A New Family of Maximally Regular T2R1-type Spatial Parallel Manipulators with Unlimited Rotation of the Moving Platform

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Abstract

This paper presents a new family of maximally regular T2R1-type spatial parallel manipulators (PMs) with three degrees of freedom. The mobile platform performs two independent translations (T2) and one independent rotation (R1) whose axis lies in the plane of translations. This family is called spatial T2R1-type to distinguish it from the planar T2R1-type PMs in which the rotation axis is perpendicular to the plane of translation. A method is proposed for structural synthesis of maximally regular T2R1-type spatial PMs based on the theory of linear transformations. A one-to-one correspondence exists between the actuated joint velocity space and the external velocity space of the moving platform. The Jacobian matrix mapping the two vector spaces of maximally regular T2R1-type PMs presented in this paper is the 3×3 identity matrix throughout the entire workspace. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission capabilities. Moreover, the moving platform has unlimited rotational capabilities. To the best knowledge of the author, this paper presents for the first time solutions of maximally regular T2R1-type PMs with unlimited rotation of the moving platform.

1 Introduction

Parallel manipulation has been the subject of study of much robotic research during the last two decades. Parallel robotic manipulators (PMs) are composed of an end effector (moving platform) connected to the base (fixed platform) by at least two kinematic chains called legs or limbs. The moving platform of a *TaRb*-type parallel robot can complete $0 \leq a \leq 3$ independent translational motions (*T*) and $0 \leq b \leq 3$ rotational motions (*R*) with $2 \leq a+b \leq 6$.

Rigidity, accuracy, high speed, and high load-to-weight ratio are the main merits of PMs. With respect to serial manipulators, disadvantages include a smaller workspace, complex command and a lower dexterity due to a high motion coupling and multiple singularities inside their workspace. Maximally regular and fully-isotropic parallel manipulators overcome these disadvantages. They have a very simple command and are highly energy-saving due to the fact that for a unidirectional motion only one motor works as in serial cartesian manipulators.

It is known that the Jacobian matrix of a robotic manipulator is the matrix mapping (i) the actuated joint velocity space and the end-effector velocity space, and (ii) the static load on the end-effector and the actuated joint forces or torques. Isotropy of a robotic manipulator is related to the condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is one. Thus, the condition number of the Jacobian matrix is an interesting performance index characterizing the distortion of a unit hypersphere under this linear mapping. The condition number of the Jacobian matrix was first used by Salisbury and Craig [1982] to design mechanical fingers and developed by Angeles [1997] as a kinetostatic performance index of the robotic mechanical systems. The isotropic design aims at ideal kinematic and dynamic performance of the manipulator [Fattah and Ghasemi, 2002]. In an isotropic configuration, the sensitivity of a manipulator is minimal with regard to both velocity and force errors and the manipulator can be controlled equally well in all directions. The concept of kinematic isotropy has been used as a criterion in the design of various parallel manipulators [Zanganeh and Angeles, 1997], [Tsai and Huang, 2003].

Five types of PMs are identified in [Gogu, 2008]: (i) maximally regular PMs, if the Jacobian J is an identity matrix throughout the entire workspace, (ii) fully-isotropic PMs, if the J is a diagonal matrix with identical diagonal elements throughout the entire workspace, (iii) PMs with uncoupled motions if J is a diagonal matrix with different diagonal elements, (iv) PMs with decoupled motions, if J is a triangular matrix and (v) PMs with coupled motions if J is neither a triangular nor a diagonal matrix. Fully-isotropic PMs give a one-to-one mapping between the actuated joint velocity space and the external velocity space. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission.

The term maximally regular parallel robot was recently coined by Merlet [2006] to define isotropic robots. In this paper, this term is used to define just one particular case of fully-isotropic PMs, when the Jacobian matrix is an identity matrix throughout the entire workspace. Various solutions of maximally regular and fully-isotropic PMs have been very recently presented in the literature [Carricato and Parenti-Castelli, 2002; Kim and Tsai, 2002; Kong and Gosselin, 2002; Gogu, 2004a;b,c; 2006, 2007].

Spatial T2R1-type PMs are used in applications that require two independent planar translations (T2) and one independent rotation (R1) of the mobile platform around an axis lying in the plane of translation. The major applications of this type of PMs are: flight and motion simulation, pointing and tracking [Dunlop and Jones, 1999], assembling and machining [Kong and Gosselin, 2005], when the rotation axis undergoes planar translation. The solutions of T2R1-type PMs known in the literature [Kong and Gosselin, 2005; Refaat *et al*., 2006; Liu *et al*., 2005] have coupled motions.

The classical methods used for structural synthesis of PMs can be divided into four approaches:
(i) the methods based on displacement group theory [Hervé 1995, 1999, 2004; Hervé and Sparacino, 1991; Karouia and Hervé, 2000; Angeles, 2004; Huynh and Hervé 2005; Lee and Hervé, 2006; Rico *et al*. 2006],
(ii) the methods based on screw algebra [Tsai, 1999; Fang and Tsai, 2002; Frisoli *et al*. 2000; Kong and Gosselin 2004a, 2004b, 2004c, 2006; 2007; Huang and Li, 2002, 2003; Carricato 2005],
(iii) the method based on velocity loop equations [Di Gregorio and Parenti-Castelli, 1998; Di Gregorio, 2002; Carricato and Parenti-Castelli, 2002, 2003] and
(iv) the method based on the theory of linear transformations [Gogu, 2004a,b,c, 2006, 2007, 2008].

The approach presented in this paper is founded on the theory of linear transformations and integrates the new formulae of mobility, connectivity, redundancy and overconstraint of parallel manipulators proposed in [Gogu 2005a,b, 2008].

The main aims of this paper are to present for the first time maximally regular overconstrained and non overconstrained solutions of T2R1-type spatial PMs with unlimited rotation of the moving platform around an axis undergoing planar translations, and to show the corresponding structural synthesis approach.

2 Kinematic Criteria for Structural Synthesis

The main kinematic criteria used for structural synthesis are associated with mobility, connectivity, redundancy and overconstraint of parallel mechanisms.

Mobility is the main structural parameter of a mechanism and also one of the most fundamental concepts in the kinematic and the dynamic modelling of mechanisms [Gogu, 2005a]. IFToMM terminology defines the mobility or the degree of freedom as the number of independent coordinates required to define the configuration of a kinematic chain or mechanism [Ionescu, 2003].

Mobility M is used to verify the existence of a mechanism $(M>0)$, to indicate the number of independent parameters in robot modelling and to determine the number of inputs needed to drive the mechanism. Earlier works on the mobility of mechanisms go back to the second half of the nineteenth century. During the twentieth century, sustained efforts were made to find general methods for the determination of the mobility of any rigid body mechanism. Various formulae and approaches were derived and presented in the literature. Contributions have continued to emerge in the last years. Mobility calculation still remains a central subject in the theory of mechanisms.

The various methods proposed in the literature for mobility calculation of the closed loop mechanisms fall into two basic categories:
a) approaches for mobility calculation based on setting up the constraint equations and calculating their rank for a given position of the mechanism with specific joint locations,
b) formulae for a quick calculation of mobility with no need to develop the set of constraint equations.

The approaches used for mobility calculation based on setting up the constraint equations and their rank calculation are valid without exception. The major drawback of these approaches is that the mobility cannot be determined quickly without setting up the kinematic or static model of the mechanism. Usually the kinematic model is expressed by the closure equations that must be analyzed for dependency. There is no way to derive information about mechanism mobility without performing position, velocity or force analysis by using analytical tools (screw theory, linear algebra, affine geometry, Lie algebra, etc). For this reason, the real and practical value of these approaches is very limited in spite of their valuable theoretical foundations. Moreover, the rank of the constraint equations is calculated in a given position of the mechanism with specific joint locations. The mobility calculated in relation to a given configuration of the mechanism is an instantaneous mobility which can be different from the general mobility (global mobility or gross mobility). Global mobility has a single value for a given mechanism; it is a global parameter characterizing the mechanism in all its configurations in a free-of-singularity branch. Instantaneous mobility is a local parameter characterizing the mechanism in a given configuration including singular ones. In a singular configuration the instantaneous mobility could be different from the global mobility.

A formula for quick calculation of mobility is an explicit relationship between the following structural parameters: the number of links and joints, the motion/constraint parameters of joints and of the mechanism. Usually, these structural parameters are easily determined by inspection without any need to develop the set of kinematic or static constraint equations.

The classical formulae for a quick calculation of mobility, known as Chebychev-Grübler-Kutzbach formulae do not fit many classical mechanisms and recent parallel robots. These formulae have been recently reviewed in [Gogu, 2005a] and their limits have been set up in [Gogu, 2005b]. New formulae for quick calculation of the mobility have been proposed in [Gogu, 2005c] and demonstrated via the theory of linear transformations. A development of these contributions can be found in [Gogu, 2008].

The connectivity between two links of a mechanism represents the number of independent finite and/or infinitesimal displacements allowed by the mechanism between the two links.

The number of overconstraints of a mechanism is given by the difference between the maximum number of joint kinematic parameters that could lose their independence in the closed loops, and the number of joint kinematic parameters that actually lose their independence in the closed loops.

The case of a parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$ in which the end-effector $n \equiv n_{Gi}$ is connected to the reference link $1 \equiv 1_{Gi}$ by k simple or complex kinematic chains G_i ($1_{Gi}\text{-}2_{Gi}\text{-}\ldots n_{Gi}$) is considered. The parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$ is characterized by:

R_{Gi} - the vector space of relative velocities between the distal (extreme) links n_{Gi} and 1_{Gi} in the kinematic chain G_i disconnected from the parallel mechanism F,

R_F - the vector space of relative velocities between the distal links $n \equiv n_{Gi}$ and $1 \equiv 1_{Gi}$ in the parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$,

$S_{Gi}=dim(R_{Gi})$ –the connectivity between the distal links n_{Gi} and 1_{Gi} in the kinematic chain G_i disconnected from the mechanism F,

$S_F=dim(R_F)$ - the connectivity between the distal links $n \equiv n_{Gi}$ and $1 \equiv 1_{Gi}$ in the mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$.

The new formulae demonstrated in [Gogu, 2008] for mobility M, connectivity S_F, overconstraint N and redundancy T of the parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$ are used as kinematic criteria for structural synthesis of parallel robotic manipulators:

$$M = \sum_{i=1}^{p} f_i - r \qquad (1)$$

$$N=6q-r \qquad (2)$$

$$T=M-S_F \qquad (3)$$

where

$$S_F = dim(R_F) = dim(R_{G1} \cap R_{G2} \cap \ldots \cap R_{Gk}) \qquad (4)$$

$$r = \sum_{i=1}^{k} S_{Gi} - S_F + r_l \qquad (5)$$

and

$$r_l = \sum_{i=1}^{k} r_l^{Gi} \qquad (6)$$

In Eqs. (1)-(6) p represents the total number of joints, q is the total number of independent closed loops in the sense of graph theory, f_i is the mobility of the ith joint, r - the number of joint parameters that lose their independence in the mechanism F, r_l^{Gi} - the number of joint parameters that lose their independence in the closed loops of limb G_i, r_l - the total number of joint parameters that lose their independence in the closed loops that may exist in the limbs of the mechanism F.

The intersection in Eq. (4) is consistent only if the operational velocity spaces R_{Gi} are defined by the velocities of the same point situated on the end-effector. This point is called the characteristic point, and is denoted in this paper by H. It is a point with the most restrictive motion of the end-effector.

The connectivity S_F of the end-effector $n \equiv n_{Gi}$ in the mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$ is less than or equal to the mobility M of the mechanism F.

The basis of the vector space R_F of relative velocities between the end-effector $n \equiv n_{Gi}$ and the reference link $1 \equiv 1_{Gi}$ in the mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$ dos not vary with the position of the characteristic point on the moving platform $n \equiv n_{Gi}$.

When there are various ways to choose the basis of the operational spaces, the bases of R_{Gi} in Eq. (4) are selected such that the minimum value of S_F is obtained. This value must be compatible with the motion state of the mechanism which can be a moving kinematic structure or fixed (immobile) structure. By this choice, the result of Eq. (4) fits in with the general mobility definition as the minimum value of the instantaneous mobility in a free of singularity branch.

The finite displacements and the velocities in the actuated joints are denoted by q_i and $\dot{q}_i$, and the linear and angular velocities of the characteristic point H situated on the moving platform by $v_1 = \dot{x}$, $v_2 = \dot{y}$ and $\omega_\alpha = \dot{\alpha}$.

With these notations, the linear mapping between the actuated joint velocity space and the end-effector velocity space for the $T2R1$-type PMs is defined by:

$$\begin{bmatrix} v_1 \\ v_2 \\ \omega_\alpha \end{bmatrix} = [J] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \qquad (7)$$

where J is the Jacobian matrix.

If the following conditions for mobility, connectivity between the mobile and the fixed platform and for the base (R_F) of the vector space of relative velocities of the mobile platform are fulfilled, than the parallel mechanism F has uncoupled motions.

a) General conditions for any position of the mechanism when $\dot{q}_1 \neq 0$, $\dot{q}_2 \neq 0$, and $\dot{q}_3 \neq 0$:

$$3 \leq M_{Gi}=S_{Gi} \leq 6 \quad (i=1,2,3) \qquad (8)$$

$$M=3 \qquad (9)$$

$$S_F=3 \qquad (10)$$

$$(R_F)=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha) \qquad (11)$$

b) Particular conditions when $\dot{q}_1 = 0$

$$S_F=2 \tag{12}$$
$$(R_F)=(\boldsymbol{v}_2, \boldsymbol{\omega}_\alpha). \tag{13}$$
c) Particular conditions when $\dot{q}_2=0$
$$S_F=2 \tag{14}$$
$$(R_F)=(\boldsymbol{v}_1, \boldsymbol{\omega}_\alpha). \tag{15}$$
d) Particular conditions when $\dot{q}_3=0$
$$S_F=2 \tag{16}$$
$$(R_F)=(\boldsymbol{v}_1, \boldsymbol{\omega}_\alpha). \tag{17}$$

Condition $M_{Gi}=S_{Gi}$ indicates the fact that each limb is non-redundant and $M=S_F$ the fact that the parallel mechanism is also non redundant. In the previous notations, the bold letters stand for the independent vectors of the base of the operational vector space.

If the Jacobian J is a diagonal matrix the parallel manipulator has uncoupled motions and the singular values are equal to the diagonal elements. The Jacobian J of a fully isotropic mechanism has non zero identical singular values and unit condition number. Consequently, if all elements of a diagonal Jacobian matrix are identical, than the PM is fully isotropic. The existence of this mechanism involves:
$$v_1=\lambda\ \dot{q}_1 \tag{18}$$
$$v_2=\lambda\ \dot{q}_2 \tag{19}$$
$$\omega_\alpha=\lambda\ \dot{q}_3 \tag{20}$$
where λ is the value of the diagonal elements. The parallel mechanism respecting conditions (18)-(20) is fully-isotropic and implicitly it has uncoupled motions. This mechanism realizes a homothetic transformation of coefficient λ between the velocity of the actuated joints and the velocity of the moving platform. When $\lambda=1$ the Jacobian matrix becomes the 3×3 identity matrix and the PM is maximally regular. This paper focuses on the structural synthesis of this type of PMs.

3 Maximally Regular T2R1-type PMs

The basic kinematic structure of a maximally regular T2R1-type PM is obtained by concatenating three limbs G_1 $(1_{G1}\equiv 0\text{-}\ldots\text{-}n_{G1}\equiv n)$, G_2 $(1_{G2}\equiv 0\text{-}\ldots\text{-}n_{G2}\equiv n)$, and G_3 $(1_{G3}\equiv 0\text{-}\ldots\text{-}n_{G3}\equiv n)$. The first link 1_{Gi} of each limb is the fixed platform denoted by 0 and the final link is the moving platform denoted by n.

An evolutionary morphology (EM) approach is used for structural synthesis of each limb G_i $(i=1,2,3)$. EM is formalized by a 6-tuple of design objectives, protoelements (initial components), morphological operators, evolution criteria, morphologies and a termination criterion. The final objectives are expressed by equations (8-17) and (18)-(20) where $\lambda=1$. The protoelements are the revolute and prismatic joints. The morphological operators are: (re)combination, mutation, migration and selection. Cylindrical (C) and spherical (S) joints are also introduced in the evolutionary process by the mutation operator. The morphological operators are deterministic and are applied at each generation of EM. Evolutionary morphology is a complementary method with respect to evolutionary algorithms that start from a given initial population to obtain an optimum solution with respect to a fitness function. EM creates this initial population to enhance the chance of obtaining a "more global optimum" by non-quantified diversification of the initial population. Evolutionary algorithms are optimization oriented methods; EM is a conceptual design oriented method. More details on evolutionary morphology are presented in [Gogu, 2008].

3.1 Overconstrained solutions

Two of the simplest kinematic structures of maximally regular overconstrained T2R1-type spatial PMs are presented in (Figs. 1 and 2). The solution in Fig. 1 has limb G_1 of type $\underline{P}PR$ ($\underline{P}\perp P\perp^{\parallel} R$), limb G_2 of type $\underline{P}C$ ($\underline{P}\perp C$) and the limb G_3 is a homokinetic double universal joint with a telescopic shaft $\underline{R}UPU$-type, where U stands for the universal joint ($R\perp R$). The solution in Fig. 2 uses the same limbs G_2 and G_3 and the limb G_1 is of type $\underline{P}RRR$ ($\underline{P}\parallel R\parallel R\parallel R$). The first joint of each limb G_i $(i=1,\ldots,3)$ is actuated (the underlined joint). The direction of the actuated prismatic joint is parallel to the x-axis in limb G_1 and the y-axis in limb G_2. The input and the output shaft of the homokinetic double universal joint in limb G_3 are parallel to the x-axis. The notations $\perp$ or $\parallel$ between two joints indicates that the joints have orthogonal or parallel axes/directions. In $\underline{P}\perp P\perp^{\parallel} R$, the notation $\perp^{\parallel}$ indicates that the axis of the revolute joint is perpendicular to the direction of the second prismatic joint and parallel to the direction of the actuated prismatic joint. The use of the homokinetic double universal joint with a telescopic shaft gives an unlimited rotational motion to the moving platform 7. This is an important advantage when the moving platform is a rotating tool in assembling and manufacturing applications.

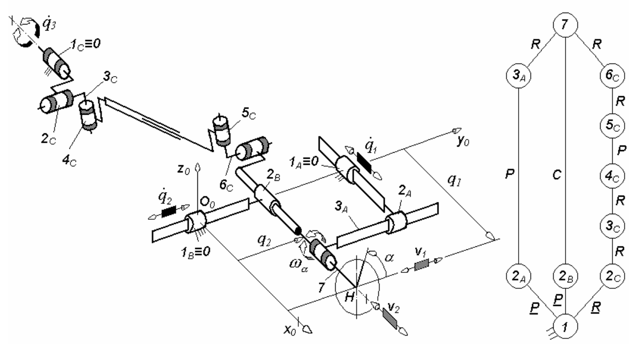


Figure 1. $\underline{P}PR$-$\underline{P}C$-$\underline{R}UPU$-type basic solution of maximally regular T2R1-type PM

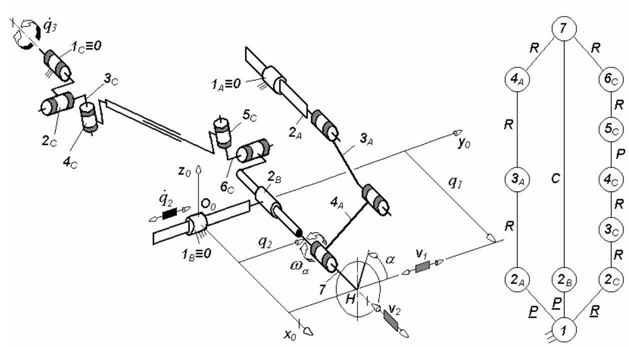
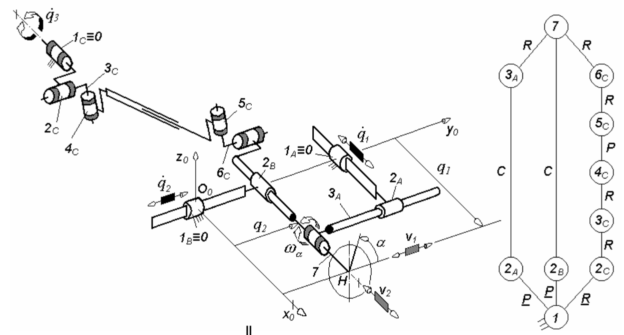
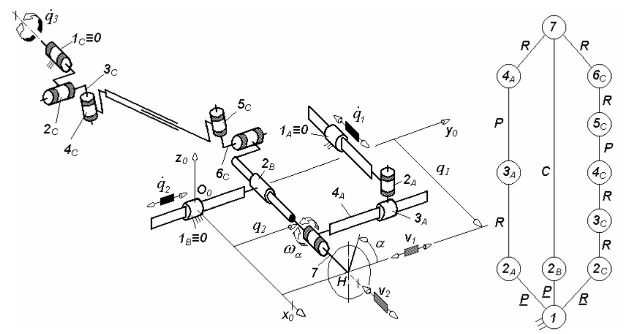


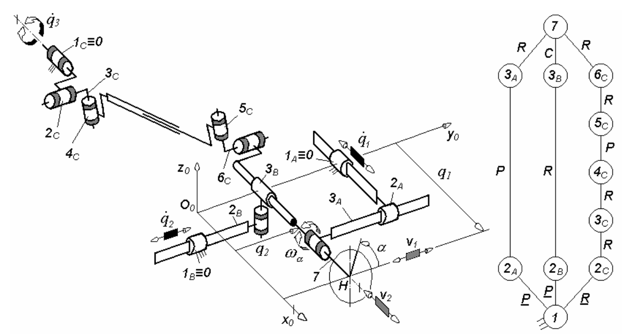
Figure 2. $\underline{P}RRR$-$\underline{P}C$-$\underline{R}UPU$-type basic solution of maximally regular T2R1-type PM



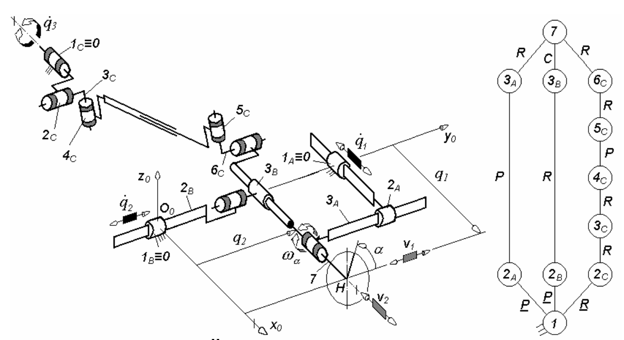
(a) $\underline{P}C{*}R\ (\underline{P} \perp C{*} \perp^{\parallel} R)$-$\underline{P}C$-$\underline{R}UPU$



(b) $\underline{P}R{*}PR\ (\underline{P} \perp R{*} \perp^{\perp} P \perp^{\perp} R)$-$\underline{P}C$-$\underline{R}UPU$

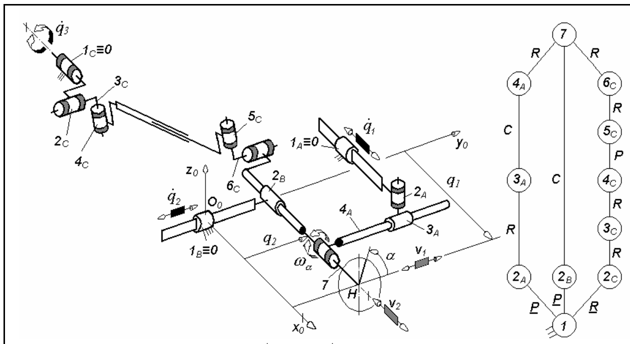


(c) $\underline{P}PR$-$\underline{P}R{*}C\ (\underline{P} \perp R{*} \perp^{\perp} C)$-$\underline{R}UPU$

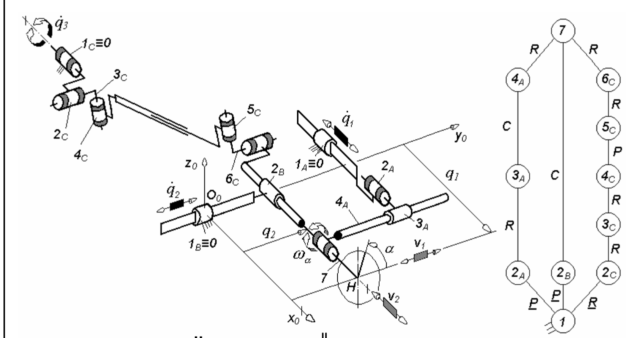


(d) $\underline{P}PR$-$\underline{P}R{*}C\ (\underline{P} \parallel R{*} \perp C)$-$\underline{R}UPU$

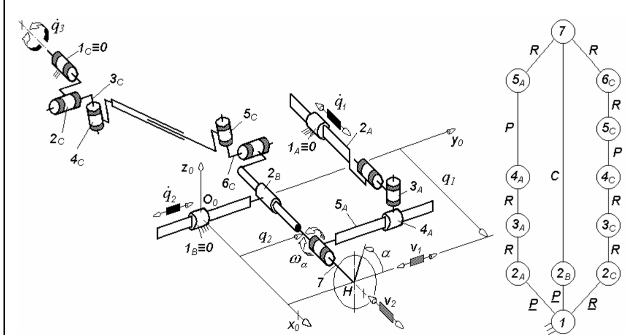
Table 1. Kinematic structures of maximally regular *T2R1*-type spatial PMs with two degrees of overconstraint derived from the basic solution $\underline{P}PR$-$\underline{P}C$-$\underline{R}UPU$ by combining one idle mobility in limb G_1 (**a-b**) or in limb G_2 (**c-d**)



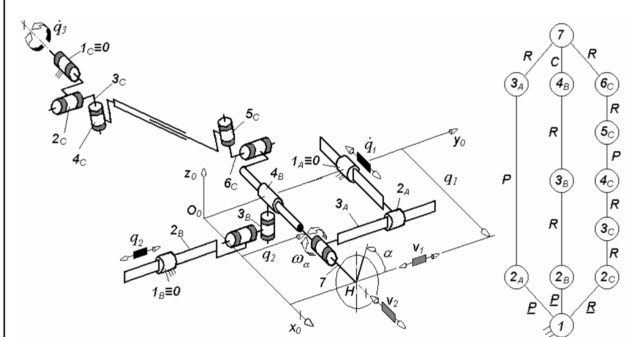
(a) $\underline{P}R{*}C{*}R\ (\underline{P} \perp R{*} \perp^{\perp} C{*} \perp^{\perp} R)$-$\underline{P}C$-$\underline{R}UPU$



(b) $\underline{P}R{*}C{*}R\ (\underline{P} \parallel R{*} \perp C{*} \perp^{\parallel} R)$-$\underline{P}C$-$\underline{R}UPU$

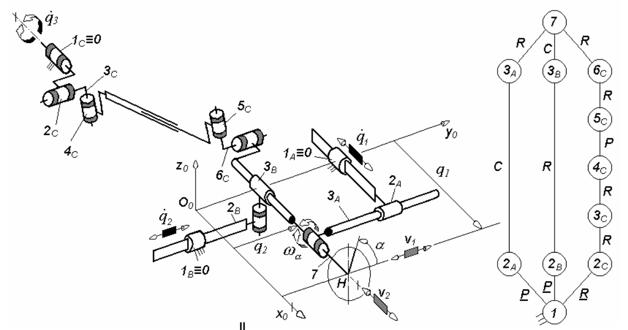


(c) $\underline{P}R{*}R{*}PR\ (\underline{P} \parallel R{*} \perp R{*} \perp^{\perp} P \perp^{\perp} R)$-$\underline{P}C$-$\underline{R}UPU$

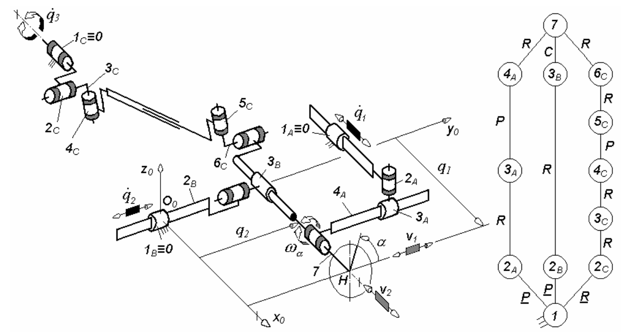


(d) $\underline{P}PR$-$\underline{P}R{*}R{*}C\ (\underline{P} \parallel R{*} \perp R{*} \perp^{\perp} C)$-$\underline{R}UPU$

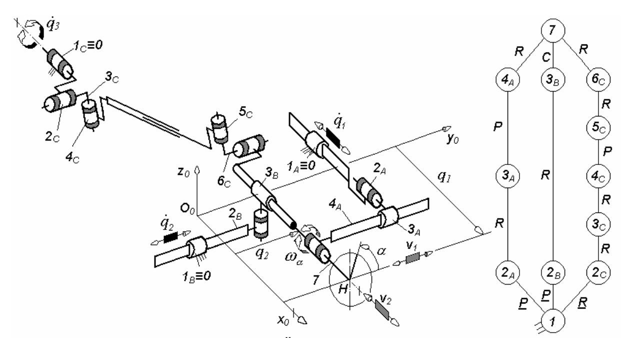
Table 2. Kinematic structures of maximally regular *T2R1*-type spatial PMs with one degrees of overconstraint derived from the basic solution $\underline{P}PR$-$\underline{P}C$-$\underline{R}UPU$ by combining two idle mobilities in limb G_1 (**a-c**) or in limb G_2 (**d**)



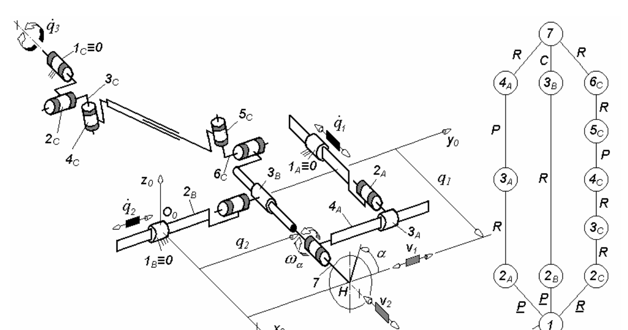
(a) $\underline{P}C{*}R\ (\underline{P}\perp C{*}\perp^{\parallel}R)\text{-}\underline{P}R{*}C\ (\underline{P}\perp R{*}\perp^{\perp}C)\text{-}\underline{R}UPU$



(b) $\underline{P}R{*}PR\ (\underline{P}\perp R{*}\perp^{\perp}P\perp^{\perp}R)\text{-}\underline{P}R{*}C\ (\underline{P}\parallel R{*}\perp C)\text{-}\underline{R}UPU$

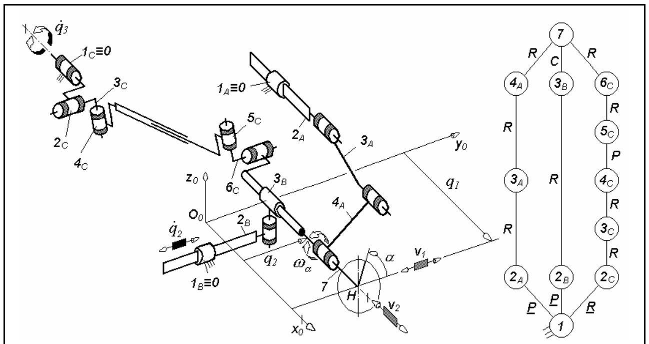


(c) $\underline{P}R{*}PR\ (\underline{P}\parallel R{*}\perp P\perp^{\parallel}R)\text{-}\underline{P}R{*}C\ (\underline{P}\perp R{*}\perp^{\perp}C)\text{-}\underline{R}UPU$

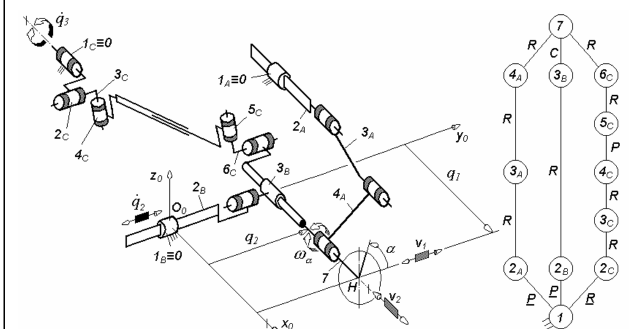


(d) $\underline{P}R{*}PR\ (\underline{P}\parallel R{*}\perp P\perp^{\parallel}R)\text{-}\underline{P}R{*}C\ (\underline{P}\parallel R{*}\perp C)\text{-}\underline{R}UPU$

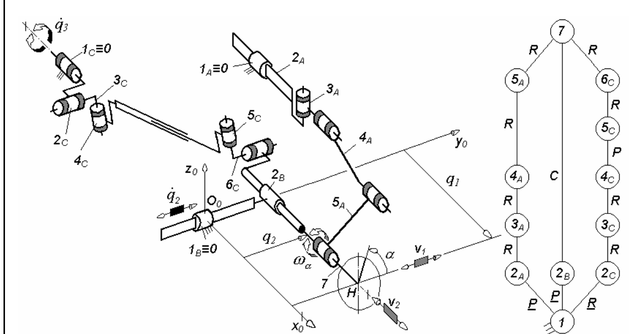
Table 3. Kinematic structures of maximally regular *T2R1*-type spatial PMs with one degree of overconstraint derived from the basic solution $\underline{P}PR$-$\underline{P}C$-$\underline{R}UPU$ by combining one idle mobility in each limb G_1 and G_2



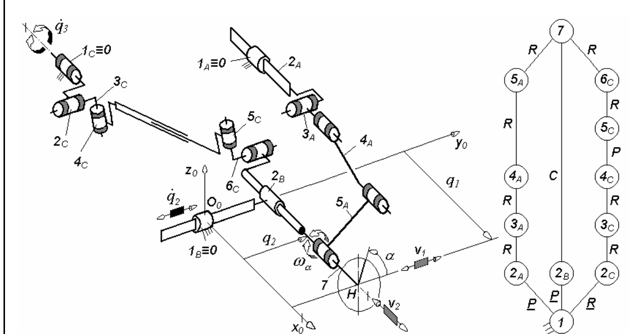
(a) $\underline{P}RRR\text{-}\underline{P}R{*}C\ (\underline{P}\perp R{*}\perp^{\perp}C)\text{-}\underline{R}UPU$



(b) $\underline{P}RRR\text{-}\underline{P}R{*}C\ (\underline{P}\parallel R{*}\perp C)\text{-}\underline{R}UPU$



(c) $\underline{P}R{*}RRR\ (\underline{P}\perp R{*}\perp^{\parallel}R\parallel R\parallel R)\text{-}\underline{P}C\text{-}\underline{R}UPU$



(d) $\underline{P}R{*}RRR\ (\underline{P}\perp R{*}\perp^{\parallel}R\parallel R\parallel R)\text{-}\underline{P}C\text{-}\underline{R}UPU$

Table 4. Kinematic structures of maximally regular *T2R1*-type spatial PMs with one degrees of overconstraint derived from the basic solution $\underline{P}RRR$-$\underline{P}C$-$\underline{R}UPU$ by combining one idle mobility in limb G_1 (**a-b**) or in limb G_2 (**c-d**)

Limbs G_i ($i=1$ and 2) of the basic solution in Fig. 1 have $M_{Gi}=S_{Gi}=3$ and $(R_{Gi})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$. Limb G_3 has $M_{G3}=S_{G3}=6$ and $(R_{G3})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$. Limb G_1 in Fig. 2 has $M_{Gi}=S_{Gi}=4$ and $(R_{Gi})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$. To simplify the notations of links e_{Gi} of limbs Gi ($i=1,\ldots,3$ and $e=1,\ldots,n$), by avoiding the double index in Fig. 1 and the following figures and tables, the links belonging to limb G_1 are denoted by e_A ($e_A \equiv e_{G1}$) and the links of limbs G_2 and G_3 by e_B ($e_B \equiv e_{G2}$) and e_C ($e_C \equiv e_{G3}$).

No.	Topology	Bases of the vector spaces R_{Gi} ($i=1,2$)
1	PC*R-PC-RUPU Table 1a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
2	PR*PR-PC-RUPU Table 1b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
3	PPR-PR*C-RUPU Table 1c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
4	PPR-PR*C-RUPU Table 1d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
5	PR*C*R-PC-RUPU Table 2a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
6	PR*C*R-PC-RUPU Table 2b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
7	PR*R*PR-PC-RUPU Table 2c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
8	PPR-PR*R*C-RUPU Table 2d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$
9	PC*R-PR*C-RUPU Table 3a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
10	PR*PR-PR*C-RUPU Table 3b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
11	PR*PR-PR*C-RUPU Table 3c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
12	PR*PR-PR*C-RUPU Table 3d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
13	PRRR-PR*C-RUPU Table 4a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
14	PRRR-PR*C-RUPU Table 4b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
15	PR*RRR-PC-RUPU Table 4c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
16	PR*RRR-PC-RUPU Table 4d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$

Table 5. Bases of the vector spaces R_{Gi} ($i=1,2$) of the overconstrained maximally regular T2R1-type spatial PMs presented in Tables 1-4.

The solutions in Figs. 1 and 2 have $k=3$, $q=2$. In a simple (serial) limb, no closed loops exist and $r_l=0$. Equations (1)-(3) indicate that these solutions are non-redundant ($T=0$) with three degrees of mobility ($M=3$). The solution in Fig. 1 has three degrees of overconstraint ($N=3$) and the solution in Fig. 2 has $N=2$.

Other overconstrained solutions with one and two degrees of overconstraint can be derived from the basic solutions in Figs. 1 and 2 by introducing the required idle mobilities in the joints of limbs G_1 and/or G_2 (see Tables 1-4). For these solutions, equations (2) and (5) give the following values**:** $\sum_{i=1}^{k} S_{Gi}=13$ for the solutions with two degrees of overconstraint and $\sum_{i=1}^{k} S_{Gi}=14$ for the solutions with one degree of overconstraint. The joints in which the idle mobilities are introduced are denoted by *. In the revolute and cylindrical joints denoted by R^* and C^*, the rotation is an idle mobility.

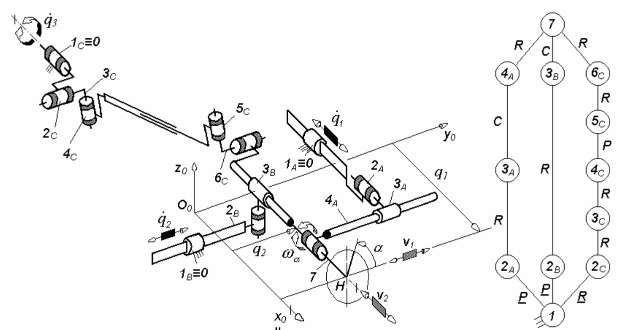
For the overconstrained solutions in Tables 1-4, the basis of the vector space of relative velocities between the mobile platform 7 and the fixed base in the kinematic chain associated with limb G_3 disconnected from the parallel mechanism is $(R_{G3})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$. The bases (R_{G1}) and (R_{G2}) are systematized in Table 5.

Worrying: Special attention must be paid when introducing idle mobilities so as not to modify the connectivity of the moving platform and/or the mobility of the PM. After introducing idle mobilities, equations (1)-(5) must always give $M=S_F=3$. For example, if a second idle mobility is introduced in the solution in Table 1d by changing the prismatic joint between links 2_A and 3_A by a cylindrical joint, the moving platform 7 can peform a second rotational motion around the y-axis and the mechanism is characterized by the following structural parameters: $M_{Gi}=S_{Gi}=4$, $(R_{Gi})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$, $(i=1,2)$ $M_{G3}=S_{G3}=6$, $(R_{G3})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$. Equations (1)-(5) give, in this case, r=10, $M=4$ and $S_F=4$, $(R_F)=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$. This mechanism does not remain a *T2R1*-type PM. Many other similar combinations of inadequate idle mobilities exist and must be avoided in the structural synthesis. This can easily be done by using equations (1)-(5).

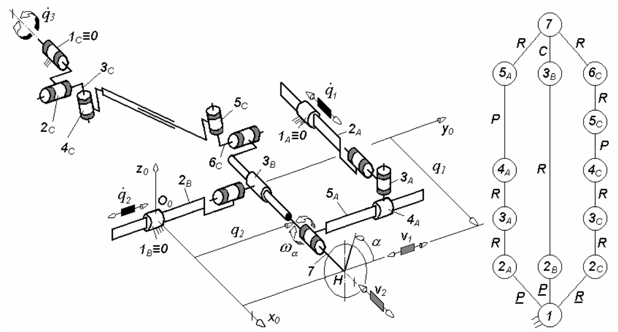
3.2 Non overconstrained solutions

For the non overconstrained solutions, Eqs. (2) and (5) give $\sum_{i=1}^{k} S_{Gi}=15$. These solutions can be obtained by introducing complementary idle mobilities in the overconstrained solutions presented in the previous section (see tables 7 and 8). Two rotational idle mobilities exist in the spherical joint denoted by S^*.

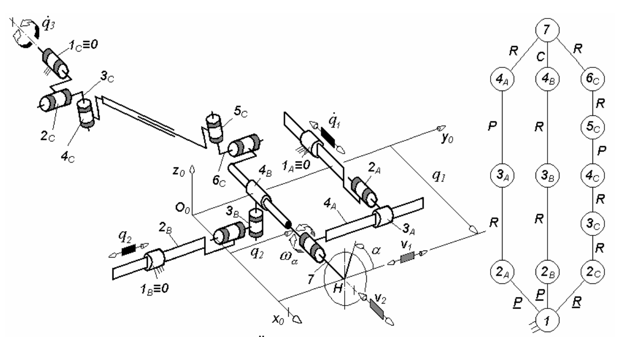
For the non overconstrained solutions in Tables 7 and 8, the basis of the vector space of relative velocities between the mobile platform 7 and the fixed base in the kinematic chain associated with limb G_3 disconnected from the



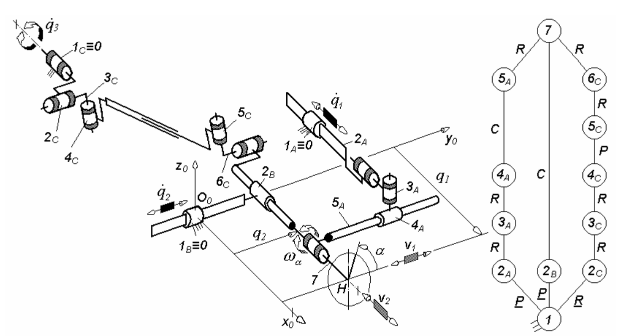
(a) $\underline{P}R^*C^*R$ $(\underline{P}\parallel R^*\parallel C^*\perp{}^{\parallel}R)$-$\underline{P}R^*C$ $(\underline{P}\perp R^*\perp^{\perp}C)$-$\underline{R}UPU$



(b)$\underline{P}R^*R^*PR$ $(\underline{P}\parallel R^*\perp R^*\perp^{\perp}P\perp^{\perp}R)$-$\underline{P}R^*(\underline{P}\parallel R^*\perp C)$-$\underline{R}UPU$

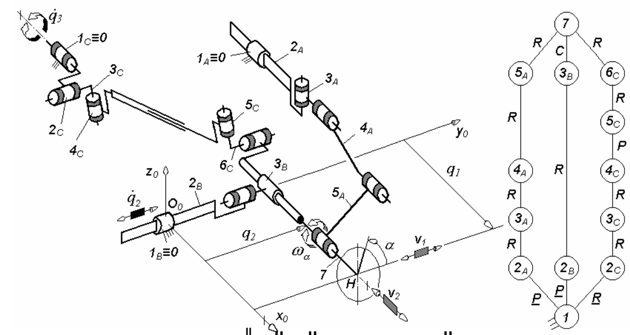


(c) $\underline{P}R^*PR$ $(\underline{P}\parallel R^*\perp P\perp{}^{\parallel}R)$-$\underline{P}R^*R^*C$ $(\underline{P}\parallel R^*\perp R^*\perp^{\perp}C)$-$\underline{R}UPU$

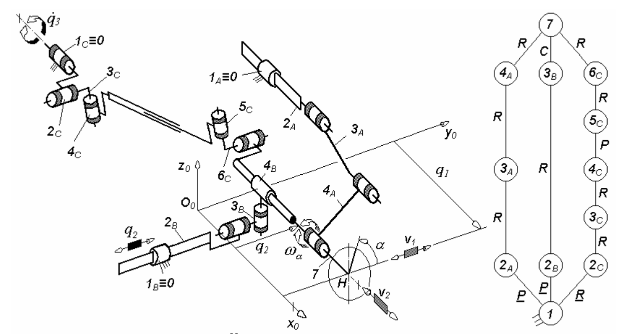


(d) $\underline{P}R^*R^*C^*R$ $(\underline{P}\parallel R^*\perp R^*\perp^{\perp}C^*\perp^{\perp}R)$-$\underline{P}C$-$\underline{R}UPU$

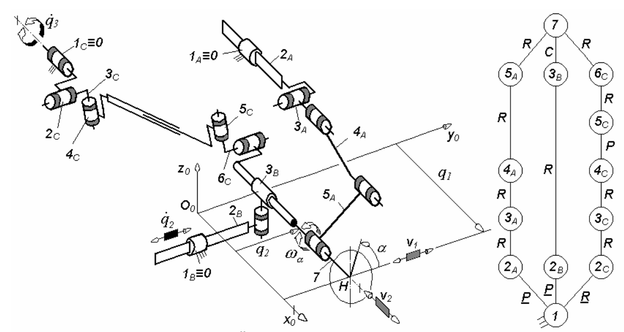
Table 7. Kinematic structures of non overconstrained maximally regular *T2R1*-type spatial PMs derived from the basic solution $\underline{P}PR$-$\underline{P}C$-$\underline{R}UPU$ by combining three idle mobilities in limbs G_1 and G_2 (**a-c**) or just in limb G_1 (**d**)



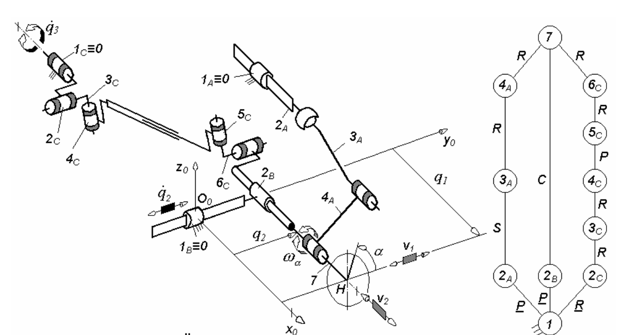
(a) $\underline{P}R^*RRR$ $(\underline{P}\perp R^*\perp{}^{\parallel}R\parallel R\parallel R)$-$\underline{P}R^*C$ $(\underline{P}\parallel R^*\perp C)$-$\underline{R}UPU$



(b) $\underline{P}RRR$-$\underline{P}R^*R^*C$ $(\underline{P}\parallel R^*\perp R^*\perp^{\perp}C)$-$\underline{R}UPU$



(c)$\underline{P}R^*RRR$ $(\underline{P}\perp R^*\perp{}^{\parallel}R\parallel R\parallel R)$-$\underline{P}R^*C$ $(\underline{P}\perp R^*\perp^{\perp}C)$-$\underline{R}UPU$



(d) $\underline{P}S^*RR$ $(\underline{P}\text{-}S^*\text{-}{}^{\parallel}R\parallel R)$-$\underline{P}C$-$\underline{R}UPU$

Table 8. Kinematic structures of non overconstrained maximally regular *T2R1*-type spatial PMs derived from the basic solution $\underline{P}RRR$-$\underline{P}C$-$\underline{R}UPU$ by combining three idle mobilities in limbs G_1 and G_2 (**a-c**) or just in limb G_1 (**d**)

No.	Topology	Bases of the vector spaces R_{Gi} ($i=1,2$)
1	PR*C*R-PR*C-RUPU Table 6a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
2	PR*R*PR-RUPU Table 6b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
3	PR*PR-PR*R*C-RUPU Table 6c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$
4	PR*R*C*R-PC-RUPU Table 6d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta\ \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$
5	PR*RRR-PR*C-RUPU Table 7a	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$
6	PRRR-PR*R*C-RUPU Table 7b	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta\ \boldsymbol{\omega}_\delta)$
7	PR*RRR-PR*C-RUPU Table 7c	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\delta)$
8	PS*RR-PC-RUPU Table 7d	$(R_{G1})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta\ \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{\omega}_\alpha)$

Table 8. Bases of the vector spaces R_{Gi} ($i=1,2$) of the non overconstrained maximally regular T2R1-type spatial PMs presented in Tables 6 and 7.

parallel mechanism is $(R_{G3})=(\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{\omega}_\alpha, \boldsymbol{\omega}_\beta, \boldsymbol{\omega}_\delta)$. The bases (R_{G1}) and (R_{G2}) are systematized in Table 8.

4 Conclusions

Mobility, connectivity, redundancy and overconstraint represent the main kinematic criteria for structural synthesis of parallel robots. An approach has been proposed for structural synthesis of overconstrained and non overconstrained maximally regular T2R1-type spatial parallel robots with unlimited rotational capabilities. The Jacobian matrix mapping the articulated and the operational vector spaces of the maximally regular parallel robotic manipulators proposed in this paper is the 3×3 identity matrix throughout the entire workspace. Basic and derived solutions with various degrees of overconstraint are presented for the first time. These solutions are obtained by a systematic approach of structural synthesis founded on the theory of linear transformations and an evolutionary morphology. The approach integrates the new formulae for mobility, connectivity, redundancy and overconstraint of parallel manipulators recently proposed by the author. The method proposed in this paper can be easily extrapolated to structural synthesis of other types of parallel robot with various combinations of translational and rotational motions of the moving platform.

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