A Virtual Odometer for a Quadrotor Micro Aerial Vehicle

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Abstract

This paper describes the synthesis and evaluation of a "virtual odometer" for a Quadrotor Micro Aerial Vehicle. Availability of a velocity estimate has the potential to enhance the accuracy of mapping, estimation and control algorithms used with quadrotors, increasing the effectiveness of their applications. As a result of the unique dynamic characteristics of the quadrotor, a dual axis accelerometer mounted parallel to the propeller plane provides measurements that are directly proportional to vehicle velocities in that plane. Exploiting this insight, we encapsulate quadrotor dynamic equations which relate acceleration, attitude and the aerodynamic propeller drag in an extended Kalman filter framework for the purpose of state estimation. The result is a drift free estimation of lateral and longitudinal components of translational velocity and roll and pitch components of attitude of the quadrotor. Real world data sets gathered from two different quadrotor platforms, together with ground truth data from a Vicon system, are used to evaluate the effectiveness of the proposed algorithm and demonstrate that drift free estimates for the velocity and attitude can be obtained.

1 Introduction

Quadrotor is a Vertical Take-Off and Landing (VTOL) capable platform which recently has gained much popularity as a Micro Aerial Vehicle(MAV). Much of this popularity stems from their mechanical simplicity compared to other VTOL platforms such as helicopters. As such, quadrotor MAVs are increasingly being deployed in both indoor and outdoor environments for tracking, exploration and mapping tasks [Achtelik *et al.*, 2008; Fang *et al.*, 2010; Waslander *et al.*, 2005]. These tasks require precise localisation, navigation and control of the MAV, and those in-turn require an accurate estimate of the rotational and translational states of the MAV. Typically for autonomous vehicles, a state estimator is a sensor fusion algorithm, which optimally combines measurements from both interoceptive and exteroceptive sensors. Algorithms for GPS aided inertial navigation and Simultaneous Localisation and Mapping (SLAM) are well established within the Unmanned Ground Vehicles (UGV) community. Gradually, these algorithms are being adapted and put to use in MAV applications [Bryson and Sukkarieh, 2007; Bachrach *et al.*, 2011].

Many existing UGV applications exploit odometry information which are typically obtained using wheel and steering encoders. Even though inertial sensors such as accelerometers and gyroscopes can be used to obtain an estimate of the attitude of a MAV, velocity estimates obtained from these rely on the integration of accelerations [Blosch *et al.*, 2010], making them susceptible to drift. If an estimate of the velocities, similar to what is used in ground vehicles is available, the accuracy of most existing MAV state estimators can be improved. For example, [Taylor, 2009] presents a SLAM algorithm for a fixed-wing MAV where the localisation accuracy is made an order of a magnitude better by using an air speed measurement gathered from a pitot tube. While a similar approach is not suitable for rotary wing platforms, [Martin and Salaun, 2009] have demonstrated that a quadrotor in translational motion experience a drag force which is proportional to the translational velocity of the MAV. As a result, a dual axis accelerometer mounted parallel to the propeller plane provides a measurement directly proportional to the drag force, hence the velocity. This rather counterintuitive behaviour is unique to quadrotors and can be exploited to directly infer information about lateral and longitudinal translational velocity components using the measurements from an IMU, without the need for integration.

In this paper we exploit the results from [Martin and Salaun, 2009] and our previous work [Abeywardena and Munasinghe, 2010] on a Kalman filter based "generic attitude estimator", to design a virtual odometer for a quadrotor MAV. While Martin et. al. have demonstrated the potential of their dynamic model in obtaining velocity estimates, their focus was on the design of an improved controller for the quadrotor using a low pass filter to infer velocity information from

the accelerometer measurements. A comprehensive state estimator for the purpose of generating an accurate velocity estimate was not discussed in their work. Algorithms presented in this paper is capable of estimating the MAV inclination to the horizontal plane and the horizontal components of translational velocity, within a time-independent error bound. This is achieved by optimally fusing the gyroscope and accelerometer measurements in an Extended Kalman Filter (EKF) based state estimator. The main contribution of our work is a drift free velocity and attitude estimator, which exploits the unique dynamics of quadrotor MAVs. Our design is captured within a standard framework that can be easily extended to complement the existing estimation and navigation algorithms for quadrotor MAVs. We also present the results of real world experiments performed by us and also by using a MAV benchmarking data set [Lee *et al.*, 2010] to demonstrate the effectiveness of the proposed estimator.

In section 2 we briefly present the dynamic equations of the quadrotor which are of interest for the estimator design. In section 3 the design of a novel EKF based attitude and velocity estimator is presented. In section 4, data sets used to validate the estimator design are explained. Section 5 presents the results of the experiments performed with the said data sets. Finally section 6 concludes the paper with a discussion on presented results and future research drives.

2 Dynamic Model of the Quadrotor MAV

Derivation of non-linear dynamics of the quadrotor appears in much of the literature focusing on the development of various control schemes. Most of the work follow the same basic approach while some have extended the basic model to include effects of gyroscopic torque [Pounds *et al.*, 2002] , blade flapping and aerodynamic drag components [Bristeau *et al.*, 2009]. The dynamic model for this research closely follows the derivation of [Pounds *et al.*, 2002] and also includes the aerodynamic drag derivation in [Bristeau *et al.*, 2009]. In this section we only summarise the equations which are necessary for the state estimator design.

Let $\{E\}$ be the earth fixed inertial frame, and a vector $[\ x \quad y \quad z\ ]^T$ denote the position of the centre of mass of the quadrotor as expressed in $\{E\}$. (See Figure.1) Let $\{B\} \equiv [\ b_1 \quad b_2 \quad b_3\ ]^T$ be a body fixed frame positioned at the centre of mass of the quadrotor.

The orientation of $\{B\}$ with respect to $\{E\}$ is defined using a cumulative rotation of Euler angles ψ (Yaw) , θ (Pitch) and ϕ (Roll) in that order, around b_3, b_2 and b_1, respectively. R is defined as the rotational transformation matrix from $\{B\}$ to $\{E\}$. The kinematic equation relating the instantaneous angular velocity $\Omega \equiv [\ \omega_x \quad \omega_y \quad \omega_z\ ]$ of $\{B\}$ with respect to $\{E\}$, to Euler rates can be expressed as:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (1)$$

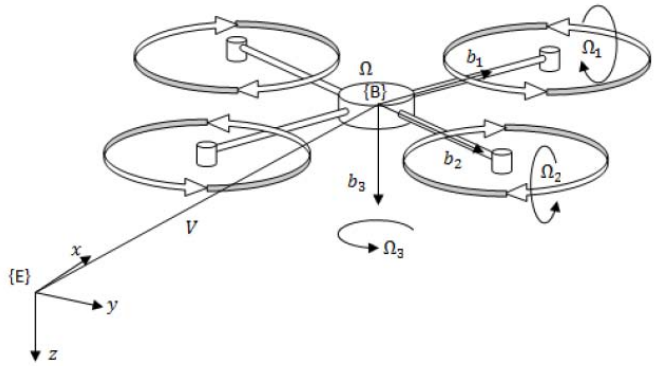


Figure 1: Coordinate frame definitions for the quadrotor dynamic model

For the purpose of the estimation, we are interested in the translational motion equation of the quadrotor as derived in [Martin and Salaun, 2009].

$$m\dot{\mathbf{V}} = m\mathbf{g} - k_T \sum_{i=1}^{4} \omega_i^2 \mathbf{b_3} - \lambda_1 \sum_{i=1}^{4} \omega_i \tilde{\mathbf{V}} \quad (2)$$

where

$\mathbf{V}$ = Velocity of $\{B\}$ as observed from an inertial frame
$\mathbf{g}$ = gravity vector
k_T = thrust coefficient of propellers
λ_1 = a positive coefficient known as rotor drag coefficient
ω_i = rotational velocity of i^{th} rotor, $i \in \{1,2,3,4\}$
$\tilde{\mathbf{V}}$ = projection of $\mathbf{V}$ on the propeller plane
m = mass of the quadrotor

For the design of the estimator, we wish to express the velocity in (2) in the $\{B\}$ frame. (i.e. velocity of $\{B\}$ as observed from an inertial frame, but expressed in $\{B\}$ frame). In doing so, we use the following relationship of vector differentiation.

$$\begin{aligned} {}^e\mathbf{V} &= R\,{}^b\mathbf{V} \\ {}^e\dot{\mathbf{V}} &= R\,{}^b\dot{\mathbf{V}} + R({}^b\Omega \times {}^b\mathbf{V}) \end{aligned} \quad (3)$$

where the leading superscript e and b denotes the earth and body frames of reference respectively. λ_1 can be replaced by a positive constant k_1 assuming that the summation of propeller speeds are constant during a smooth flight. Therefore, the first two components of ${}^b\dot{\mathbf{V}} \in \{{}^b\dot{v}_x, {}^b\dot{v}_y, {}^b\dot{v}_z\}$ can be written as:

$$ {}^{b}\dot{v}_x = -g\sin\theta - \frac{k_1}{m}{}^{b}v_x + \omega_z {}^{b}v_y - \omega_y {}^{b}v_z \quad (4) $$

$$ {}^{b}\dot{v}_y = g\cos\theta\sin\phi - \frac{k_1}{m}{}^{b}v_y + \omega_x {}^{b}v_z - \omega_z {}^{b}v_x \quad (5) $$

Considering the typical flight characteristics of the quadrotor, it can be assumed that the second order velocity terms in the above equations are relatively small (close to zero). Then the translational velocities can be simplified to

$$ {}^{b}\dot{v}_x = -g\sin\theta - \frac{k_1}{m}{}^{b}v_x \quad (6) $$

$$ {}^{b}\dot{v}_y = g\cos\theta\sin\phi - \frac{k_1}{m}{}^{b}v_y \quad (7) $$

We assume that the quadrotor is equipped with a triad of gyroscopes and accelerometers aligned with B, and make use of the inertial sensor error models presented in [Park, 2004] and [Park and Gao, 2008]. Similar to their work, we assume zero cross-correlation between noise of different sensors. Further, gyroscope sensors are assumed to be corrupted by a varying bias (modelled as a first order Gaussian Markov process) and zero mean White Gaussian Noise (WGN). The accelerometers are assumed to be corrupted by zero mean WGN and a deterministic bias term which can be compensated for, offline.

The error model equations for i^{th} gyroscope and accelerometer are therefore expressed as:

$$ g_i = \Omega_i + \beta_{gi} + w_{gi} \quad (8) $$

$$ \dot{\beta}_{gi} = -\frac{1}{\tau_{gi}}\beta_{gi} + w_{\beta gi} \quad (9) $$

$$ a_i = \tilde{a}_i + w_{ai} \quad (10) $$

where $\tilde{a}_i$ is the acceleration that would be measured by an ideal accelerometer, β_{gi} is the bias of i^{th} gyroscope and τ_{gi} is the time constant of i^{th} gyroscope bias. w_{gi}, $w_{\beta gi}$ and w_{ai} are zero mean WGN terms.

3 EKF based Estimator Design

We propose a six state, Extended Kalman Filter based state estimator for the quadrotor. The filter states are:

ϕ – Roll angle in current orientation estimate

θ – Pitch angle in current orientation estimate

β_x – Bias in X axis gyroscope

β_y – Bias in Y axis gyroscope

${}^{b}v_x$ – X velocity component of quadrotor in body frame

${}^{b}v_y$ – Y velocity component of quadrotor in body frame

A Kalman filter based design was adopted firstly because it provides an optimal estimate minimising the mean square estimation error [Maybeck, 1979]. Secondly, it provides us with a solid framework within which we can tune and analyse the performance of the designed estimator.

3.1 Process Model

(1) and (8) form the first part of the process equation for the EKF. Out of the three Euler angles we only estimate ϕ and θ as the observability of ψ is poor, given the process and measurement equations [Abeywardena and Munasinghe, 2010].

$$ \begin{aligned} \dot{\phi} &= (g_x - \beta_{gx} + w_{gx}) + \tan\theta\sin\phi(g_y - \beta_{gy} + w_{gy}) \\ &\quad + \tan\theta\cos\phi(g_z - \beta_{gz}) \\ \dot{\theta} &= \cos\phi(g_y - \beta_{gy} + w_{gy}) - \sin\phi(g_z - \beta_{gz}) \end{aligned} \quad (11) $$

It should be noted that g_x, g_y, g_z and β_{gz} are not states and are considered as control parameters in the process equation. Strictly speaking, β_{gz} is a random variable which can be included in the estimator given the second order dependence of 4 and 5 on ω_z. Theoretically, this should enable us to estimate the remaining Euler angle ψ as well. However, in practise the effect of the above mentioned second order terms on 4 and 5 are negligible. Due to this reason β_{gz} is not included as a state in the estimator. An approximation of β_{gz} is made offline and that value is used during the operation of the estimator.

Second part of the process equation is made up by the gyroscope biases.

$$ \begin{aligned} \dot{\beta}_{gx} &= -\frac{1}{\tau_{gx}}\beta_{gx} + w_{\beta gx} \\ \dot{\beta}_{gy} &= -\frac{1}{\tau_{gy}}\beta_{gy} + w_{\beta gy} \end{aligned} \quad (12) $$

The final part of the process equation is the translational motions equations (6) and (7), which describe the evolution of the body frame velocity of the quadrotor. The model imperfections are compensated by adding two WGN noise terms to those two equations.

$$ \begin{aligned} {}^{b}\dot{v}_x &= -g\sin\theta - \frac{k_1}{m}{}^{b}v_x + w_{\alpha x} \\ {}^{b}\dot{v}_y &= g\cos\theta\sin\phi - \frac{k_1}{m}{}^{b}v_y + w_{\alpha y} \end{aligned} \quad (13) $$

Equations (11) , 12) and (13) together makeup the process dynamics for the estimator. The resulting system can be represented as a non-linear function of states, control inputs and noise terms.

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}, \mathbf{w}) \tag{14}$$

3.2 Measurement Model

Observations of the EKF are the measurements from X and Y accelerometers, which are aligned with b_1 and b_2, respectively. Assuming that the accelerometers are located at the centre of gravity of the quadrotor, their measurements are given by,

$$\tilde{\mathbf{a}} = \dot{V} - \mathbf{g}$$

Substituting (6) and (7) in above, and incorporating the accelerometer noise in to the equation,

$$\begin{aligned} a_x &= -\frac{k_1}{m}\, {}^b v_x + w_{ax} \\ a_y &= -\frac{k_1}{m}\, {}^b v_y + w_{ay} \end{aligned} \tag{15}$$

where a_x and a_y are respectively the measurements from the X and Y axis accelerometers on-board the quadrotor.

3.3 EKF Mechanization Equations

For the mechanization of the Extended Kalman Filter, the discrete state transition matrix A_k should be calculated. For this we first calculate F, which is the Jacobian matrix of partial derivatives of f with respect to $\mathbf{x}$. Then A_k is calculated by discretization of the Jacobian matrix as.

$$\begin{aligned} F(t) &= \frac{\partial f(\mathbf{x}, \mathbf{u}, \mathbf{w})}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_k, \mathbf{u}_k} \\ &= \begin{bmatrix} a_{11} & a_{12} & -1 & -t\theta s\phi & 0 & 0 \\ a_{21} & 0 & 0 & -c\phi & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau_{gx}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_{gy}} & 0 & 0 \\ 0 & -gc\theta & 0 & 0 & \frac{-k_1}{m} & 0 \\ gc\theta c\phi & -gs\theta s\phi & 0 & 0 & 0 & \frac{-k_1}{m} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} a_{11} &= t\theta c\phi(\bar{\omega}_y - \beta_y) - t\theta s\phi(\bar{\omega}_z - \beta_z) \\ a_{12} &= sec^2\theta s\phi(\bar{\omega}_y - \beta_y) + sec^2\theta c\phi(\bar{\omega}_z - \beta_z) \\ a_{21} &= -s\phi(\bar{\omega}_y - \beta_y) - c\phi(\bar{\omega}_z - \beta_z) \end{aligned}$$

For the discretization we used a truncated Taylor series approximation and a sample time of T_s, which results in,

$$A_k = I + F(t)T_s \tag{16}$$

In deriving the discrete process noise matrix Q_k, we chose to neglect the non-linearities in the noise terms of (11) to keep the complexities of the designed estimator to a minimum. This assumption is justified by the fact that for small ϕ and θ angles, the non-linearities in the noise terms becomes negligible. With this assumption, the continuous process noise matrix $Q(t)$ is derived as

$$\begin{aligned} \mathbf{w} &= \begin{bmatrix} w_{gx} & w_{gy} & w_{\beta gx} & w_{\beta gy} & w_{\alpha x} & w_{\alpha y} \end{bmatrix}^T \\ Q(t) &= E[\mathbf{w}\mathbf{w}^T] \\ &= diag\begin{bmatrix} \sigma_{gx}^2 & \sigma_{gy}^2 & \sigma_{\beta gx}^2 & \sigma_{\beta gy}^2 & \sigma_{\alpha x}^2 & \sigma_{\alpha y}^2 \end{bmatrix} \end{aligned} \tag{17}$$

where $E[.]$ denotes the expectation operator. The first four terms of the process noise matrix above are the noise variances of gyroscope sensors and their biases. These can be found by experimentation with actual sensors. Last two terms, which correspond to the uncertainty in 13, were approximated first and then fine tuned for optimum performance of the estimator. Discretization of $Q(t)$ results in Q_k.

$$\begin{aligned} Q_k &= \int_0^{T_s} AQ(\tau)A^T\, d\tau \\ &= T_s \,\mathrm{diag}\begin{bmatrix} \sigma_{gx}^2 & \sigma_{gy}^2 & \sigma_{\beta gx}^2 & \sigma_{\beta gy}^2 & \sigma_{\alpha x}^2 & \sigma_{\alpha y}^2 \end{bmatrix} \end{aligned} \tag{18}$$

where we have approximated Q_k by neglecting 2^{nd} and higher order terms of T_s in the result.

Measurement matrix H required for the EKF can be directly obtained from (15).

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & -k_1/m & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_1/m \end{bmatrix} \tag{19}$$

Assuming zero correlation between the two accelerometer measurements, measurement noise matrix R_k consists of the noise variances of the X and Y accelerometers.

$$R_k = diag\begin{bmatrix} \sigma_{ax}^2 & \sigma_{ay}^2 \end{bmatrix} \tag{20}$$

For initialization, all states of the filter are set to zero and their error covariances are set to small positive values reflecting the uncertainty in initial estimate. (The final filter design is capable of properly converging from a wide range of initial values and noise variances)

For the EKF, state projection is carried out with the use of a 2^{nd} order Runga-Kutta integrator as follows

$$\begin{aligned} \hat{x}_k^- &= \hat{x}_{k-1} + 1/2T_s[f_{k-1} + f_k] \\ \text{where} \quad f_{k-1} &= f(\hat{x}_{k-1}, u_{k-1}) \\ f_k &= f(\hat{x}_{k-1} + T_s f_{k-1}, u_k) \end{aligned}$$

Covariance projection, Kalman gain calculation, state update and covariance update equations of the estimator take their standard form [Grewal and Andrews, 2001], as listed below.

$$
\begin{aligned}
P_k^- &= A_k P_{k-1} A_k^T + Q_k \\
K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \\
\hat{x}_k &= \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \\
P_k &= (I - K_k H_k) P_k^-
\end{aligned}
$$

4 Data Sets for Experimental Validation

Two data sets were used to analyse the performance of the EKF state based estimator presented in the previous section. First data set was collected using a Parrot AR Drone quadrotor MAV (See Figure 2). AR Drone weighs about 420g including the protective hull and is equipped with a triad of gyroscopes and accelerometers which are sampled at a rate of 200Hz. The second data set is from a publicly available "MAV benchmarking tool" by [Lee *et al.*, 2010]. MAV platform used for this data set is a Pelican quadrotor from Ascending Technologies, which is larger in size and weighs about 750g. The use of two data sets from two very different quadrotor MAVs enables an insight in to the robustness of the designed estimator.

Figure 2: AR Drone Quadrotor used to collect the first data set

A critical parameter that needs to be estimated for both data sets is the rotor drag coefficient λ_1. Since a theoretical calculation of this parameter is a complex task, we resorted to an experimental estimation method. The basic methodology adopted here is to obtain the accelerometer measurements and ground truth velocity data of a few flight tests. A rough estimate of the parameter k_1 (which incorporates λ_1) can then be obtained by formulating equation (15) as a least-squares problem.

For both data sets, ground truth position and attitude data gathered from a Vicon motion capture system were available. In Vicon motion capture systems, a set of reflective markers rigidly attached to the quadrotor body are observed with the use of multiple, fixed IR cameras to directly compute the MAV pose. An estimate of the ground truth velocity in global coordinate frame was obtained from Vicon position estimates using backward difference. Vicon attitude estimates were then used to transform that velocity to MAV coordinate frame.

5 Experimental Results

5.1 Results of First Data Set

For this experiment, the AR Drone was manually operated within an indoor space of about 6 $\times$ 4m. During the experiment, both inertial data and Vicon data were captured at a rate of 200Hz and processed offline. Figures 3 and 4 present the attitude and velocity estimates from the EKF together with the "ground truth" obtained from the Vicon system.

Comparing the two figures, we observe that attitude estimates agree more closely with the ground truth than the velocity estimates. The reason for this behaviour can be explained by analysing equations 13 and 15. A small error in attitude estimate manifests as an accumulating error in velocity and thus can be easily detected via the velocity measurements provided by the accelerometers. In contrast, an error in velocity estimate is not immediately apparent from gyroscope measurements. What is more important to note is that the errors do not grow with time as typically observed in velocity and attitude estimates obtained by directly integrating measurements from an IMU.

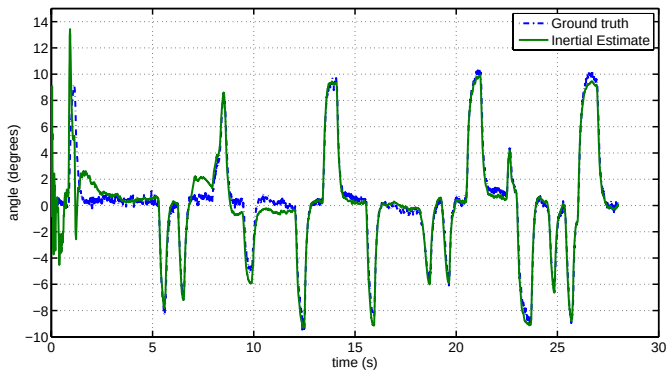
The true estimation errors and their 3σ bounds as estimated by the filter for ϕ angle of attitude and v_x of velocity are presented in figure 5. The actual errors are well within the filters estimate of error bounds, indicating that the filter is consistent. The spikes in velocity estimation error standard deviation (and the corresponding variation in angle estimate) are due to dropped inertial measurements during wireless transmission from the AR Drone to the data logging ground station.

5.2 Results of Second Data Set

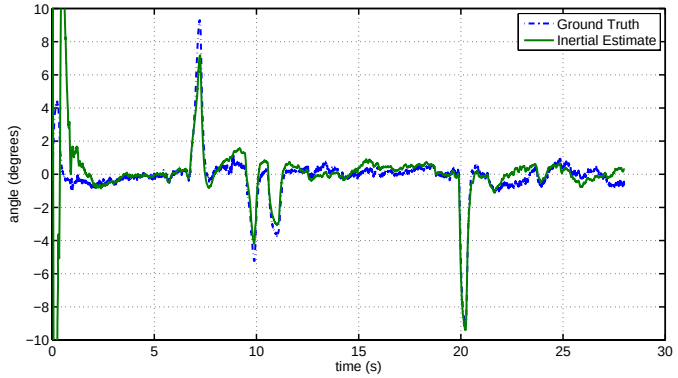
Data from five different experiments are available from the second data set. The results obtained from the "two loops down" data are presented here. In this experiment, the MAV loops twice around a rectangular area of about 6 $\times$ 2m. Figures 6 and 7 present the estimated and ground truth attitude and velocity data.

Velocity estimates are in reasonable agreement with the ground truth, except for few areas where there is a clear difference. (eg. from 15 - 25sec and 40 - 55sec in 6(a)) Comparing the corresponding Euler angles (θ angle for v_x and ϕ angle for v_y) it appears that these shifts are caused by the errors in attitude estimation, which are most likely due to erroneous gyroscope measurements.

However, closer examination also show that ground truth measurements corresponding to some of these mismatches

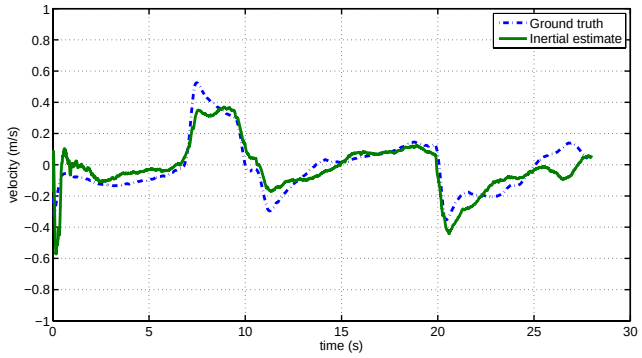


(a)

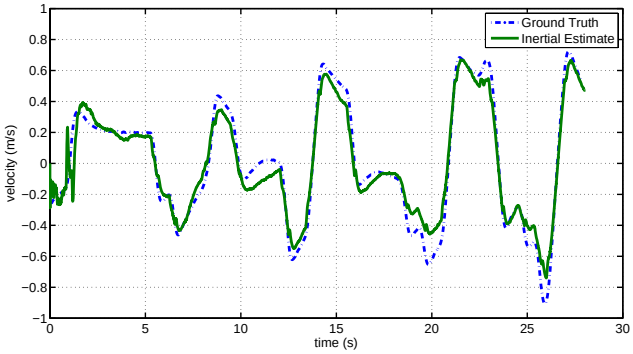


(b)

Figure 3: Comparison of ground truth and inertial attitude estimates of AR Drone, (a) - Roll angle (ϕ), (b) - Pitch angle (θ)



(a)



(b)

Figure 4: Comparison of ground truth and inertial velocity estimates of AR Drone, (a) - X Velocity (V_x), (b) - Y Velocity (V_y)

appear not to agree with the quadrotor motion model presented in equations 6 and 7. For example, while one would expect a negative θ angle to induce a forward acceleration, ground truth data between 15sec and 25sec of figure 6(b) violates this expectation. These either point to possible errors in the reported ground truth measurements, or the fact that we may have misinterpreted this data. We are currently coordinating with the authors of [Lee *et al.*, 2010] to further explore this issue.

To conclude this section, Table 1 presents the RMS estimation errors of both experiments detailed above.

Table 1: RMS estimation errors of experiments performed.

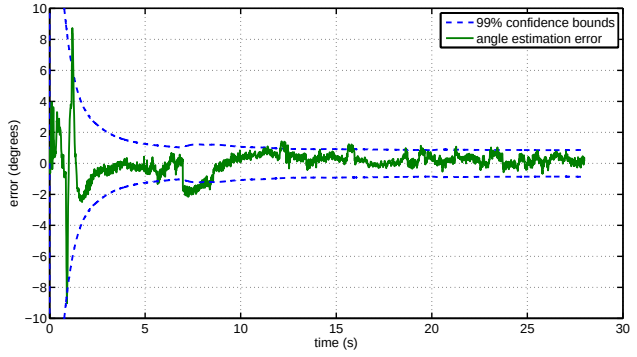
RMS Error	Data Set 1	Data Set 2
Velocity (ms^{-1})	0.14	0.2
Attitude (degrees)	0.5	2

6 Conclusion

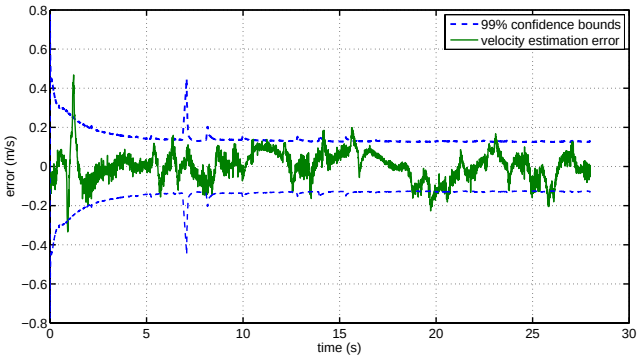
In the paper, we presented the design of a virtual odometer for a quadrotor MAV. Our design is based on a EKF based state estimator, which is capable of estimating roll and pitch angles of the attitude in addition to X and Y components of the translational velocities within a bounded error. This estimator is applied in the context of two different quadrotor MAVs. The resulting attitude and velocity estimates obtained for both quadrotors are drift free.

Moving forward, our research will focus on two aspects. First, we expect to perform further experiments with the Vicon system to analyse and improve the estimator design. The possibility of integrating a magnetometer in to the virtual odometer design will also be explored. This will not only improve the estimation accuracy, but will also enable estimation of the MAV heading (ϕ) angle.

Secondly, we expect to fuse the virtual odometry information with exteroceptive sensors such as vision and GPS. Our ultimate goal is to improve the accuracy, reliability and affordability of quadrotor MAV localisation and navigation systems.

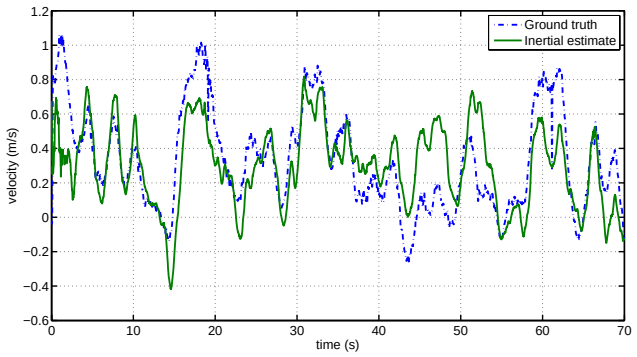


(a)

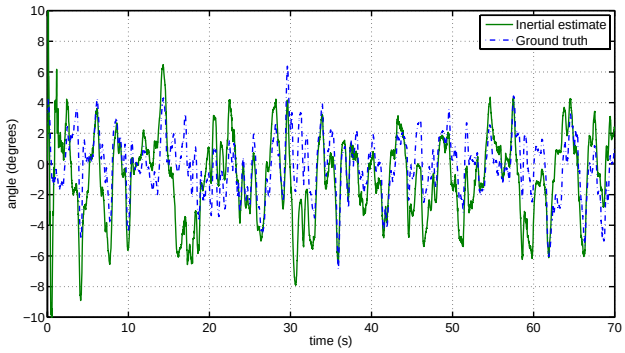


(b)

Figure 5: Estimation errors and their 3σ bounds, (a) - Roll angle (ϕ), (b) - X velocity (V_x)



(a)



(b)

Figure 6: Comparison of ground truth and inertial estimates corresponding to pitch axis, for second data set, (a) - X Velocity ($^b v_x$), (b) - Pitch angle (θ)

7 Acknowledgements

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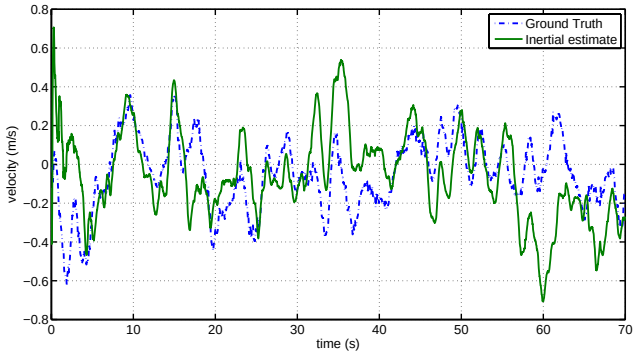
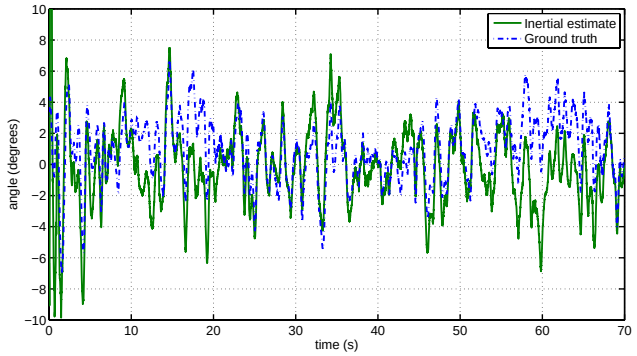


(a)



(b)

Figure 7: Comparison of ground truth and inertial estimates corresponding to roll axis, for second data set, (a) - Y Velocity (${}^{b}v_{y}$), (b) - Roll angle (ϕ)

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