Trajectory Tracking of Robot Manipulators Using Linear and Nonlinear PD-type Controllers

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Abstract

This paper presents trajectory tracking of robot manipulators using linear and nonlinear proportional-derivative (PD) type controllers. It is shown that the controller gains which are nonlinear functions of system states can improve tracking performance and prevent actuator saturation. Furthermore, it is shown that global asymptotic stability of the closed-loop system can be achieved with the proposed PD+ type controllers. Simulation results on a single-link robot and a typical two-degrees-of-freedom robot demonstrate excellent performance using the proposed control schemes.

1 Introduction

Robot manipulator systems are generally nonlinear systems with a large envelope of control space. One important task in controlling a robot system is trajectory tracking control. Trajectory tracking control demands robot arms to move in a free space following a desired trajectory without interacting with the environment [Spong and Vidyasagar, 1989]. How to find an effective controller to achieve accurate tracking of a desired motion has been a major research issue during the past decades. A lot of control schemes have been applied to robot trajectory tracking control, computed torque control, PD control, robust control and adaptive control are four common strategies for controlling robot manipulator systems [Koditschek, 1985; Paden and Panja, 1988; Spong and Vidyasagar, 1989; Wen, 1990;]. Computed torque control is a nonlinear control scheme. It is an effective scheme for controlling of robot manipulators. However, it is difficult to maintain good performance in high-speed operation due to complications of nonlinear effects [Lewis and Frank, 2004]. The closed-loop robot system together a PD controller with gravity compensation has been shown to be globally asymptotically stable provided that the proportional and derivative gains are positive constants [Takegaki and Arimoto, 1981]. A so-called PD+ controller was presented by Koditschek, Paden and Panja in [Koditschek, 1985] and [Paden and Panja, 1988]. Variable-gain PD controllers for position control and motion control of manipulators have been implemented in [Kelly and Carelli, 1996]. Furthermore, sufficient conditions for global asymptotic stability of a class of nonlinear PD-type controllers are also given in [Kelly and Carelli, 1996]. A motion control strategy for robot manipulators with inverse dynamics and nonlinear PD gains is presented in [Morales and Carelli, 2003].

Robust control techniques have been investigated for robot manipulator control in [Battilotti and Lanari, 1997; Ortega and Spong, 1989; Spong, 1992; Wang and Peng, 2007]. These controllers are also globally asymptotically stable. It is quite common to use a regressor matrix in the conventional approach to analyse and design an adaptive control system for robot manipulators [Ortega and Spong, 1989; Battilotti and Lanari, 1997]. An adaptive and robust tracking controller is proposed for robotic system under dynamics uncertainties and external disturbances in [Wang and Feng, 2007], this controller can guarantee robustness to parametric and dynamics uncertainties. Recently, much research effort has been directed towards design of intelligent and hybrid controllers in the field of controlling robot manipulators [Neo and Er, 1995; Feng, 1997; Huerta and Antonio, 1999; Er and Kang, 1997; Sun and Er, 2004]. In [Neo and Er, 1995], an adaptive fuzzy robot control algorithm which consists of a fuzzy controller and an adaptive law to update unknown dynamics online is developed. A new stable tracking controller for robot manipulators based on computed torque method and radial basis function (RBF) neural networks is proposed in [Feng, 1997]. A modified PD-type control system using RBF neural networks compensator is proposed in [Huerta and Antonio, 1999]. A robot learning controller based on recurrent neural networks (RNN) is proposed in [Yan and Li, 1997]. RNN is used to model the inverse dynamics while a PD controller is added to handle unmodeled dynamics and disturbances.

In this paper, we present linear and nonlinear PD+ type controllers, which are applied for trajectory tracking control problem of robot manipulators. The organization of the paper is as follows. Section 2 describes the robot dynamics and useful properties along with linear and nonlinear PD+ type control schemes. Stability analysis of these proposed controllers is addressed using the Lyapunov method in Section 3. Experimental results of these proposed controllers on a single-link and a typical two-link manipulator are presented in Section 4, and finally, brief concluding remarks are given in Section 5.

2 Model of Manipulator Dynamics, Properties and Control Schemes

2.1 Model of Manipulator Dynamics

In the absence of friction and other disturbances, the dynamics of a serial n - link rigid robot can be written as [Spong and Vidyasagar, 1989]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where q is the $n\times 1$ vector of joint displacements, τ is the $n\times 1$ vector of applied torques or forces, $M(q)$ is an $n\times n$ symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})$ denotes an $n\times n$ matrix of centrifugal and Coriolis torques, and $G(q)$ denotes the $n\times 1$ vector of gravitational torques. Assume that the links are joined together with revolute joints and matrix $C(q,\dot{q})$ is defined using the Christoffel symbols [Spong and Vidyasagar, 1989].

2.2 Properties of Manipulator Dynamics

The model (1) exhibits some important properties of [Spong and Vidyasagar, 1989] which can be exploited to facilitate controller design.

- Property 1: The inertia matrix $M(q)$ is symmetric and positive definite.
- Property 2: Using the following expression for the elements c_{kj} of matrix $C(q,\dot{q})$

$$c_{kj} = \sum_{i=1}^{n} c_{ijk}(q)\dot{q}_i \tag{2}$$

where c_{ijk} are the Christoffel's symbols given by

$$c_{ijk} = \frac{1}{2}\left\{\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ki}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k}\right\} \tag{3}$$

where m_{ij} is the ij -element of matrix $M(q)$. It can be shown that the matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric. This implies that

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T \tag{4}$$

2.3 Linear and Nonlinear PD+ type Control Schemes

The goal of tracking control for robot manipulators is to design a control law $\tau(q,\dot{q},q_d,\dot{q}_d,\ddot{q}_d)$ so that $(q,\dot{q})$ tracks $(q_d,\dot{q}_d)$ in some sense, where q_d is the desired trajectory. For position robot control problem, it is a constant vector; for motion control, it may be a continuous vector function of time. A simple control scheme for trajectory control is a PD+ controller with gravity compensation given by

$$\tau = -K_p\tilde{q} - K_v\dot{\tilde{q}} + M(q)\ddot{q}_d + C(q,\dot{q})q_d + G(q) \tag{5}$$

where K_p and K_V are the proportional and derivative gain matrix respectively and $\tilde{q} = q - q_d$ is the position error while $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ is the derivative of the error. It has been proven by Takegaki that the closed-loop system is globally asymptotically stable provided that K_p and K_v are symmetric positive definite matrices with a PD controller [Takegaki and Arimoto, 1981].

Because the system performance depends on controller gains K_p and K_v , which are assumed to be constant by using controller (5), it is important to develop a class of PD type controllers with nonlinear PD gains to improve tracking performance and satisfy constraints on the control torque. The nonlinear controller is given by [Kelly and Carelli, 1996]

$$\tau = -K_p(\tilde{q})\tilde{q} - K_v(q_d,\tilde{q},\dot{\tilde{q}})\dot{\tilde{q}} + M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) \tag{6}$$

where $K_p(\tilde{q})$ and $K_v(q_d,\tilde{q},\dot{\tilde{q}})$ are nonlinear functions of the robot states.

3 Stability Analysis of Proposed Controllers

3.1 Stability of Linear Gain PD Controller

Consider model (1) together with a linear gain PD controller given by (5), by using Matrosov's theorem, it has been proven by Paden and Panja [Paden and Panja, 1988] that the closed-loop system is globally asymptotically stable provided that K_p, K_v are symmetric asymptotically stable matrices.

3.2 Stability of Nonlinear Gain PD Controller

Consider the robot dynamics model (1) together with the control law given by (6), let the gain matrices $K_p(\tilde{q})$ and $K_v(q_d,\tilde{q},\dot{\tilde{q}})$ have the following structure [Kelly and Carelli, 1996]

$$K_p(\tilde{q}) = \begin{bmatrix} k_{p1}(\tilde{q}_1) & 0 & \cdots & 0 \\ 0 & k_{p2}(\tilde{q}_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & k_{pn}(\tilde{q}_n) \end{bmatrix} \tag{7}$$

$$K_V(q_d,\tilde{q},\dot{\tilde{q}}) = K_V(q_d,\tilde{q},\dot{\tilde{q}})^T > 0 \tag{8}$$

If there exists a class of function $\gamma_i(\cdot)$ such that for all $x \in \Re$ and $i = 1,\cdots,n$, the closed-loop system is globally asymptotically stable and the control aim $\lim_{t\to\infty} q(t) = q_d$ is achieved.

Before stating the stability result, we present the following lemma.

Lemma 1: Let $\gamma(\cdot)$ be a class $\Re$ function and $f:\Re \to \Re$ be a continuous function. If

$$f(x) \geq \gamma(|x|) \quad \forall x \in \Re \tag{9}$$

then

$$\int_0^x f(\sigma)d\sigma > 0 \quad x \neq 0 \in \Re \tag{10}$$

and

$$\int_0^x f(\sigma)d\sigma \to \infty \quad as \quad |x| \to \infty \tag{11}$$

The closed-loop system equation obtained by combining the robot model (1) and nonlinear control law given by (6) can be written as follows:

$$\frac{d}{dt}\begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix} = \begin{bmatrix}\dot{\tilde{q}}\\ M(q)^{-1}[-K_p(\tilde{q})\tilde{q} - K_v(q_d, \tilde{q}, \dot{\tilde{q}})\dot{\tilde{q}} - C(q,\dot{q})\dot{\tilde{q}}]\end{bmatrix} \tag{12}$$

The origin of the state space is its unique equilibrium point. Let the derivative gain matrix $K_v(q_d, \tilde{q}, \dot{\tilde{q}})$ be symmetric positive definite and the proportional gain matrix $K_p(\tilde{q})$ be symmetric and have the structure given by (7).

In order to prove the stability of the closed-loop robotic system, consider the following Lyapunov function candidate of [Kelly and Carelli, 1996]:

$$V(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^T M\dot{q} + \int_0^{\tilde{q}} \delta^T K_p(\delta)d\delta \tag{13}$$

where

$$\int_0^{\tilde{q}} \delta K_p(\delta)d\delta = \int_0^{\tilde{q}_1} \delta_1 K_{p1}(\delta_1)d\delta_1 + \cdots + \int_0^{\tilde{q}_n} \delta_n K_{pn}(\delta_n)d\delta_n \tag{14}$$

The first term of $V(\tilde{q},\dot{q})$ is positive definite function with respect to $\dot{q}$ because $M(q)$ is a positive definite matrix. In addition, by applying Lemma 1, we can show that the integral term $\int_0^{\tilde{q}} \delta^T K_p(\delta)d\delta$ is positive for $\tilde{q} \in \Re$. Therefore, $V(\tilde{q},\dot{q})$ is a globally positive definite function. We consider the time derivative of the Lyapunov function candidate as

$$\dot{V}(\tilde{q},\dot{q}) = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \tilde{q}^T K_p(\tilde{q})\dot{\tilde{q}} \tag{15}$$

Note that

$$\dot{\tilde{q}} = \dot{q} \tag{16}$$

$$\begin{aligned} M(q)\ddot{q} &= \tau - C(q,\dot{q})\dot{q} - G(q) \\ &= -K_p(\tilde{q})\tilde{q} - K_v(q_d, \tilde{q}, \dot{\tilde{q}})\dot{q} - C(q,\dot{q})\dot{q} \end{aligned} \tag{17}$$

By substituting (16) and (17) in (15) we have

$$\begin{aligned} \dot{V}(\tilde{q},\dot{q}) &= \dot{q}^T[-K_p(\tilde{q})\tilde{q} - K_v(q_d, \tilde{q}, \dot{\tilde{q}})\dot{q} - C(q,\dot{q})] \\ &+ \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \tilde{q}^T K_p(\tilde{q})\dot{q} \\ &= \frac{1}{2}\dot{q}^T[\dot{M}(q) - 2C(q,\dot{q})]\dot{q} - \dot{q}^T K_v(q_d, \tilde{q}, \dot{\tilde{q}})\dot{q} \\ &= -\dot{q}^T K_v(q_d, \tilde{q}, \dot{\tilde{q}})\dot{q} \end{aligned} \tag{18}$$

Since $K_v(q_d, \tilde{q}, \dot{\tilde{q}})$ is a symmetric positive definite matrix, $\dot{V}(\tilde{q},\dot{q})$ is a globally negative semi-definite function, hence, the stability of the equilibrium is established.

We apply the LaSalle's theorem [Spong and Vidyasagar, 1989] to prove asymptotic stability. In the region

$$\Omega = \left\{ \begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix} \Big/ \dot{V}(\tilde{q},\dot{\tilde{q}}) = 0 \right\} = \left\{ \begin{bmatrix}\tilde{q}\\ \dot{\tilde{q}}\end{bmatrix} = \begin{bmatrix}\tilde{q}\\ 0\end{bmatrix} \in \Re^{2n} \right\} \tag{19}$$

the origin is the unique invariant set, the unique invariant is $\tilde{q} = 0$. Therefore by applying the LaSalle's theorem we conclude that the origin of the space state is asymptotically stable.

4 Simulation Results

4.1 Linear Gain PD+ Controller

Simulation studies are carried out to verify these two kinds of proposed PD+ type controllers. Two simulation models based on a single-link robot and a typical two-link robot are used to execute the linear PD+ type control scheme. Models are shown in Figure 1 and Figure 5 respectively. The dynamic model of the single-link manipulator is

$$ml^2\ddot{q} + mgl\cos(q) = \tau \tag{20}$$

The planned task is to design a linear PD+ type controller applied to the single-link robot manipulator to follow a desired trajectory. The mass, $m = 2kg$, is considered to be located at the end of the link as shown in Figure 1, the length of the link is $l = 0.25m$ and g is gravity acceleration,

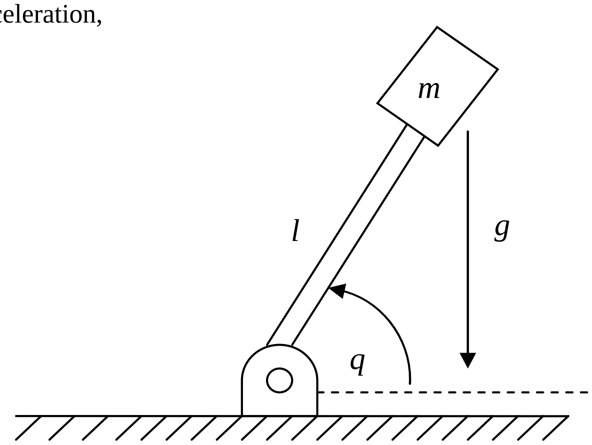

Figure 1 Single-link Robot

$g = 9.8m/s^2$. The desired trajectory is specified as: $q_d = \sin(2\pi t)$.

As stated in the previous section, we can obtain the linear PD+ controller as follows:

$$\tau = -K_v\dot{\tilde{q}} - K_p\tilde{q} + ml^2\ddot{q}_d + mgl\cos(q) \tag{21}$$

The PD gains are chosen as: $K_v = 6.25$, $K_p = 2.5$. Simulation results are shown in Figure2 ~ Figure 4.

Figure 2 shows that the tracking error converges to zero after 12 seconds. Figure 3 shows a comparison of the desired trajectory and a actual trajectory of the single-link robot while Figure 4 shows a comparison of the desired velocity and actual velocity of the single-link robot joint. It is evident from these figures that excellent trajectory tracking is achieved.

A simulation example with a typical two-link robot manipulator (Figure 5) is also performed for the purpose of evaluating the performance of the proposed linear PD+ type control scheme.

The dynamic equation of the proposed manipulator is further derived from [Slotine and Li, 1991] as shown below:

$$
\begin{aligned}
\tau_1 &= [(I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_1 l_1^2) + \\
&2(m_e l_1 l_{ce} \cos\delta_e)\cos q_2 + 2(m_e l_1 l_{ce} \sin\delta_e)\sin q_2]\ddot{q}_1 \\
&+[(I_e + m_e l_{ce}^2) + (m_e l_1 \cos\delta_e)\cos q_2 \\
&+(m_e l_1 l_{ce} \sin\delta_e)\sin q_2]\ddot{q}_2 \\
&-[(m_e l_1 l_{ce} \cos\delta_e)\sin q_2 \\
&-(m_e l_1 l_{ce} \sin\delta_e)\cos q_2]\dot{q}_1\dot{q}_2 \\
&-[(m_e l_1 l_{ce} \cos\delta_e)\sin q_2 \\
&-(m_e l_1 l_{ce} \sin\delta_e)\cos q_2](\dot{q}_1 + \dot{q}_2)\dot{q}_2 \\
&+(m_1 l_{c1} + m_e l_1) g \cos q_1 + m_e g l_{ce} \cos(q_1 + q_2) \\
\tau_2 &= [(I_e + m_e l_{ce}^2) + (m_e l_1 l_{ce} \cos\delta_e)\cos q_2 \\
&+(m_e l_1 l_{ce} \sin\delta_e)\sin q_2]\ddot{q}_1 + (I_e + m_e l_{ce}^2)\ddot{q}_2 \\
&+[(m_e l_1 l_{ce} \cos\delta_e)\sin q_2 - (m_e l_1 l_{ce} \sin\delta_e)\cos q_2]\dot{q}_1^2 \\
&+ m_e g l_{ce} \cos(q_1 + q_2)
\end{aligned} \tag{22}
$$

The parameters of the two-link planar manipulator used for simulation studies are as follows:

m_1	mass of link 1 = 1kg;
L_1	length of link 1 = 1m;
m_e	mass of link 2 and payload = 2kg;
δ_e	angle of payload with respect to link 2 = 30° ;
I_1	centroidal moment of inertia of link 1 = 0.12kg.m^2;
L_{c1}	length of centre of gravity of link 1 from the axis of rotation = 0.5m;
I_e	centroidal moment of inertia of link 2 and payload = 0.25kg.m^2;
L_{ce}	length of centre of gravity of link 2 and payload from the axis of rotation = 0.6m.

By using Property 3, we can obtain the matrix $C(q,\dot{q})$ as follows:

$$
C(q,\dot{q}) = \begin{bmatrix} h\dot{q}_2 & h(\dot{q}_1 + \dot{q}_2) \\ -h\dot{q}_1 & 0 \end{bmatrix} \tag{23}
$$

where

$$
h = -m_e l_1 l_{ce} \cos\delta_e \sin(q_2) + m_e l_1 l_{ce} \sin\delta_e \cos(q_2) \tag{24}
$$

The gravitational matrix $G(q)$ is as follows:

$$
G(q) = \begin{bmatrix} (m_1 l_{c1} + m_e l_1) g \cos(q_1) + m_e l_{ce} \cos(q_1 + q_2) \\ m_e l_{ce} \cos(q_1 + q_2) \end{bmatrix} \tag{25}
$$

Substituting the parameters into the dynamic equations, we obtain the following matrices:

$$
\begin{aligned}
&M(q) \\
&= \begin{bmatrix} 3.34 + 2.08\cos q_2 + 1.2\sin q_2 & 0.97 + 1.04\cos q_2 + 0.6\sin q_2 \\ 0.97 + 1.04\cos q_2 + 0.6\sin q_2 & 0.97 \end{bmatrix} \\
&C(q,\dot{q}) \\
&= \begin{bmatrix} (-1.04\sin q_2 + 0.6\cos q_2)\dot{q}_2 & (-1.04\sin q_2 + 0.6\cos q_2)(\dot{q}_1 + \dot{q}_2) \\ (1.04\sin q_2 - 0.6\cos q_2)\dot{q}_1 & 0 \end{bmatrix} \\
&G(q) \\
&= \begin{bmatrix} 24.5\cos q_1 + 11.76\cos(q_1 + q_2) \\ 11.76\cos(q_1 + q_2) \end{bmatrix}
\end{aligned} \tag{26}
$$

The desired trajectories in terms of link angular positions are chosen to be $\cos(\pi t)\ rad$ and $\sin(\pi t)\ rad$ for q_1 and q_2 respectively. The linear PD+ control law is given as:

$$
\tau = -K_p\tilde{q} - K_v\dot{\tilde{q}} + M(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + G(q) \tag{27}
$$

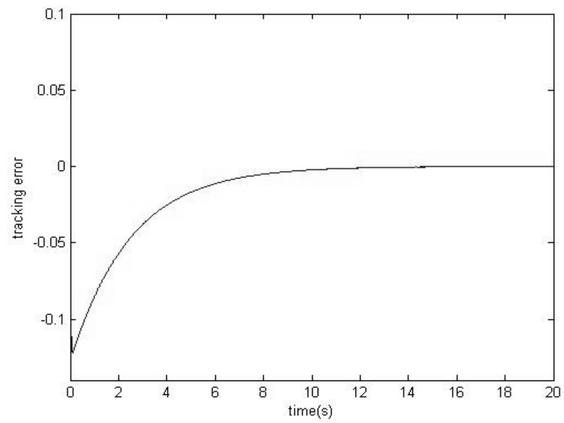

Figure 2 Tracking Error

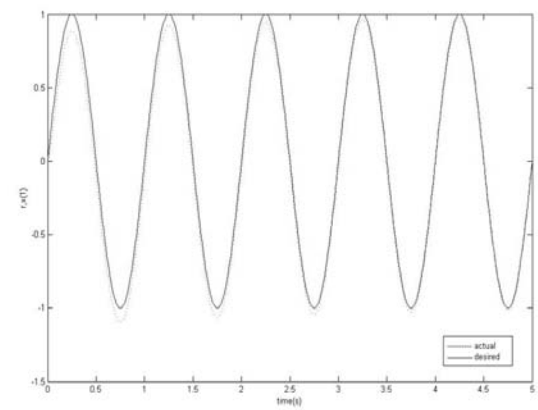

Figure 3 Tracking performance – position comparison

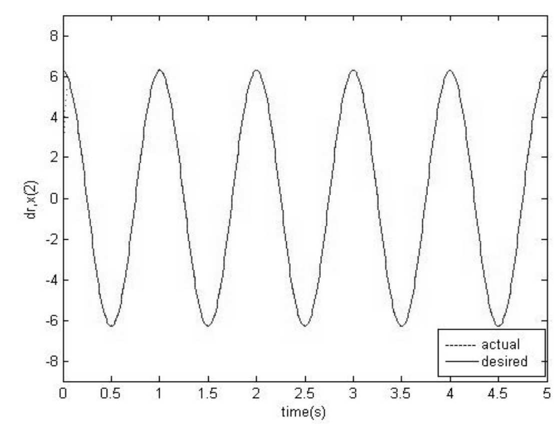

Figure 4 Tracking performance – velocity comparison

where $K_p = diag(K_{p11}, K_{p22})$ and $K_v = diag(K_{v11}, K_{v22})$. In this simulation, we choose the linear PD gains matrices as : $K_p = diag(5, 15), K_v = diag(10, 25)$.

Simulation results are shown in Figure 6 ~ Figure 11. Figure 6 ~ Figure 9 show comparisons of the desired trajectories and actual trajectories of the two-link manipulator. Figure 10 shows that the tracking position error of joint 1 converges to zero after about 10 seconds. Figure 11 shows that the tracking position error of joint 2 converges to zero after about 15 seconds.

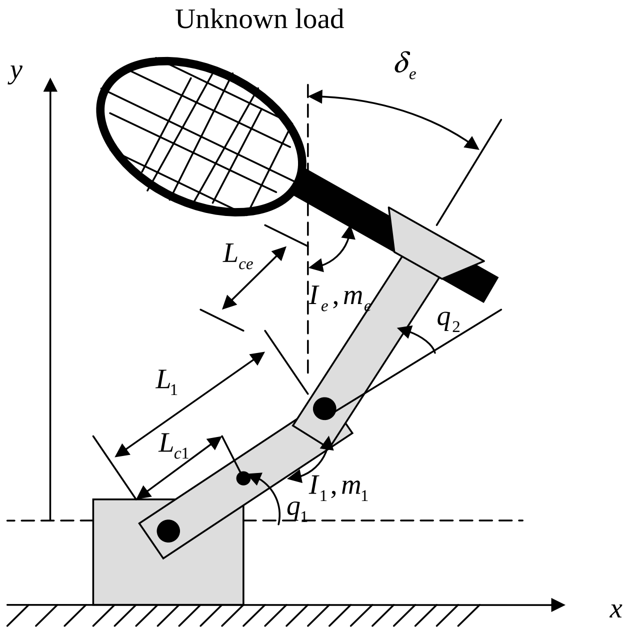

Figure 5 Articulated two-link robot manipulator

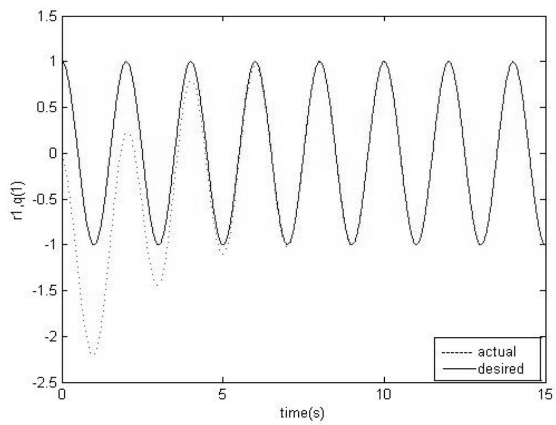

Figure 6 Position tracking of link 1 – linear gains

From these experimental simulation results, we can conclude that the proposed linear PD+ type controller can be applied to single-link and typical two-link robot manipulators. Both of these manipulators can tracking the desired trajectories very well.

4.2 Nonlinear Gain PD+ Controller

As foreshadowed earlier, considering the nonlinear PD+ type control law given by (6), we choose the nonlinear PD gain matrices as follows:

$$K_p(\tilde{q}) = K_{p1}B_p(\tilde{q}), \quad K_v(q_d, \tilde{q}, \dot{q}) = K_{v1}B_v(\dot{\tilde{q}}) \qquad (28)$$

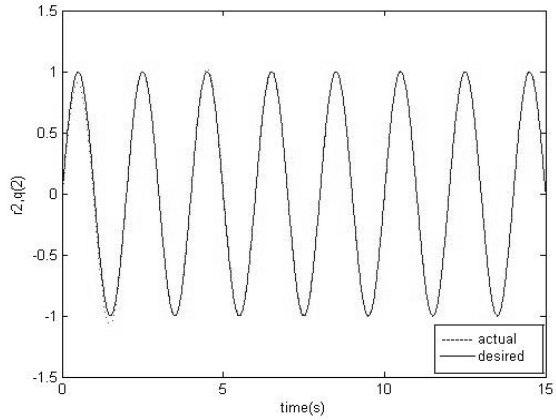

Figure 7 Position tracking of link 2 – linear gains

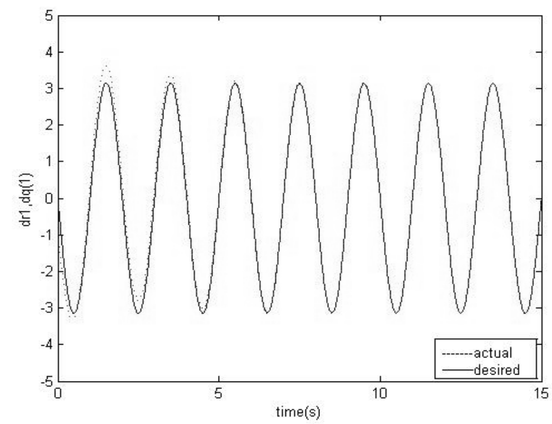

Figure 8 Velocity tracking of joint 1 – linear gains

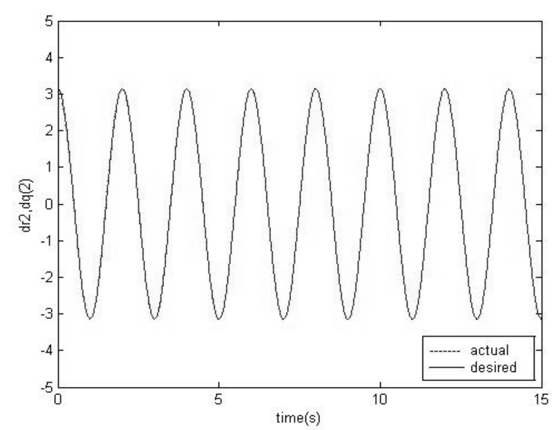

Figure 9 Velocity tracking of joint 2 – linear gains

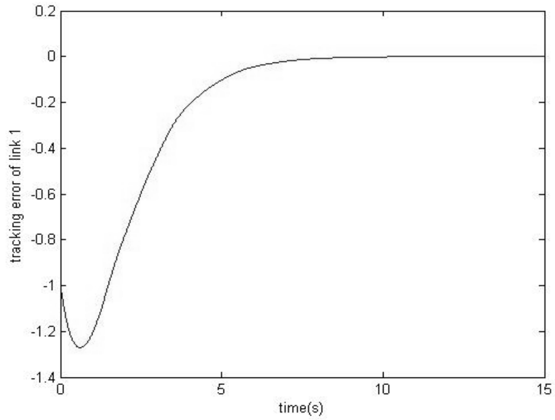

Figure 10 Tracking error of joint 1 – linear gains

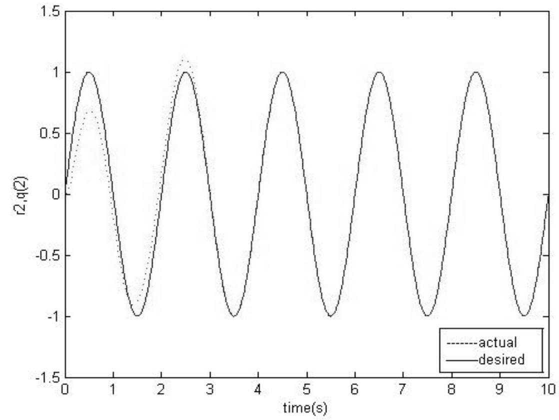

Figure 13 Position tracking of link 2 – nonlinear gains

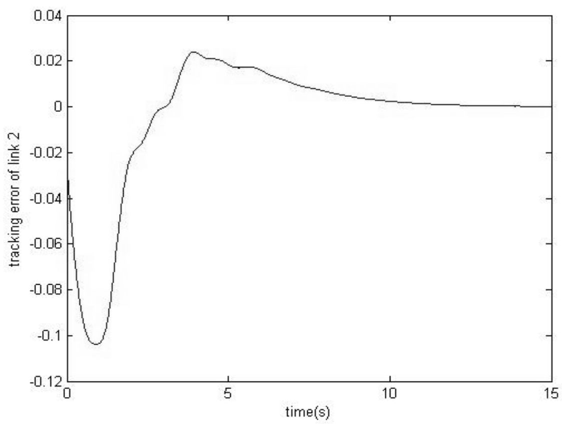

Figure 11 Tracking error of joint 2 – linear gains

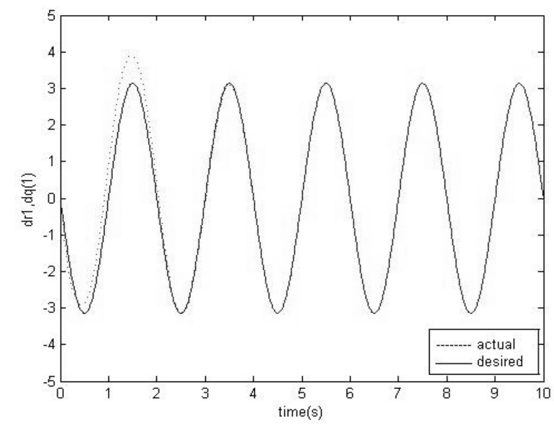

Figure 14 Velocity tracking of joint 1 – nonlinear gains

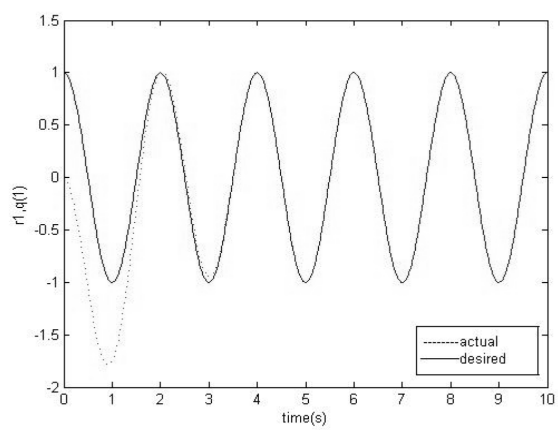

Figure 12 Position tracking of link 1 – nonlinear gains

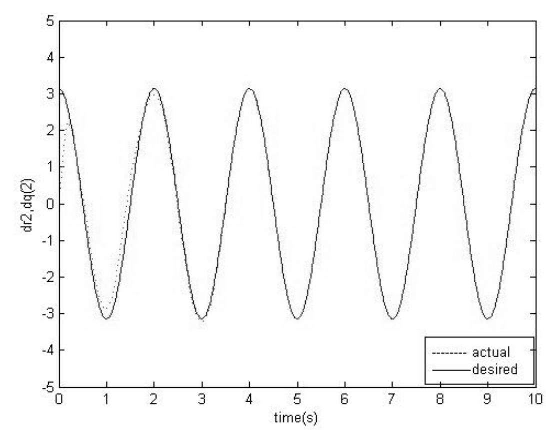

Figure 15 Velocity tracking of joint 2 – nonlinear gains

where

$$
\begin{aligned}
&K_{p1} = diag(K_{p11}, \cdots, K_{p1n}) \\
&K_{v1} = diag(K_{v11}, \cdots, K_{v1n}) \\
&B_p(\tilde{q}) = diag\left(\frac{1}{\alpha_1 + |\tilde{q}_1|}, \cdots, \frac{1}{\alpha_n + |\tilde{q}_n|}\right) \\
&B_v(\dot{\tilde{q}}) = diag\left(\frac{1}{\beta_1 + |\dot{\tilde{q}}_1|}, \cdots, \frac{1}{\beta_1 + |\dot{\tilde{q}}_n|}\right)
\end{aligned}
\tag{29}
$$

Here, $K_{p1i}, K_{v1i}, \alpha_i, \beta_i, (i = 1, \cdots, n)$ are all positive constants. There functions satisfy conditions of Lemma l with the class $\Re$ functions $\gamma_i(|\tilde{q}_i|) = \varepsilon_i \tilde{q}_i / [\alpha_i + |\tilde{q}_i|]$ and $K_{p1i} > \varepsilon_i > 0$. On the other hand, $K_v(q_d, \tilde{q}, \dot{q})$ in (28) is a symmetric positive definite matrix. According to Lemma 1, we conclude that the closed-loop robot system is globally asymptotically stable.

We use the model shown in Fig. 5 and choose the same parameters of the manipulator which are given in the simulation of linear gain PD+ controller. In the simulated

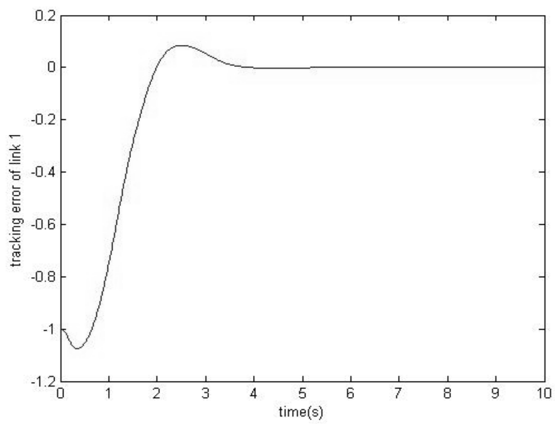

Figure 16 Tracking error of joint 1 – nonlinear gains

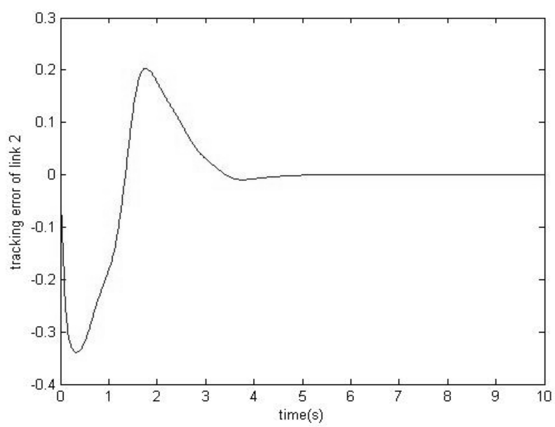

Figure 17 Tracking error of joint 2 – nonlinear gains

trajectory control of robot manipulator by the nonlinear PD+ type conroller, the desired trajectories in terms of link angular positions are chosen to be $\cos(\pi t)\, rad$ and $\sin(\pi t)\, rad$ for q_1 and q_2 respectively. The initial angular position and velocity of both links are all set to zero. The parameters of the proportional gain matrix K_p and the derivative gain matrix K_v are set as follows:

$$K_{p11} = 30, K_{p12} = 25, K_{v11} = 20, K_{v12} = 10 \tag{30}$$

other parameters are set as:

$$\alpha_1 = \alpha_2 = 1, \beta_1 = \beta_2 = 1 \tag{31}$$

Simulation results are shown in Figure 12 ~ Figure 17. Figure 12 ~ Figure 15 depict the tracking performance of the manipulator. Figure 16 and Figure 17 depict the tracking error of joint 1 and joint 2 respectively. It is clear from these figures that the proposed nonlinear PD+ type controller shows good tracking performance and the errors converge to zero quickly.

5 Conclusions

The problem of trajectory tracking for robot manipulators with linear and nonlinear PD+ type controllers has been investigated in this paper. It has been proven that the two proposed PD+ type controllers are globally asymptotically stable. Simulation results show excellent performance by two proposed controllers and trajectory tracking errors converge to zero quickly.

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