Motion Safety for an Autonomous Vehicle Race in an Urban Environment

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Abstract

The 2007 Darpa grand challenge is a 100 km race between autonomous vehicles in an urban environment. This paper presents the results of kinematic motion study in order to quantify the minimum perception range requirements of a canonical set of manoeuvres that are required for the event. The paper also considers the potential advantage that a course reversal strategy may provide competitors that field small, tight-turning vehicles.

1 Introduction

The 2007 DARPA Urban Grand Challenge is a race between fully autonomous vehicles over a series of waypoints covering approximately 100 kms of urban terrain. Putting aside the fully autonomous requirement, the types of vehicles[1] eligible to enter the race are sufficiently open to encourage a range of technical solutions. Amongst other things, the race rules also require vehicles to obey Californian traffic laws, stay within minimum and maximum speed limits that are capped at 30 mph and maintain minimum mandatory vehicle separations. Fortunately, the race vehicles are not required to detect and avoid pedestrians, which are not allowed on the course, nor are they required to interpret street signs or traffic lights as stop point locations are provided for this purpose.

This paper quantifies the results of kinematic analysis of a range of fundamental vehicle manoeuvres: panic stop, merging, crossing a road, overtaking, right-angle-turns and course reversal that are required for the race. This enables the minimum perception range requirement of candidate vehicles to be determined and thus the size of the safety region needed to avoid entering an inevitable collision state [Fraichard and Asama, 2004].

2 Kinematic Analysis of Vehicle Manoeuvres

Vehicle operations in an urban environment require a robust ability to detect and track other vehicles in order to avoid collisions. Of the wide range of manoeuvres that multiple-vehicle movement in traffic can be decomposed into, those that commence from a stationary start are most challenging from a range perspective because: a) the velocity differentials are maximum and b) the responsibility for any collision rests with the vehicle departing a stationary position. Also of importance is the perception range related to emergency braking for collision avoidance, which must be continuously maintained. This section discusses the results of kinematic simulation of six canonical manoeuvres: panic stop, merging, crossing a road, overtaking, right-angle turns and 180-degree turns.

2.1 Representative Vehicle and Environment

The following vehicle and environment characteristics was used in the simulations:

length	l	4.6 m
width	w	1.8 m
wheelbase	b	2.66 m
steering response	t_{steer}	1 s
longitudinal acceleration	a^{+}	2.8 ms^{-2}
longitudinal deceleration	μg	10 ms^{-2}
speed limit	$v_{\max}$	13.4 m/s
path width	p	3.6 m,

where t_{steer} is the response time for the vehicle to transition between minimum and maximum curvature, $\kappa_{\max}$.

2.2 Panic Stop

Collision free movement requires the maintenance of at least a single feasible escape trajectory at all times. When the forward path is totally blocked this requires a vehicle to sense significant obstacles beyond the distance, s_{stop}, at which it can stop. This distance is comprised of

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[1] 'Safe', 'Unladen manufactured weight' > 900 kg, Wheelbase > 72", Height < 12', Width < 9', at least four wheels, Speed > 30 mph [Darpa, 2006a]

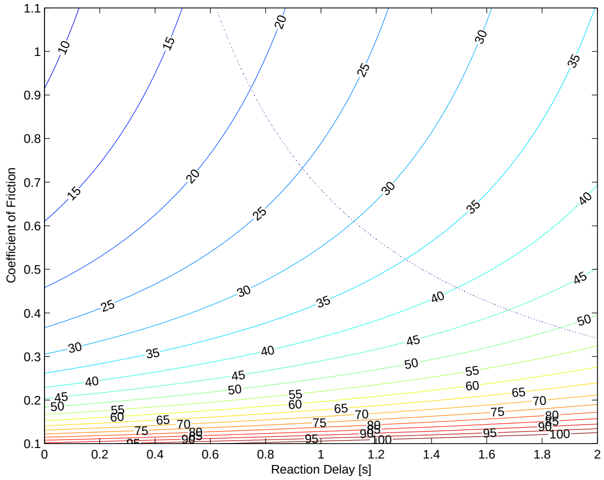

Figure 1: Braking distance [m] for an initial velocity of 13.4 m/s.

two components: reaction distance, which is the distance lost before the brakes applied, and braking distance:

$$s_{stop} = t_d v_s + \frac{v_s^2}{2\mu g}, \tag{1}$$

where t_d is the system delay before an obstacle is detected and the brakes are applied, v_s is the starting velocity, which is equal to $v_{\max}$ in this instance, μ is the coefficient of sliding friction, and g the acceleration due to gravity [Kelly and Stentz, 1997]. Figure 1 plots (1) over the operating region of the race environment and systems delays of up to 2 seconds. As indicated by the dotted line in Figure 1 the reaction distance and braking distance are roughly equal for modern passenger vehicles operating on high friction surfaces. However, even at the relatively low 13.4 m/s maximum speed of the race the braking distance component of the stopping distance will tend to dominate the reaction distance component on lower friction surfaces, such as the dirt/gravel roads that the race route is likely to include. In these instances, a swerving manoeuvre may be required to avoid obstacles that are detected within the stopping distance afforded by the friction properties of the surface [Urmson, 2006].

2.3 Straight Line Acceleration

This subsection considers manoeuvres featuring straight line acceleration, including: merging into a traffic flow, crossing a road and overtaking.

Merging

From a stationary position on the side of the road, a vehicle requires a gap to pull into to enable it to accelerate up to the speed of the traffic. This requires a region to the rear of the vehicle that is free of approaching traffic. Ignoring distance lost in transitioning from the kerb-side parking lane into the adjacent traffic lane[2], this distance, s_{merge}, is defined by

$$s_{merge} = \frac{v_{\max}^2}{2a}. \tag{2}$$

For traffic travelling at 13.4 m/s, this equates to an initial minimum safety region that extends out to between 9 and 90 m for average rates of acceleration between 10 and 1 ms^{-2}. For our representative vehicle with it's average acceleration rate of 2.8 ms^{-2} the length of this rearwards safety region is 32 m. [(2) is presented in Figure 14 as a 'bidirectional vehicle'.]

Crossing a Road

Crossing a road from a stationary position requires a region clear of approaching traffic to each side of the vehicle. This distance, s_{cross}, is a function of the time taken for the trailing edge of the vehicle to leave each traffic lane and the maximum traffic speed:

$$s_{cross} = \begin{cases} v_{\max}\sqrt{2\frac{pi+l+k}{a}} & \text{if } (pi+l+k) < \frac{v_{\max}^2}{2a} \\ \frac{v_{\max}^2}{2a} + pi + l + k & \text{otherwise} \end{cases}, \tag{3}$$

where p is the width of the path or traffic lane, i is the traffic lane number, k is the distance between the starting position of the front of the vehicle and the closest edge of the first traffic lane. Figure 2 shows a plan view of the representative vehicle accelerating across a four lane road at 2.8 ms^{-2} from a standing start 1-m from the kerb on the southern side of the road. Also shown in Figure 2 are the safety regions needed to each side of the vehicle for each traffic lane from (3). Figure 3 presents (3) for the representative vehicle crossing one to four 3.6-m wide lanes versus rates of acceleration. Note that in these and following figures the labels represent the lane number in a compass direction, for example W3 is the third of east-west lanes in the westerly direction.

The analysis suggests that depending on vehicle acceleration and number of traffic lanes a gap of between 20 and 70 metres is required for a vehicle to cross a road without collision. For the rate of acceleration of the representative vehicle ($a = 2.8$ms^{-2}) a gap of between 30 and 43 metres is required for this vehicle to cross a one to four lane road.

Overtaking

Overtaking a vehicle requires a safety region to the front of the vehicle free of approaching vehicles. The distance needed to overtake with a maximum forward speed of 13.4 m/s, vehicle lengths of 4.6 m and an additional safety buffer of 8 m around the overtaken vehicle can

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[2] For a detailed kinematic analysis of vehicles merging into traffic flows and changing lanes the reader is referred to [Jula *et al.*, 2000].

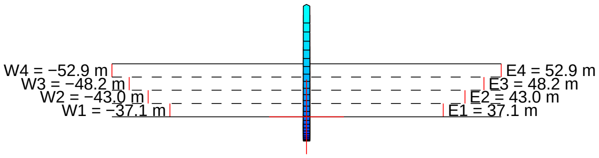

Figure 2: Safety region to each side of a representative vehicle starting from a position 1-m from the southern edge of a four-lane east-west road (ie $k = 1$ m) that accelerates at 2.8 ms^{-2} to 13.4 m/s.

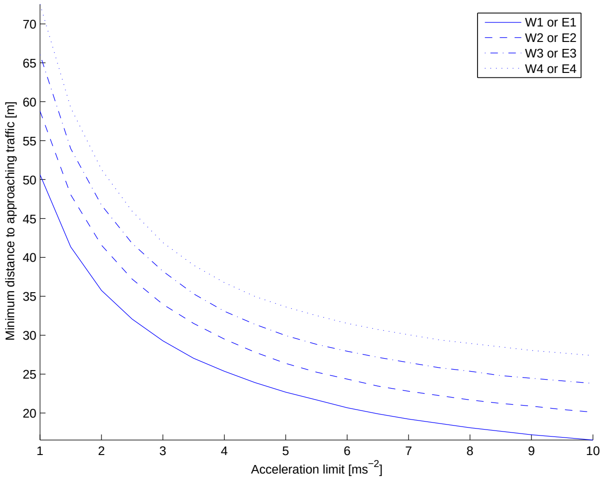

Figure 3: Safety region versus $a_{\max}$ for a representative vehicle with a maximum speed of 13.4 m/s to cross a four-lane road without colliding with any traffic travelling at 13.4 m/s.

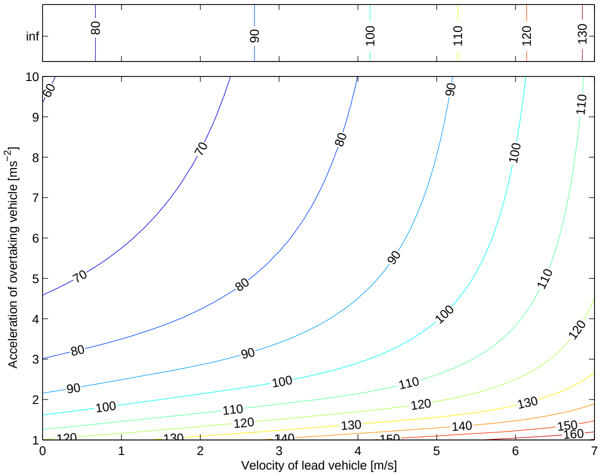

Figure 4: Forward safety region [m] required for overtaking a vehicle with a maximum speed of 13.4 m/s.

be found in Figure 4. The upper set of contour lines is for a vehicle overtaking at maximum velocity (ie $a = \infty$) that is able to stop (at 10 ms−2) before colliding with the rear of the stationary vehicle and the lower set of contour lines is for an overtaking vehicle that starts with a forward speed that matches the overtaken vehicle.

A gap of around 80 metres in approaching traffic is required for a vehicle to overtake a near stationary vehicle at full speed. If such a gap is not available when the vehicle approaches it's panic stop distance (1) then the vehicle would have to decelerate to match the speed of the lead vehicle. For the vehicle to be able to overtake from this position it would need to accelerate around the lead vehicle, which at the performance of the representative vehicle will require a gap in approaching traffic of at least 80-m.

2.4 Right-Angle-Turn

Right-angle-turns are inherently risky with approximately 25% of all US traffic crashes being crossing path collisions [Chan, 2005] and are too complex for a simple analytical approach. So, a recently developed vehicle motion model to automate multiple-partion-turns [Robertson and Durrant-Whyte, 2006] was adapted to model acceleration-bounded time-optimal right-angle turns. In this regard, acceleration is the magnitude of the second derivative of position with respect to time and includes the net combination of linear acceleration, braking and lateral acceleration. A simulated right-angle-turn is presented in Figure 5 that shows the control input, net acceleration and plan view of a time optimal right-angle-turn into the closest traffic lane from an initial stationary starting position 1-m from the nearest edge of the first lane. Figure 6 presents the plan view of a right-angle-turn from the same position into

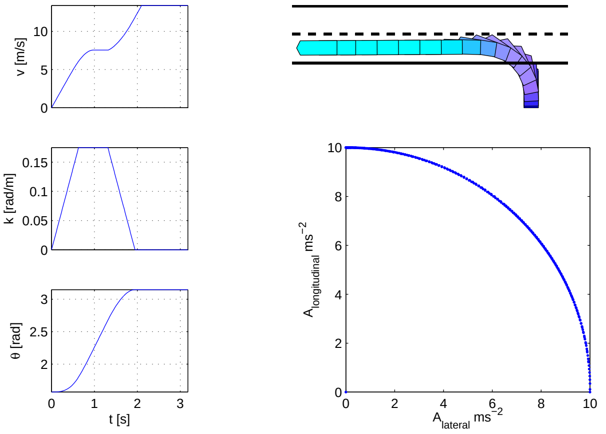

Figure 5: Velocity, curvature and theta profile (left), plan view (upper right) and lateral vs longitudinal acceleration (lower right) for the representative vehicle turning left into the first lane of a two lane road with lanes $w = 3.6$-m wide, $t_{steer} = 1$ s, $|a_{\max}| = 10\text{ms}^{-2}$ and $v_{\max} = 13.4$ m/s.

the fourth closest traffic lane with the required safety region for each region indicated. The maximum extent of the safety regions to the front, outside and inside of turn from lane S1 into lanes 'W1' through to 'W4' versus rate of acceleration are presented in Figures 7, 8 and 9, respectively.

These results show that without any margin for error a safety region ranging between 30 and 140-m is required, depending on the achievable level of acceleration. With the representative vehicle requiring between 50 and 80 metres, depending on the direction of the approaching traffic. In practice, a cross path vehicle separation of less than a second is considered a near miss and a buffer of less than two seconds classified as a potential conflict [Chan, 2006], so these distance will need to be extended to achieve an acceptable level of risk.

2.5 180° Turn

Urban road networks comprise single- and multiple-lane connecting paths that support traffic movement in one or two directions. Subject to local rules and conditions, paths with at least one lane in each direction can usually support course reversal, or vehicle reorientation, for some part of their length. The ability for a path to support vehicle reorientation can substantially reduce the physical path length of the route needed to travel between any two points, with bi-directionality of the route at the start and end locations of the journey being most critical. To consider an extreme, yet everyday, example of parking a vehicle on the other side of a multiple-lane road. This task requires either a series of 90 degree turns around each corner of nearby block, a multiple-partition-

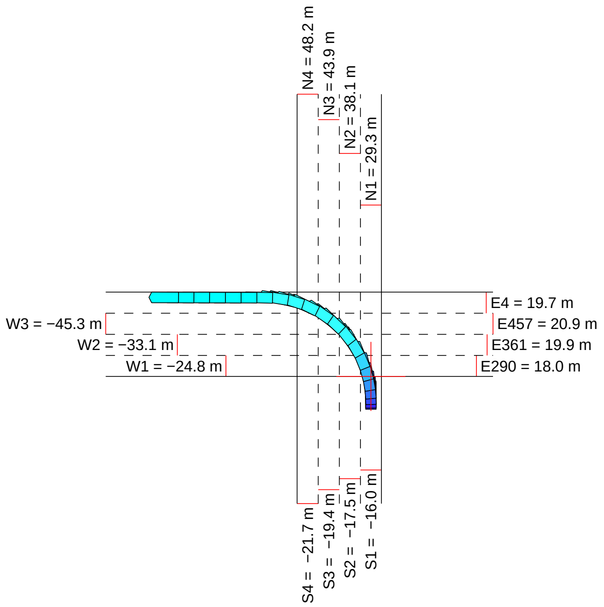

Figure 6: Safety region marked for a 90° turn with $v_{\max} = 13.4$ m/s, $|a_{\max}| = 10$ ms^{-2}, $\kappa_{\max} = 0.06$ and $t_{steer} = 1$ s from a stationary starting position 1-m from the southern edge of the E1/W1 lane.

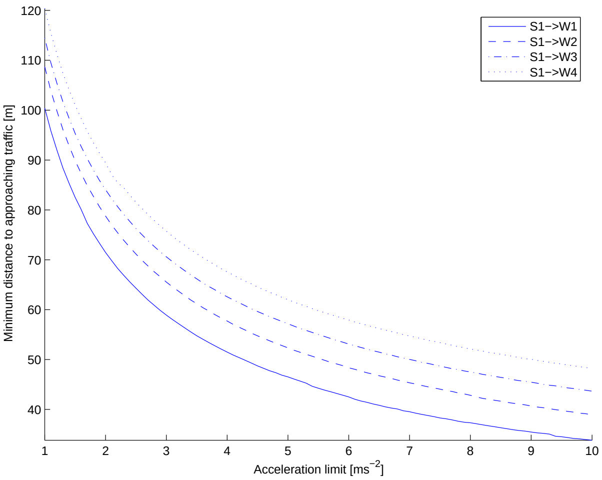

Figure 7: Safety region versus $|a_{\max}|$ for 90 degree turns into lane indicated with traffic approaching from the northerly direction at 13.4 m/s.

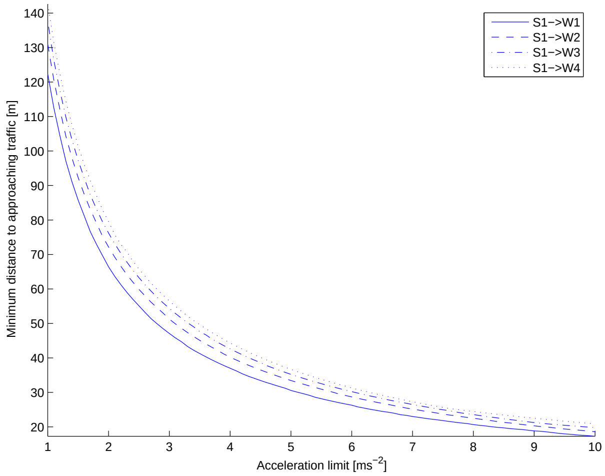

Figure 8: Safety region versus $|a_{\max}|$ for 90 degree turns into lane indicated with traffic approaching from the easterly direction, or from the outside of the turn, at 13.4 m/s.

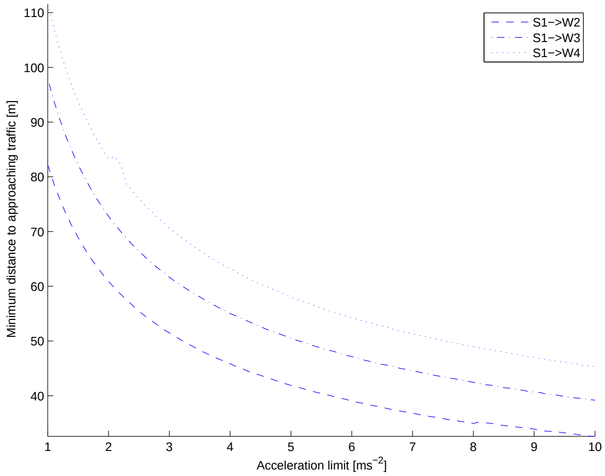

Figure 9: Safety region versus $|a_{\max}|$ for 90 degree turns into lane indicated with traffic approaching from the westerly direction, or from the inside of the turn, at 13.4 m/s.

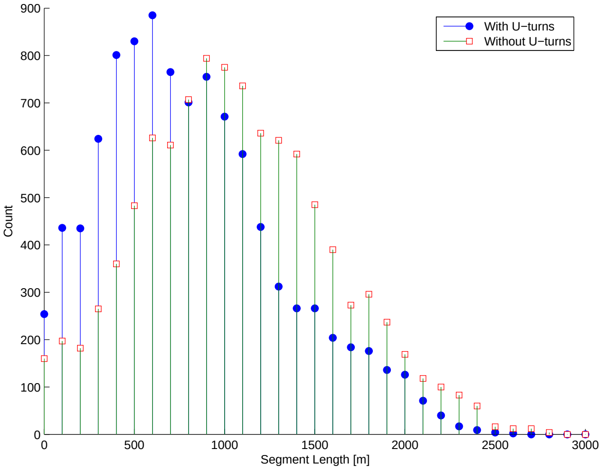

Figure 10: Histogram of length-optimal paths between 10,000 waypoints with and without u-turns.

turn [Robertson and Durrant-Whyte, 2006] or, if space is available, a u-turn.

To explore the path efficiency available by mid-path reorientation, length-optimal routes[3] between 10,000 randomly selected waypoint pairs were compared over two subtly different graphs: the first graph being the example route network, which features 13 road segments comprised of 146 nodes and 173 edges [Darpa, 2006b] with the addition of 59 edges that were manually inserted by the author to enable lane changing on adjacent lanes travelling in the same direction; and the second graph including an additional 70 edges that were manually inserted by the author to connect adjacent lanes with opposing directions of traffic.

Histograms of the distance between consecutive waypoints along the two length-optimal routes are shown in Figure 10. The average of the shortest paths without u-turns was 1064 m and with u-turns it was 817 m, or about 23% shorter. The remainder of this section quantifies the time penality and necessary safety region for four manoeuvres that can be used for course reversal: reversing, u-turn, reverse-hook and three-point-turn.

A multiple-partition turn algorithm for the reorientation of a vehicle between two parallel lines has recently been developed that enables the turn duration and thus required safety region to be determined as a function of a vehicle's characteristic dimensions, minimum turning radius and net acceleration [Robertson and Durrant-Whyte, 2006]. An example of a three partition path generated by this algorithm and the necessary steering and velocity controls required for this turn can be found in Figure 11. This algorithm has been slightly extended to simulate reverse-hook turns that may be slightly faster

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[3] Determined using Dijkstra's algorithm [Dijkstra, 1959].

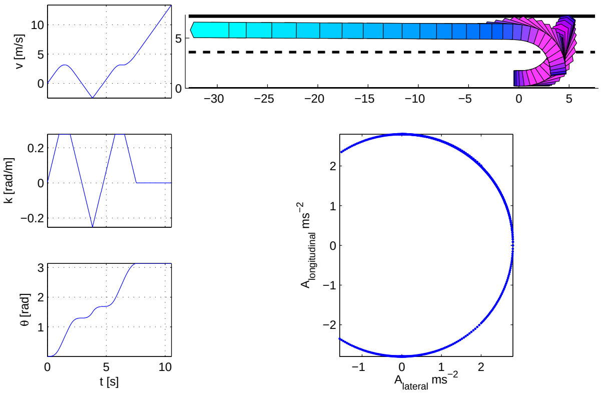

Figure 11: Velocity, curvature and theta profile (left), plan view (upper right) and lateral vs longitudinal acceleration (lower right) for a small vehicle ($w = 1.5$ m and $l = 3.5$ m) undertaking a 180 degree three-point-turn on a 7.2-m wide two lane road, with $t_{steer} = 1$ s, $\kappa_{\max} = 0.2778$, $|a_{\max}| = 2.8\text{ms}^{-2}$ and $v_{\max} = 13.4$ m/s.

from a stationary start, Figure 12. Other manoeuvres to reorient a vehicle include a u-turn, which requires path-widths greater than the turning circle diameter of the vehicle, and reversing, which requires a bidirectional vehicle. The time required for a small vehicle ($w = 1.5$ m and $l = 3.5$ m) to conduct each of the four types of manoeuvres from a stationary start and accelerate up to $v_{\max}$ $= 13.4$ m/s on a standard 7.2-m wide two-lane road are presented in Figure 13. Additional time, $v_{\max}/a$, (and presented as the solid line in the figure) must be allowed for the vehicle to decelerate from a moving start. Figure 14 presents the safety regions in the direction of initial and final orientations, as shown by the positive and negative vertical axis, respectively. Similarly, in order to gauge the feasibility of employing vehicle reorientation to shorten the path between consecutive waypoints, Figure 15 presents the same results from a moving start.

The simulation results enable the relative performance of the four manoeuvres to be compared. In general, the simpler the manoeuvre, the faster it can be completed and thus the smaller the necessary safety region. The results indicate that if route reversal is to be employed as a competitive strategy to reduce the length of path between consecutive waypoints then due consideration should be given to the size and minimum turning radius of candidate vehicles. Clearly, a small vehicle with a tight turning radius that is capable of using a simple u-turn to turn around would provide advantages over a larger vehicle that would need to resort to using a multiple-partition path to turn around.

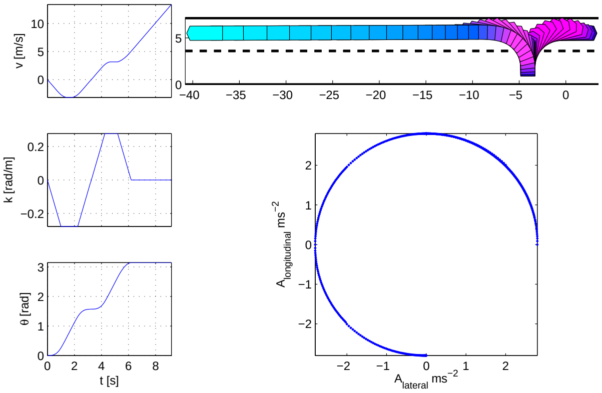

Figure 12: Velocity, curvature and theta profile (left), plan view (upper right) and lateral vs longitudinal acceleration (lower right) for a small vehicle ($w = 1.5$ m and $l = 3.5$ m) undertaking a 180 degree reverse-hook-turn on a 7.2-m wide two lane road, with $t_{steer} = 1$ s, $\kappa_{\max} = 0.2778$, $|a_{\max}| = 2.8\text{ms}^{-2}$ and $v_{\max} = 13.4$ m/s.

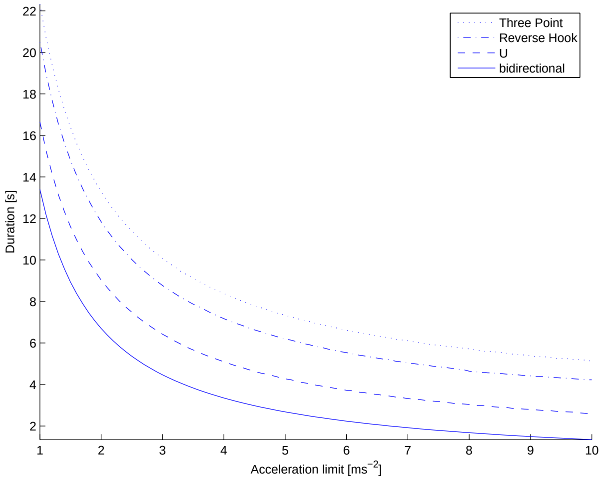

Figure 13: Duration versus $|a_{\max}|$ for 180 degree turns for a small vehicle ($w = 1.5$ m and $l = 3.5$ m) operating with $v_{\max}$ at 13.4 m/s.

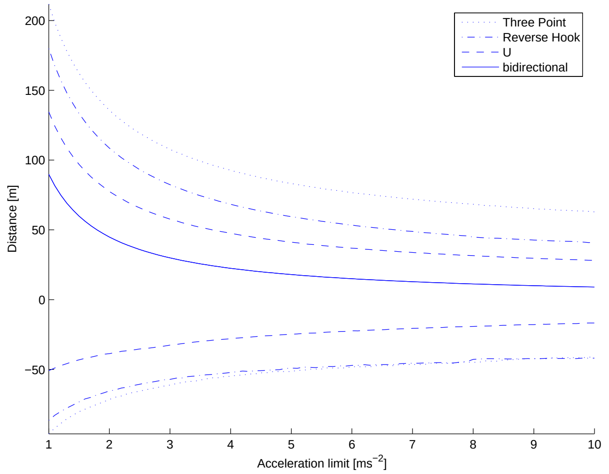

Figure 14: Safety region required for four types of 180 degree turns by a small vehicle ($w = 1.5$ m and $l = 3.5$ m) from a stationary start and a maximum speed of 13.4 m/s.

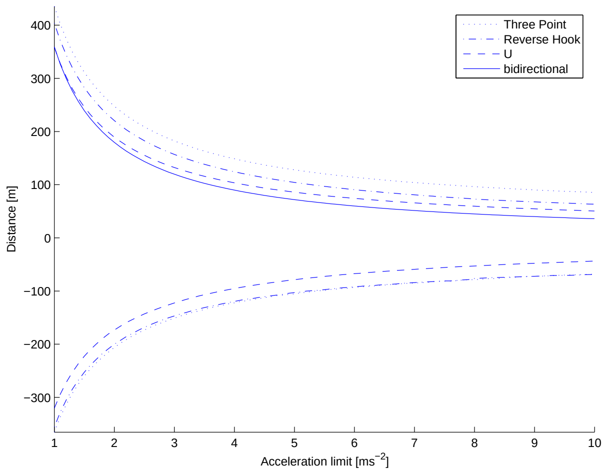

Figure 15: Safety region required for four types of 180 degree turns by a small vehicle ($w = 1.5$ m and $l = 3.5$ m) from a moving start and a maximum speed of 13.4 m/s.

3 Discussion

Net acceleration has a diminishing, but important, effect on minimum perception range requirements for safe manoeuvring in uncontrolled environments. Safe passage requires a collision free path for at least the distance required to stop, this distance is of the order of 10 to 30 m. Merging into a traffic flow requires a safety region to the rear of the vehicle of between 9 and 90 m. Crossing a road requires a safety region in the direction of approaching traffic of 15 to 75 m. Overtaking of slower moving vehicles which requires the use of the wrong side of the road poses severe demands on a vehicle's perception system. Without speeding it is only feasible to overtake vehicles travelling substantially below the speed limit. Fortunately, the race rules explicitly exclude the need to overtake non-stationary vehicles that require driving on the wrong side of the road. Overtaking a stationary vehicle, which is within scope of the event, requires a forward safety region of at least 80-m or more depending on the initial velocity of the overtaking vehicle and the rate of acceleration of the overtaking vehicle. Right-angle turns require a safety region in the directions of approaching traffic of between 20 to 140 m. The time needed for a vehicle to reorient is strongly dependent on the vehicle's inherent characteristics and the width of the road which prescribe the type of manoeuvre [Robertson and Durrant-Whyte, 2006]. On roads less than 9-m wide mid-size vehicles are likely to resort to three-point manoeuvres to reorientate, which requires a forward safety region of between 75 and 200 metres and a rear safety region of between 50 and 100 metres. Note that whilst vehicle reorientation is essential to recover from a blocked path, which by definition should reduce the probability of meeting on-coming traffic, vehicle reorientation can substantially reduce the path length between checkpoints. In this regard, vehicle reorientation from a moving start requires safety region of the order of 60 to 400 m.

The extent of the safety regions for a vehicle operating at net acceleration limit of 3 ms^{-2} are summarised in Figure 16. Note that the distances quoted for a panic stop and merge are independent of vehicle dimension, whereas the 90 degree corner and overtaking distances are for the representative vehicle and the 180 degree reorientation distances are for a smaller vehicle. The analysis shows that a minimum perception range of at least 80 m is required in each direction of oncoming for a vehicle to avoid entering into an inevitable collision state. A perception system with this range would provide no margin for error or any safety buffer between vehicles and would also be unlikely to be able support either the overtaking of moving vehicles or course reversal in unconstrained traffic flows.

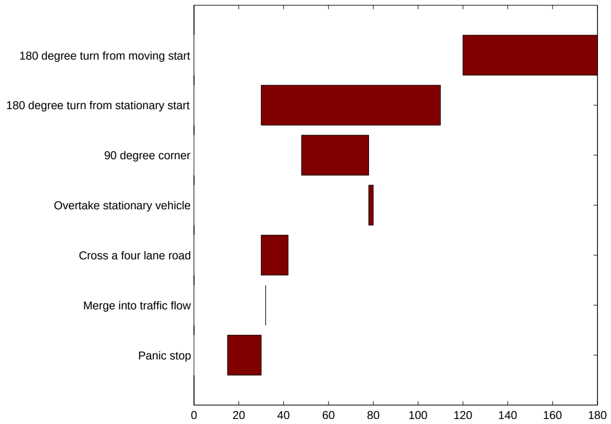

Figure 16: Extent of safety regions for a vehicle operating at within a net acceleration limit of 3 ms^{-2} and maximum speed of 13.4 m/s.

4 Conclusion

This paper quantified the minimum perception range requirements that are needed for a vehicle to avoid entering an inevitable collision state in an urban road network for nine different manoeuvres. The paper also used a modified version of an urban road route network to gauge the potential advantage of adopting a course reversal strategy in an urban environment. Adoption of a course reversal strategy, which was shown to lead to a substantial reduction in total path length, requires a small, tight turning vehicle that is capable of using a u-turn, otherwise the additional perception range needed to avoid entering an inevitable collision state would in all likelihood preclude the use of such a strategy.

5 Acknowledgements

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