delta_t}$.

Based on the estimation functions (4) input matrices are formed as: $\mathbf{H}_k^1 = [\cos\theta_t, -\sin\theta_t]$ and $\mathbf{H}_k^2 = [1, \tan\delta_t, \frac{1}{\cos\delta_t}]$. Then, after initialisation step the RLS algorithm is able to estimate the vectors of parameters, $\widehat{\mathbf{B}}_k^1$ and $\widehat{\mathbf{B}}_k^2$, for the functions (4). The estimated slip velocities are calculated as follows:

$$\begin{aligned}\widehat{V}_{lr} &= V - \widehat{b}_1^1; \\ \widehat{V}_{sr} &= \widehat{b}_2^1; \\ \widehat{V}_{sf} &= l_t\widehat{b}_3^2; \\ \widehat{V}_f &= \frac{V - \widehat{V}_{lr} + \widehat{V}_{sf}\sin\delta_t}{\cos\delta_t}; \\ \widehat{\beta}_f &= \tan^{-1}\Big(\frac{\widehat{V}_{sf}}{\widehat{V}_f}\Big).\end{aligned} \tag{5}$$

3.2 Controller for the Tractor

In this section, aDOB-SMC ('a' stands for adaptation) is derived based on the tractor's error model (2). We consider the estimations of the slips in the kinematics of the the tractor are available from RLS section. So for simplicity we ignore the use of *hat*, $\widehat{\ }$, on the estimated variables: $\widehat{V}_{lr}$, $\widehat{V}_{sr}$ and $\widehat{\beta}_f$ or $\widehat{V}_{sf}$.

The concept presented in [Yang *et al.*, 2011] is used in developing the aDOB-SMC for this system. The estimated slip velocities calculated in (5) are used in the aDOB-SMC. The controller is design to eliminate any residual error in the estimated values.

Assumption 1: V is always greater than V_{lr}, hence $-\sigma|V - V_{lr}|$ is equal to $-\sigma(V - V_{lr})$.

This assumption is valid in most practical scenarios although it may be violated in unusual circumstances. The assumption means that the tractors are expected to move forward despite slipping.

Assumption 2: We can linearise $\tan(\delta_t + \beta_f)$; so that we have $\tan(\delta_t + \beta_f) = \tan\delta_t + \tan\beta_f$.

The assumption is based on practical experience [Huynh *et al.*, 2012; Fang *et al.*, 2006]. Thus the range of β_f is expected as -5 to 5 degrees.

Accepting the assumptions 1 and 2 we denote l_{os} as x_1, $\Big(-\sigma(V - V_{lr})\sin(\theta_{os}) - \sigma V_{sr}\cos(\theta_{os})\Big)$ as x_2, and $\tan(\delta_t)$ as u_t; then we re-write (2) as:

$$\begin{aligned} \dot{x}_1 &= x_2 + d, \\ \dot{x}_2 &= a + b\, u_t, \end{aligned} \tag{6}$$

where d is additive mismatched disturbance to the system and is bounded; Ψ is an intermediate variable, which is defined as $\Psi = \Big(-\sigma(V - V_{lr})\cos\theta_{os} + \sigma V_{sr}\sin\theta_{os}\Big)$; $a = \left(\frac{(V-V_{lr})}{l_t}\tan\beta_f\Psi + \frac{V_{sr}}{l_t}\Psi - \frac{\sigma(V-V_{lr})}{\frac{1}{c_d}+l_{os}}\cos\theta_{os}\Psi\right)$; and $b = \left(\frac{(V-V_{lr})}{l_t}\Psi\right)$.

By re-arranging (6) as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u_t + \mathbf{h}\, d, \tag{7}$$

where $\mathbf{x} = [x_1, x_2]^T$, $\mathbf{f}(\mathbf{x}) = [x_2, a(\mathbf{x})]^T$, $\mathbf{g}(\mathbf{x}) = [0, b(\mathbf{x})]^T$ and $\mathbf{h} = [1, 0]^T$ are defined, the following control law is declared:

$$u_t = -b^{-1}(\mathbf{x})\Big[a(\mathbf{x}) + c(x_2 + \widehat{d}) + k_t\ \mathrm{sgn}(s)\Big], \tag{8}$$

where s is a sliding surface and is defined as,

$$s = x_2 + cx_1 + \widehat{d}. \tag{9}$$

In (8) and (9) $\widehat{d}$ is calculated as estimated disturbance using (10) given below [Yang *et al.*, 2011; Chen, 2003].

$$\begin{aligned} \dot{p} &= -\mathbf{r}\,\mathbf{h}\,p - \mathbf{r}\,[\mathbf{h}\,\mathbf{r}\,\mathbf{x} + \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u_t], \\ \widehat{d} &= p + \mathbf{r}\,\mathbf{x}, \end{aligned} \tag{10}$$

where $\mathbf{r} = [r_1, r_2]$ is adjusted for the estimation of the distance while p and $\widehat{d}$ calculate the internal state of the nonlinear observer.

The control law (8) can be smoothed by using hyperbolic tangent function instead of *signum* function as follows:

$$u_t = -b^{-1}(\mathbf{x})\Big[a(\mathbf{x}) + c(x_2 + \widehat{d}) + k_t\ \tanh(\frac{k_t s}{\varrho_t})\Big], \tag{11}$$

where ϱ_t, $\varrho_t > 0$ and c, $c > 0$ as the controller's parameters and k_t is controller's gain.

Stability analysis of the control law (8) by nominated sliding surface (9) has been explained in [Yang *et al.*, 2011]. Thus for a tractor with a kinematic model as given in (1), the proposed control law (8) brings Lyapunov stability if the assumptions 1 and 2 are satisfied and k is chosen such that: $k_t > (c + \mathbf{r}\,\mathbf{g})|d - \widehat{d}|$.

3.3 Controller for the Trailer

Based on the error model of the hitch angle, Φ_{os} in (3), a sliding mode controller is designed. Similar to the tractor's controller design, we consider the estimation of the slip velocities in the kinematics of the tractor are available; however, the side slip velocity on the trailer, V_{si} is not available. For simplicity we ignore the use of *hat*, $\widehat{\ }$, on the estimated variables: $\widehat{V}_{lr}$, $\widehat{V}_{sr}$ and $\widehat{\beta}_f$ or $\widehat{V}_{sf}$.

The aim of the trailer controller is to align the trailer rear axle middle point to follow the desired path accurately. In order to maintain the path following it is necessary to control the orientation of the trailer; so, we define $(\Phi_{os} + \Phi_d) = (\theta_i - \theta_t + \theta_{os}) = \Phi'_{os}$.

In addition, we denote $\tan(\delta_i)$ as u_i; hence the re-arranged equation of (3) is:

$$\begin{aligned} \dot{\Phi}_{os} =& \Big(-\alpha_1 \sin\Phi'_{os} - \alpha_2\cos\Phi'_{os} - \alpha_3\Big) \\ &+ \Big(-\alpha_1\cos\Phi'_{os} + \alpha_2\sin\Phi'_{os}\Big)u_i - \beta_1, \end{aligned} \tag{12}$$

where $\alpha_1 = \frac{(V-V_{lr})}{l_i}$, $\alpha_2 = \Big(-\frac{V_{sr}}{l_i} + \frac{(V-V_{lr})a_i}{l_t l_i}\tan\delta_t + \tan\beta_f + \frac{V_{sr}a_i}{l_t l_i}\Big)$, $\alpha_3 = \Big(\frac{(V-V_{lr})(\tan\delta_t + \tan\beta_f)}{l_t} + \frac{V_{sr}}{l_t}\Big)$, and $\beta_1 = \frac{V_{si}}{l_i\cos\delta_i}$.

If we define $\chi = \sin\Phi'_{os}$, the offset model (12) can be re-written as:

$$\dot{\chi} = \upsilon + \omega\, u_i + \xi, \tag{13}$$

when $\upsilon = \Big(-\alpha_1\chi\sqrt{1-\chi^2} - \alpha_2(1-\chi^2) - \alpha_3\sqrt{1-\chi^2}\Big)$, $\omega = \Big(-\alpha_1(1-\chi^2) + \alpha_2\chi\sqrt{1-\chi^2}\Big)$ and $\xi = -\beta_1\sqrt{1-\chi^2}$.

Assumption 3: We assume δ_i is in the range of [-22,22] degrees, based on practical reasons and system limitations in real system, and the side slip on the rear wheels of the trailer is bounded and the maximum value of it is: $V^*_{si} = |(V - V_{lr})|$; so $\beta^*_1 = \sup|\frac{V_{si}}{l_i\cos(22^\circ)}| = \frac{(V-V_{lr})}{l_i\cos(22^\circ)}$.

The sliding surface is defined as:

$$\vartheta = \chi. \tag{14}$$

In order to guarantee Lyapunov stability, the control law is proposed as:

$$u_i = -\omega^{-1}\Big[\upsilon + k_i\ \mathrm{sgn}(\vartheta) + \beta^*_1\chi\Big]. \tag{15}$$

Theorem: For the trailer with offset model as given in (3), the proposed control law (15) brings Lyapunov stability if the Assumption 3 is satisfied and k_i is chosen such that: $k_i > 0$.

Proof: We define a Lyapunov function as:

$$V_{ly}(\vartheta) = \frac{\vartheta^2}{2}; \tag{16}$$

and the Lyapunov function's derivative is:

$$\dot{V}_{ly}(\vartheta) = \dot{\vartheta}\vartheta. \tag{17}$$

The derivative of the sliding surface (14) is:

$$\dot{\vartheta} = \dot{\chi}. \tag{18}$$

By substituting (13) in (18), and substituting u_i by the control law (15) we get:

$$\dot{V}_{ly} = -k_i \, |\chi| - (\beta_1^* - \beta_1)\chi^2. \tag{19}$$

Since $\beta_1^* >= +\beta_1$ the whole closed loop is Lyapunov stable if $k_i > 0$.

The control law (15) can be smoothened by use of hyperbolic tangent function instead of *signum* function as follows:

$$u_i = -\omega^{-1}\Big[\upsilon + k_i \, \tanh(\frac{k_i\vartheta}{\varrho_i}) + \beta_1^*\chi\Big], \tag{20}$$

where $\varrho_i, \, \varrho_i > 0$.

4 CONTROL VALIDATION

In this section the presented RLS algorithm and the controllers for both tractor and trailer are verified in simulations. A kinematic model is formed based on (1) and the slip velocities unknown to the controller are simulated as a function of the desired path's curvature:

$$\begin{aligned} V_{sr} &= -7c_d \cos\theta_t + 0.15; \\ V_{sf} &= -7c_d \cos\theta_t + 0.15; \\ V_{si} &= -7c_d \cos\theta_i + 0.20; \\ V_{lr} &= c_d V_{sr} + c_d V_{sf} + 0.30. \end{aligned} \tag{21}$$

Estimated slips (5) are used in the control law of the tractor (11), aDOB-SMC, as well as the control law of the trailer (20), a-SMC. However, as indicted in the previous section, the side slip velocities of the steered wheels of the trailer are not estimated.

In the following, the performance of the proposed aDOB-SMC for the tractor is compared to two other control approaches, namely back stepping controller presented in [Huynh *et al.*, 2012], and the modified SMC presented in [Hao, 2004] both of which have been used in tractor path-following applications. The controller for the trailer, a-SMC, remains the same in different experiments in order to compare the performance of aDOB-SMC on the tractor with other control techniques.

The initial position and orientation of the tractor is selected away from the desired path, $x_t = -5$ m, $y_t = -15$ m and $\theta_t = 0°$. The desired path starts at the origin of the plane. The smooth transition from the starting position to the desired path is achieved by including a Bezier 4^{th} order curve generated within the implementation of the controller. Thus, regardless of the initial position and orientation of the tractor, a smooth transition from the current position to the path is calculated and the transitional path is appended to the desired path.

The slip estimation initialisation steps based on the Algorithm 1 are done: $\widehat{\mathbf{B}}_{k-1} = \mathbf{0_{n\times 1}}$, we consider zero slips at the beginning of estimation, so $\mathbf{P}^0 = \mathbf{1_{n\times n}}$, and finally forgetting factor is set to $\lambda = 0.90$.

The constant speed of the vehicle $V = 3$ m/s, and aDOB-SMC's gains are: $c = 25, k_t = 18, \mathbf{m} = [5, 0]^T, \rho_t = 1, \varrho_t = 0.01$; a-SMC's gains are: $k_i = 3, \rho_i = 1, \varrho_i = 0.01$.

Note that the following two controllers are simulated for the purpose of comparison.

The controller parameters for back stepping controller proposed in [Huynh *et al.*, 2012] are: $c_b = 0.6, k_{1b} = 0.6, a_{1b} = 0.01, a_{2b} = 1.05, \eta_0 = 1$ and $\epsilon = 0.1$ for the control law in [Huynh *et al.*, 2012].

Finally, controller parameters for the chained system and modified SMC [Hao, 2004] are: $c_f = 0.1, \Lambda = 5, k_f = 25, \rho_f = 1, \varrho_f = 0.01$, and the control law is defined in [Hao, 2004].

In Fig. 2, three estimated slip velocities with respect to the actual values are shown. The actual slips are changing according to the curvature of the desired path (21).

In Fig. 3, the position error of the proposed controller, aDOB-SMC with respect to the desired path in comparison to the other two control approaches, back stepping and modified-SMC, are shown.

Fig. 4 shows the path followed by the tractor with respect to the desired path when the tractor is controlled using our aDOB-SMC, and the path followed by the trailer when controlled with our a-SMC.

In the Fig. 5 the offset error signal of the tractor' orientation angle, θ_{os} and in Fig. 6 the hitch angle offset error, Φ'_{os} are compared in the three different control strategies.

The tractor orientation θ_t and the trailer orientation θ_i v.s. the reference path orientation is shown in Fig. 7 for aDOB-SMC and a-SMC controllers.

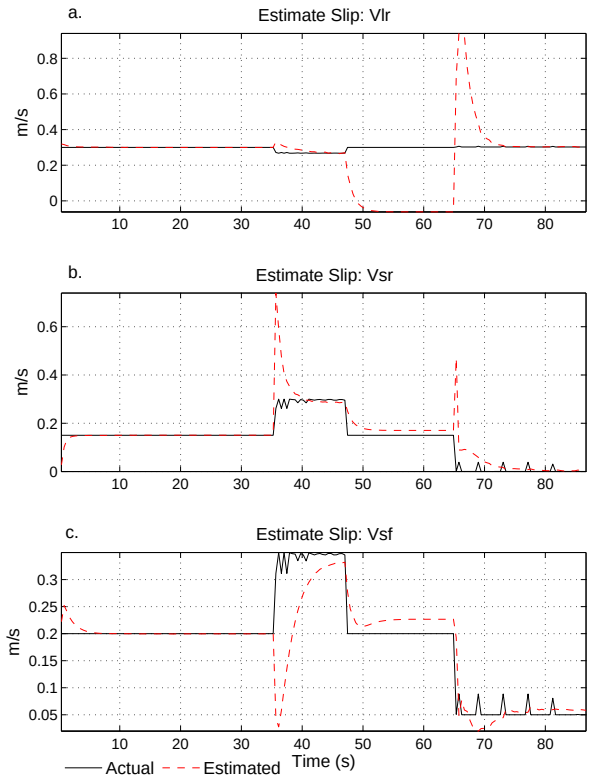

Figure 2: Estimated and actual slip velocities of the tractor. a: V_{lr}; b: V_{sr}; c: V_{sf}

5 DISCUSSION

In Fig. 2, the estimated values for the slip velocities show acceptable convergence of the RLS algorithm. Although there is residual error and some spikes in estimation, the existence of a robust sliding mode controller, the performance of which is strengthened by the observer and the aDOB-SMC provide stability and accuracy in the closed-loop system. In the Fig. 2a, there is more inaccuracy at changing points in estimation of longitudinal slip. However, the estimated longitudinal slip, $\widehat{V}_{lr}$ has minor influence on the lateral control of the vehicle.

As we see in Fig. 3, the offset error is very small and is now insensitive to slip changes. Clearly the adaptation of the aDOB-SMC as well as robustness of it has significantly improved the performance. For example, when V_{sr} and V_{sf} are changing dramatically (e.g. between 30 s to 50 s) the l_{os} remains almost steady for aDOB-SMC in contrast to a significant performance downgrade in modified-SMC and back stepping controllers.

In Fig. 4, the path-following of the tractor as well as trailer are shown. The accuracy of controllers guarantee precision along straight lines as well as turning points.

Moreover, orientation offset values of about $-3°$ to $-6°$, depending to the side-slips V_{sr} and V_{si} can be seen

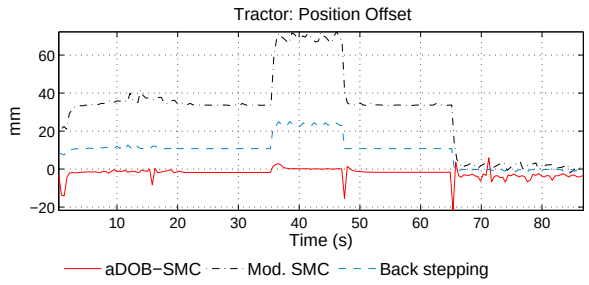

Figure 3: The position error of the proposed aDOB-SMC in comparison to the back stepping and modified-SMC for the tractor.

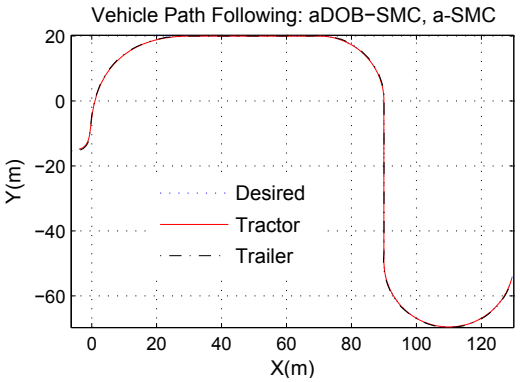

Figure 4: X-Y plane, path-following of the proposed controllers for the tractor and the trailer plotted together with the desired path. The 4^{th} Bezier path is calculated to provide smooth transition from the initial position of the vehicle to the beginning of the devised path.

in every control method in the Fig. 5 and Fig. 6. This offset angle caused by side slips is the normal way of steering for elimination of position error, which is known as *crab angle*; the angle between vehicle's heading and the vehicle's velocity to keep the tractor moving straight in presence of side slips. When there is no side slips the crab angle is zero.

As we see in Fig. 7 slips are well contained and the controller was able to guarantee orientation following as dictated by θ_r in the planned path.

Table 2 shows comparative results from all three controllers. As can be seen the aDOB-SMC is showing substantially small amounts of position error in comparison to modified-SMC and back stepping. e.g. the amount of RMS value of position error has reduced to -2.08 mm from 9.72 mm in back stepping and 20.42 mm in modified-SMC; the standard deviation for aDOB-SMC is less than 2 mm that is far better than two other approaches.

In addition, the results listed in Table 1 show almost the same performance for orientation control by all con-

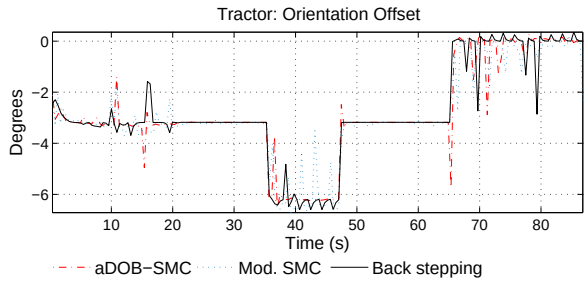

Figure 5: Orientation error of the tractor, θ_{os} in the three different control strategies.

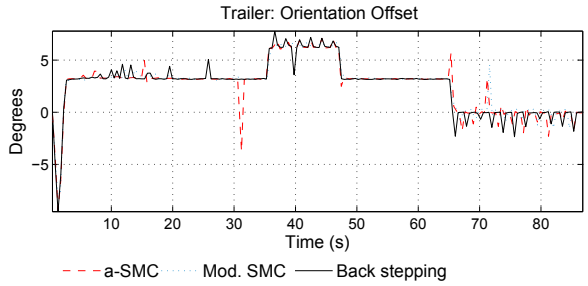

Figure 6: Hitch angle orientation error, Φ'_{os}, in the three different control strategies.

trollers. The controllers are maintaining the orientation of the tractor and hitch point precisely.

Table 1: Orientation error's mean value [RMS] & Standard Deviation.

Tractor Orientation Error (°)	Back Stepping	Modified SMC	DOB SMC
Mean Value [RMS]	-2.82 [3.40]	-2.87 [3.38]	-2.83 [3.39]
Standard Deviation	1.91	1.78	1.87

Further, smooth transition from the initial position and orientation to the starting point of the desired path can be seen in Fig. 4. Hence, the applicability of the 4^{th} order Bezier curve is also confirmed, which is useful for path-following scenarios in the case of multiple vehicles.

6 CONCLUSION

The study addressed off-road vehicle control in the presence of slips. A farm vehicle as a tractor-steerable trailer has been considered. The low acceleration and typical paths in farms with mostly straight or low curvature characteristics suggest kinematic models are sufficient.

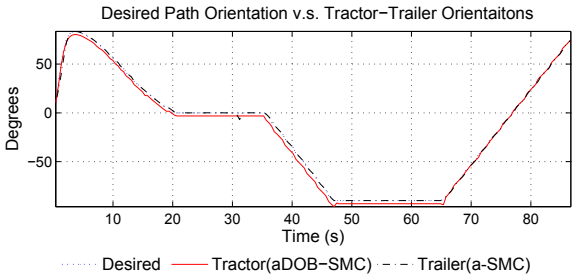

Figure 7: Orientation error of the tractor/trailer v.s. the reference orientation θ_r.

Table 2: Position error's mean value [RMS] & Standard Deviation.

Tractor Position Error (mm)	Back Stepping	Modified SMC	DOB SMC
Mean Value [RMS]	9.72 [12.09]	30.68 [36.83]	-2.08 [3.43]
Standard Deviation	7.21	20.42	2.74

These kinematic models were augmented with slip velocities that appear as disturbances. Considering these disturbances as parameters that affect the performance of the controller, they were estimated using recursive least squares estimation algorithm. In control, sliding mode controllers (SMC) have chosen; and controllers were developed for the tractor and the steerable trailer. Due to the availability of the estimations of the slips, the estimated values have been included in the control laws to bring adaptability as well as robustness for the system. Lyapunov criterion based stability analysis was also provided for the controllers. The controller performance and the slip estimation were verified in simulations; and the results have been compared to two other tractor control approaches, the modified-SMC and the back stepping approach. Better performance and robustness of the aDOB-SMC controllers in the presence of changing slips have been shown.

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