Multiple Laser Polar Scan Matching with Application to SLAM

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Abstract

Polar Scan Matching is one of the methods of point to point scan matching for Simultaneous Localization and Mapping Application. It works in the original laser polar coordinate system and therefore eliminates the need for an expensive correspondence search as in other scan matching methods by using the matching bearing association rule. However, most of the low-cost laser range finders available on the market have limited field of view and therefore the use of more than one laser range finder is necessary to cover 360 degree field of view and hence to provide a better Scan-Matching SLAM performance. This paper presents extensions of Polar Scan Matching to allow for matching with multiple laser scans simultaneously. Also a method for updating reference scans while mapping the environment is introduced. This paper also illustrates the advantages of using multiple laser range finders for SLAM. The results are illustrated with results from SLAM with a mobile robot equipped with two laser rangefinders.

1 Introduction

A mobile robot must usually know its pose (position and orientation) and the location of objects in order to perform useful operation autonomously in an initially unknown environment. The technique to solve this fundamental problem is named Simultaneous Localization and Mapping (SLAM). A popular SLAM solution is the Extended Kalman Filter SLAM (EKF-SLAM) [Dissanayake et al. 2001] where the robot's pose and landmarks' location are estimated stochastically to account for errors in sensor's data. Based on the method of representing landmarks, EKF-SLAM can be categorized into two types namely: feature-based [Kleeman, 2003; Garulli et al, 2005] and raw-scans based [Diosi and Kleeman, 2005; Nieto et al., 2007]. Recently, the raw-scans based approach has become popular due to the elimination of the requirement that a certain type of geometric feature must be present in the environment.

In raw-scans SLAM, each map landmark is a complete laser scan collected approximately every metre of travel and the measurement is carried out by a scan-matching, which computes a maximum likelihood alignment between two sets of raw-scans data (reference scan and current scan) taken from different robot poses. Examples of scan matching approach are the following: Iterative Closest Point (ICP) [Besl and McKay, 1992], Iterative Matching Range Point (IMRP) [Feng and Milios, 1994], Iterative Dual Correspondence (IDC) [Feng and Milios, 1994], and fast Polar Scan Matching (PSM) [Diosi and Kleeman, 2005]. The basic difference in these approaches lies in the method of finding correspondence between the reference scan and the current scan. ICP finds the correspondence by using the nearest neighbour algorithm where for each point of the current scan, the point with smallest Euclidean distance to the reference scans is selected. IMPR selects corresponding points by matching the range in the reference coordinate frame. IDC uses two sets of correspondence namely ICP to find translation and IMPR to find orientation. These three aforementioned methods operate in a Cartesian coordinate system, while PSM operates in the laser scanner's polar coordinate system and therefore eliminates the need for searching for correspondence by simply matching the points by their bearing. Because of the nature of the algorithm, PSM is much more efficient than the other three methods.

SLAM scan-matching works best if there is sufficient scans overlap during the scan matching process. However, most low-cost laser range finders available have a limited field of view; therefore the usage of more than one laser range finder is necessary to cover a 360 degrees field of view and to provide sufficient scan points for scan-matching in order to ensure accurate localisation.

This paper presents an approach to Polar Scan Matching EKF-SLAM that utilises multiple laser range finders where each template of landmark contains multiple sets of raw laser scans and the scan-matching for the multiple lasers is carried out simultaneously. Previous work [Diosi and Kleeman, 2005; Nieto et al., 2007] in scan-matching SLAM, used static reference scans during the whole mapping process which meant that the reference scan missed some of the information in between consecutive reference scans and did not account for occlusion. Here, the reference scans are updated in each of the scan-matching process which results in more detailed maps.

The paper presents the overview of the robot for experimental work in section 2, the motivation for this work in section 3, the Multiple Laser PSM Algorithm in section 4, the update of reference scans in section 5, the details of the SLAM approach in section 6 and the experimental results in section 7.

2 Overview of the Mobile Robot

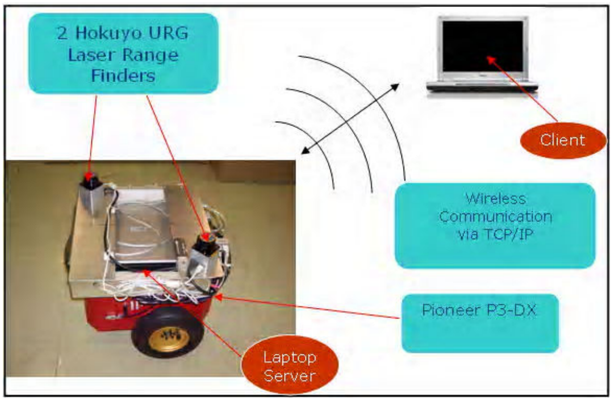

Figure 1-The experimental mobile robot with front and rear laser range finders.

The robot used for experimental work is shown in figure 1 and consists of two Hokuyo laser range finders and a laptop server mounted on an ActiveMedia Pioneer 3 DX mobile robot. The field of view of each laser is 240 degrees and the angular resolution is ~.36 degrees with a scanning refresh rate of up to 10 Hz. Distances are reported from 20 mm to 4 metres. They are mounted in such a way to give the robot 360 degrees field of view. A real-time client-server application is created to allow the robot to be tele-operated and also to allow viewing of the map building result while the robot is exploring the environment.

3 Motivation

One of the crucial factors which determine the accuracy and the reliability of scan-matching is the amount of scan overlap. A small amount of overlap can cause divergence, ambiguity and inaccuracy in the scan-matching process. This problem often occurs when only one limited field of view laser is used, for example in scans taken from opposite directions in a corridor (see figure 2). The aligned scans in figure 2 are weakly constrained and hence inaccurate, but the same situation may yield better results if more than one laser is used in the scan-matching process. This fundamental problem in scan-matching motivates the introduction of multiple laser scan-matching approach and the next section presents the algorithm.

In addition, even with more than one laser, the original scan-matching approach may still suffer from the lack of sufficient overlap. This happens when the scan-matching is performed with short range lasers and the distance between consecutive reference scans is too great. Furthermore, some objects which are occluded by other objects in the environment may not be mapped because they are not seen by the lasers when the robot captures the reference scans. One can shorten the distance between consecutive reference scans to solve this problem, but it can cause the SLAM to perform inefficiently due to the increase in the requirement of memory for storing reference scans and time for updating the SLAM state covariance matrix. The method for updating reference scans (see section 5) is proposed in this paper to avoid the abovementioned problem.

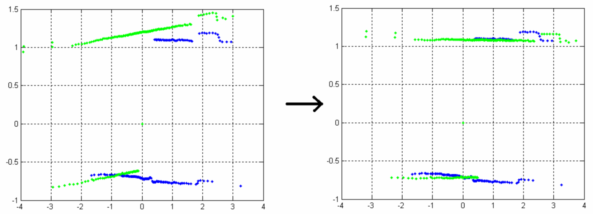

Figure 2-The examples of lack of overlap scans problem: The left picture shows current and reference scans taken from opposite direction in a corridor before scan-matching and the right picture shows the scans are not accurately aligned via PSM.

4 Multiple Laser PSM

Scan-matching is the process of aligning an observed set of points with a reference set of points. The method adopted in this paper is PSM [Diosi and Kleeman, 2005] due to its high speed and accuracy. PSM aligns the current scan with respect to the reference scan by minimising the sum of square range residual. This section describes the extension of PSM to tackle the problem of scan-matching using multiple laser range finders. The use of more than one laser range finder gives a 360 degree field of view and richer information about the environment, thus avoiding the lack of scans overlap problem and producing more accurate scan-matching results.

To allow the PSM to work with multiple laser scans, the sum of the range residuals is modified to account for the range residuals of the combinations between multiple reference scans and multiple current scans. For example, with two lasers, there will be two sets of reference scans (R_1,R_2) and two sets of current scans (C_1,C_2), and the PSM is to minimise the sum of the sum of range residual between R_1 and C_1, between R_1 and C_2, between R_2 and C_1 and between R_2 and C_2.

The algorithm works as follows. The current scan is defined as follows:

$$C = \{x_c, y_c, \phi_c \{x_{Lj}, y_{Lj}, \phi_{Lj}, \{r_{cji}, \beta_{cji}\}_{i=1}^{M}\}_{j=1}^{N}\} \tag{1}$$

and the reference scan is defined as follows:

$$R = \{x_{rk}, y_{rk}, \phi_{rk} \{\{r_{rji}, \beta_{rji}\}_{i=1}^{M}\}_{j=1}^{N}\}_{k=1}^{O} \tag{2}$$

where (x_c, y_c, ϕ_c) represents the current pose of the robot with respect to the global coordinate frame, $(x_{rk}, y_{rk}, \phi_{rk})$ represents the pose of the centroid of the k^{th} reference scan with respect to the global coordinate frame, $(x_{Lj}, y_{Lj}, \phi_{Lj})$ represents the pose of laser j with respect to the robot coordinate frame, (r_c,β_c) represents the polar coordinate of the current laser scans with respect to the laser coordinate frame and (r_r,β_r) represents the polar coordinate of the reference laser scans with respect to the reference coordinate frame. The scan-matching is carried out iteratively in three phases namely, scan projection, translation estimation and orientation estimation.

The scan projection is the process of transforming the current scans to the reference coordinate frame. The transformation works using the following equations:

$$p_{cji} = \begin{bmatrix} r_{cji}\cos(\beta_{cji}) \\ r_{cji}\sin(\beta_{cji}) \\ 1 \end{bmatrix}$$
$$p'_{cji} = T_{rk}^{-1} T_c T_{Lj} p_{cji} \tag{3}$$
$$\begin{bmatrix} r'_{cji} \\ \beta'_{cji} \end{bmatrix} = \begin{bmatrix} \sqrt{(p'_{cji}(1))^2 + (p'_{cji}(2))^2} \\ a\tan 2(p'_{cji}(2), p'_{cji}(1)) \end{bmatrix}$$

where, T_{rk} is the homogenous transformation matrix to transform from the reference coordinate frame to the global coordinate frame, T_c is the homogenous transformation matrix to transform from the current coordinate frame to the global coordinate frame and T_{Lj} is the homogenous matrix to transform from the laser coordinate frame to the robot coordinate frame. Matrix multiplication and inversion are computationally expensive but can be reduced to the following equation.

$$\begin{aligned}
T_{com} &= T_{rk}^{-1} T_c T_{Lj} \\
T_{com}(1,1) &= \sin(\phi_{Lj} + \phi_c - \phi_{rk}) \\
T_{com}(1,2) &= \cos(\phi_{Lj} + \phi_c - \phi_{rk}) \\
T_{com}(1,3) &= y_{Lj}\cos(\phi_c - \phi_{rk}) + x_{Lj}\sin(\phi_c - \phi_{rk}) + x_c\cos(\phi_{rk}) \\
&\quad + y_c\sin(\phi_{rk}) - y_{rk}\sin(\phi_{rk}) - x_{rk}\cos(\phi_{rk}) \\
T_{com}(2,1) &= -\cos(\phi_{Lj} + \phi_c - \phi_{rk}) \\
T_{com}(2,2) &= \sin(\phi_{Lj} + \phi_c - \phi_{rk}) \\
T_{com}(2,3) &= y_L\sin(\phi_c - \phi_{rk}) - x_L\cos(\phi_c - \phi_{rk}) - x_c\sin(\phi_{rk}) \\
&\quad + y_c\cos(\phi_{rk}) - y_{rk}\cos(\phi_{rk}) + x_{rk}\sin(\phi_{rk}) \\
T_{com}(3,1) &= 0 \\
T_{com}(3,2) &= 0 \\
T_{com}(3,3) &= 1
\end{aligned} \tag{4}$$

The scan projection is followed by a linear interpolation to provide the bearing matching association rule for both the translation estimation and the orientation estimation by converting the projected laser scans into the laser's original data format (constant bearing step size). The scan projection as well as the interpolation are carried out N times for N number of lasers. see [Diosi and Kleeman, 2005] for details on the interpolation algorithm.

After the scan projection and the interpolation, the next step is to estimate translation. That is to find x_c and y_c that will minimize the sum of weighted range residual. As in [Diosi and Kleeman, 2005], the range difference between a projected current range and a reference range is modeled as:

$$(r'_c - r_r) = H\begin{bmatrix} \Delta x_c \\ \Delta y_c \end{bmatrix} + noise \tag{5}$$

where

$$H = \begin{bmatrix} \dfrac{\partial r'_{c1,1}}{\partial x_c} & \dfrac{\partial r'_{c1,1}}{\partial y_c} \\ \vdots & \vdots \\ \dfrac{\partial r'_{cN,M}}{\partial x_c} & \dfrac{\partial r'_{cN,M}}{\partial y_c} \end{bmatrix} \tag{6}$$

The sum of weighted range residual for N number of lasers is:

$$\sum w\Delta r^2 = \sum_i w_{1,1,i}\Delta r_{1,1,i}^2 + \ldots + \sum_i w_{1,N,i}\Delta r_{1,N,i}^2 + \ldots + \sum_i w_{N,N,i}\Delta r_{N,N,i}^2 \tag{7}$$

where

$$w_{jr,jc,i} = \frac{C}{\Delta r_{jr,jc,i} + C} \tag{8}$$

$$\Delta r_{jr,jc,i} = (r_{r;jr,i} - r'_{c;jc,i}) \tag{9}$$

jr is the index of laser in the reference scans, jc is the index of laser in the current scans and i is the index of the range measurement of the laser. Note that $\Delta r_{jr,jc,i}$ is only included in the sum of weighted range residual if it is less than a maximum error value to reduce the effect of association error.

The position correction of the current scan is calculated by minimizing the sum of weighted range residual using the well known equation for weighted least squares [Kay, 1993]:

$$\begin{bmatrix} \Delta x_c \\ \Delta y_c \end{bmatrix} = (H^T W H)^{-1} H^T W (r'_c - r_r) \tag{10}$$

The orientation estimation is performed by rotating the projected current scan to the left and right until the projected current scan aligns with the reference scan. The rotation is done by shifting the range measurements in the array and is therefore significantly faster than rotation by transformation matrix. As in [Diosi and Kleeman, 2005] the shifting angle is $\pm 20^\circ$ with a preset interval (resolution of the lasers), and for each shift angle the average absolute range residual is calculated. The minimum is found by fitting a parabola to the three closest points to the smallest average absolute error. The sum of absolute range residual for N number of laser is:

$$\sum|\Delta r| = \sum_i|\Delta r_{1,1,i}| + \ldots + \sum_i|\Delta r_{1,N,i}| + \ldots + \sum_i|\Delta r_{N,N,i}| \tag{11}$$

and the average is:

$$AVG(|\Delta r|) = \frac{\sum |\Delta r|}{tot} \tag{12}$$

where *tot* is the total number of terms. An example of PSM alignment is shown in figure 3.

In order to fuse the scan matching results with other sensors measurements in the Kalman filter, it is necessary to obtain a covariance estimate of the alignment. Assuming correct associations are obtained, the covariance in translation estimation can be used as the covariance for PSM. The covariance for PSM is therefore as follows:

$$R = \sigma_r^2 (H^T W H)^{-1} \tag{13}$$

where, σ_r^2 is the estimated range error variance.

$$\sigma_r^2 = \frac{[r_c' - r_r]^T [r_c' - r_r]}{n-3} \tag{14}$$

and n is the total number of association.

However, there are situations where this assumption is violated due to the lack of information about the environment. For example, in long featureless corridors, the corresponding pair between reference and current points does not necessarily represent the same physical point in the environment and hence results in an overly optimistic covariance matrix. A simple solution to this problem is to use heuristic error estimation approach as in [Diosi and Kleeman, 2005]. The error estimation is achieved by recognizing the shape of the laser scans. If the shape is corridor (ie dominated by parallel walls), then a non-diagonal covariance matrix, which expresses large error in the direction of the corridor is chosen. The detection of corridor is performed by calculating the variance of orientations of line segments connecting consecutive scan points. If the variance is less than a certain value, then it is classified as corridor. The orientation of corridor is found by calculating angle histogram of the line segments [Weiss and Puttkamer, 1995]. The maximum value of the histogram corresponds to the corridor direction. The example of more sophisticated solutions to this problem are Hessian [Bengtsson and Baerveldt, 2001] and Closed-Form [Censi, 2007].

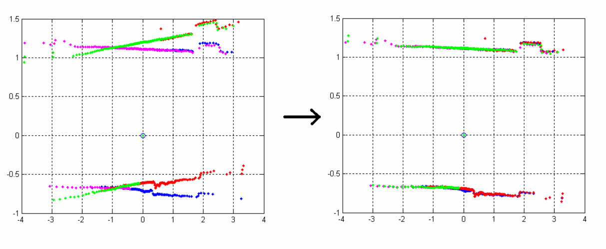

Figure 3-Example of scan alignment with 2 lasers via PSM: the left picture shows current and reference scans prior matching and the right picture shows the scans aligned using PSM.

5 Update of Reference Scans

This section presents the proposed method for updating reference scans and will be incorporated into the SLAM implementation in near future.

The update of the reference scans is to be performed after each of the successful scan-matching. Both the reference scans and the interpolated current scans are stored in array forms with the size of $360/\Delta\beta$ where $\Delta\beta$ is the resolution of the lasers, in order to cover 360 degrees field of view. All the bearings with out of range measurements are stored as a value of zero.

To tackle the mentioned problem of occlusion, each bearing can store more than one range measurement if the range difference is higher than a certain value. In previous work [Diosi and Kleeman, 2005], the occluded scan points are not included in the scan matching process. Here, the occluded points can be incorporated in the scan matching process by using nearest neighbour approach. So for each bearing, the current range measurement is matched with the nearest range measurement stored in the reference scan. Figure 4 shows the example where part of the environment is not captured by the two reference scans and by using the proposed method, the environment can be mapped more completely.

The update is then carried out by using the matching bearing association rule where the current scan is used to update the out of range data and the occluded data in the reference scans. The pseudocode for updating the reference scans is the following:

```
for (i=0;i<360/Δβ;i++)
{
    if (laser_c[i]>0)
    // current scan range is not out of range
    {
        if (laser_r[i].size = 1 and laser_r[i].range[0] = 0)
        //reference scan range is out of range
        {
            laser_r[i].range[0] = laser_c[i];      //update
        }
        else
        {
            j = nearest_range(laser_r[i]);
            //find the nearest range at bearing i in reference
            //scans
            if (abs(laser_r[i].range[j] – laser_c[i])>MAX)
            {
                laser_r[i].append(laser_c[i]);    //append
            }
        }
    }
}
```

The full process of the scan-matching is shown in figure 5.

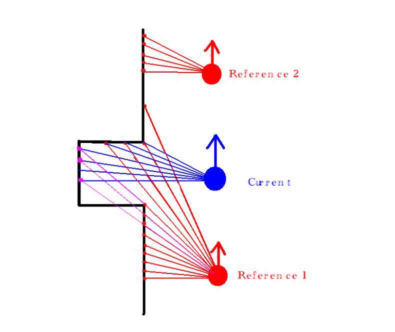

Figure 4-the examples of occlusion problem. Each bearing may store more than one range measurement to tackle the problem of occlusion. The red lines are the reference scan, the violet lines are the appended reference scan to store the previously occluded objects, and the blue lines are the current scan

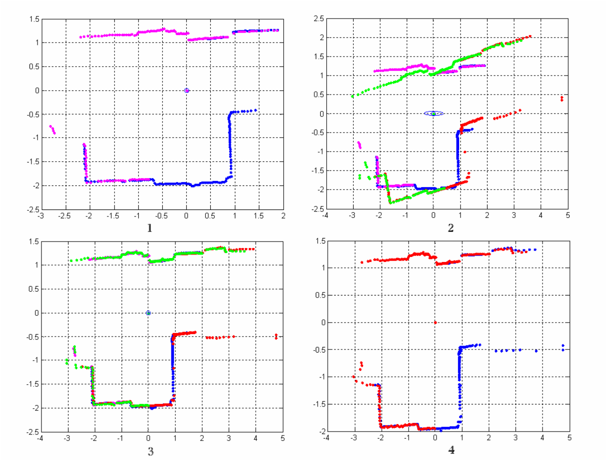

Figure 5-The full process of scan matching with 2 lasers: The picture 2 shows the scans prior matching and the picture 3 shows the scans aligned using PSM. The first picture shows the initial reference scan and the final picture shows the result of the updated reference scans.

6 EKF – Scan Matching SLAM

The implementation of EKF-SLAM is employed based on the description by Davison [Davison, 1998] where all map features are included in SLAM state vector and updated on each observation step. The augmented state vector containing both the state of the robot and the state of all landmark location is as follows:

$$X = [\theta, x, y, p_1, p_2, ..., p_n]^T \tag{15}$$

The prediction model is derived based on odometry model in [Kleeman, 2003] where the error model assumes error sources are additive white noise on the wheel separation, B and the left and right wheel distance measurements, Δl and Δr.

$$x_v(k) = [\theta(k), x(k), y(k)]^T$$

$$y_v(k) = [\Delta r(k), \Delta l(k), B]^T \tag{16}$$

$$x_v(k) = \begin{bmatrix} \theta(k-1) + \frac{\Delta r - \Delta l}{B} \\ x(k-1) + \frac{\Delta r + \Delta l}{2}\cos(\theta(k-1) + \frac{\Delta r - \Delta l}{2B}) \\ y(k-1) + \frac{\Delta r + \Delta l}{2}\sin(\theta(k-1) + \frac{\Delta r - \Delta l}{2B}) \end{bmatrix}$$

The covariance of the error in the robot pose at step k is defined as follows:

$$P_v(k) = \left(I + \frac{\partial f}{\partial x}\right) P_v(k-1) \left(I + \frac{\partial f}{\partial x}\right)^T + \frac{\partial f}{\partial y} Q(k-1) \frac{\partial f}{\partial y}^T \tag{17}$$

$Q(k)$ is the error covariance of $y(k)$ and defined as follows:

$$Q = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_l^2 & 0 \\ 0 & 0 & \sigma_B^2 \end{bmatrix} \tag{18}$$

In [Kleeman, 2003], these variances are derived in such a way to make the system consistent, which means that for a given distance or orientation change, the final covariance should be independent of the number of steps to traverse a path. This means that variances σ_r^2 and σ_l^2 must be proportional to Δr and Δl at each step and variance σ_B^2 must be proportional to the absolute value of the angle change, D. They are therefore defined as follows:

$$\sigma_r^2 = E^2|\Delta r| \qquad \sigma_l^2 = E^2|\Delta l| \qquad \sigma_B^2 = \frac{A^2B^2}{2\pi D} \tag{19}$$

where, E is the accumulated standard deviation error from 1 m of travel and A is the angle error standard deviation introduced by noise on the wheel separation B for a full 2π revolution of the robot in 1 step.

As in [Diosi and Kleeman, 2005; Nieto et al., 2007], landmarks are defined by a template of raw sensor data which are collected approximately every metre of robot travel and observed by a process of scan-matching. The result of scan matching is the global pose of the centroid of the template points, p.

$$p = [x_{L_1}, y_{L_1}, \phi_{L_1}, \ldots, x_{L_i}, y_{L_i}, \phi_{L_i}, \ldots, x_{L_n}, y_{L_n}, \phi_{L_n}]^T \tag{20}$$

The observation model for the pose of a landmark coordinate frame with respect to the robot is calculated as follows:

$$H(k) = \begin{bmatrix} x_{ki}(k) \\ y_{ki}(k) \\ \phi_{ki}(k) \end{bmatrix}$$

$$H(k) = \begin{bmatrix} (x_B(k) - x_v(k))\cos(\theta_v(k)) + (y_B(k) - y_v(k))\sin(\theta_v(k)) \\ -(x_B(k) - x_v(k))\sin(\theta_v(k)) + (y_B(k) - y_v(k))\cos(\theta_v(k)) \\ \phi_B(k) - \theta_v(k) \end{bmatrix} \tag{21}$$

The Jacobian of H is defined as follows:

$$\frac{\partial H_i}{\partial x} = \begin{bmatrix} -(x_{Li}-x_v)\sin(\theta_v)+(y_{Li}-y_v)\cos(\theta_v) & -\cos(\theta_v) & -\sin(\theta_v) & \cdots & \cos(\theta_v) & \sin(\theta_v) & 0 & \cdots \\ -(x_{Li}-x_v)\cos(\theta_v)-(y_{Li}-y_v)\sin(\theta_v) & \sin(\theta_v) & -\cos(\theta_v) & \cdots & -\sin(\theta_v) & \cos(\theta_v) & 0 & \cdots \\ -1 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots \end{bmatrix} \quad (22)$$

Each observation is associated to a feature in the SLAM state vector using nearest neighbour approach and a new landmark is appended to the SLAM state vector every time the robot gets to a position which is further than 1 metre from the closest landmark.

7 Experimental Results

The experiments were carried out in different real indoor environments. The robot was travelling at less than 0.1 m/s to avoid additional error caused by delay in the laser range finders. Figures 6 and 7 show the SLAM results where the landmarks are shown in green and their poses are drawn as robots that are numbered and coloured in white. It is worth mentioning that the consecutive laser scans were not matched against each other since it was assumed that over a short distance, the odometry is more accurate as reported in [Diosi and Kleeman, 2005]. The scan matching was performed every half a second with a running time between 15 to 30 ms.

Figure 6 shows the SLAM result of the robot travelling in a loop like environment anticlockwise. The robot can successfully close the loop without any special loop-closing algorithm [Bosse et al. 2004] because the size of the loop is relatively small. In contrast, figure 7 shows the SLAM result of the robot travelling in a corridor environment. The robot travelled to the far end of the corridor and came back to its initial position. During its return travel, the robot was always able to perform scan-matching successfully thanks to the usage of two lasers as the scan-matching would have failed if only one limited field of view laser is used due to the lack of overlap problem.

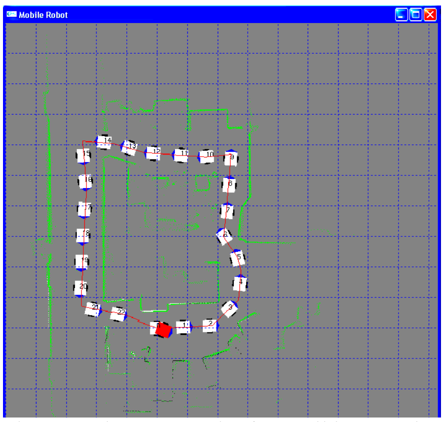

Figure 6-The map result of a small loop environment. The poses of the landmark is shown as numbered robot in white colour. The grid size is 1 metre.

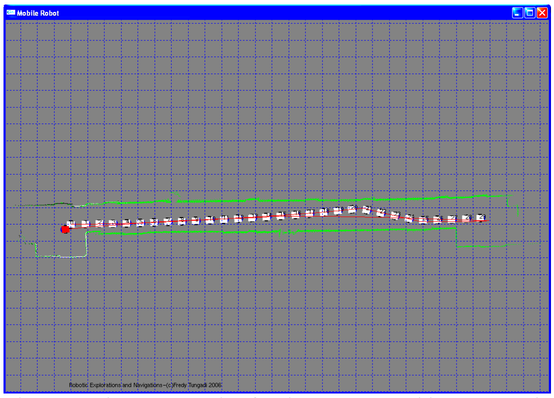

Figure 7-The map result of Polar Scan Matching SLAM in a corridor. The grid size is 1 metre.

8 Conclusion and Future Work

The multiple laser PSM has been shown to produce high quality of map without geometric interpretations with the EKF-SLAM approach. This work demonstrate the benefits of multiple laser PSM that solve the lack of overlap problem. The next step is to implement the proposed method for updating the reference scans into the SLAM implementation to handle occlusion problem and thus produce more detail maps.

The current implementation of multiple laser PSM SLAM is only reliable when it is assumed that the working environment is static. The future work will concentrate on tracking moving objects to improve the robustness in PSM.

It is also found that in an extremely long corridor, even with heuristic error covariance estimation, the error in the robot position along the direction of the corridor can become significant. In future, the current PSM implementation will be incorporated with an Advanced Sonar Ring [Fazli and Kleeman, 2006], which has been reported to be able to detect small features on the corridor such as small wall moldings and door frames, in order to correct the robot's pose along the direction of the corridor.

9 Acknowledgements

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