Passivity-based Control of Robot Manipulators Subject to Constraints

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Abstract

This paper presents a passivity-based controller design capable of achieving autonomous obstacle avoidance for robot manipulators subject to joint position and joint rate constraints. The control objectives are achieved by exploiting the passivity properties of the system and utilizing barrier function ideas to reshape the control Lyapunov function. The final control Lyapunov function is reminiscent of those used in the artificial potential field method.

1 Introduction

Robot manipulators have become an integral part in almost all modern manufacturing processes, performing tasks that are considered too dull, repetitive, and hazardous for humans, or that require strength, skill, and precision beyond the capability of humans. The control problem for robot manipulators is therefore a well-studied one. The recognised 'classical' techniques for controlling a manipulator include: feedback linearization or inverse dynamics [Kreutz, 1989], variable structure control [Slotine and Li, 1991], computed torque feedforward control [An *et al.*, 1989; Khosla and Kanade, 1989], and passivity-based control (PBC) [Takegaki and Arimoto, 1981]. However, although these early works solved the global asymptotic tracking and set-point regulation problems, they did not address the problems of obstacles in the workspace and/or physical constraints on the robot's joint positions and joint rates.

The environments in which robot manipulators operate are often constrained and cluttered. Due to factors such as safety and economy of operations, it is imperative that the robots avoid collisions with obstacles while performing their work. There is a great amount of research devoted to the obstacle avoidance problem. Lozano-Perez [1987], Brooks [1984], and others [Faverjon, 1984; Kawarazaki and Taguchi, 1995] proposed off-line algorithms, using free-space, to plan collision-free motions for general robot manipulators. However, these off-line methods are typically computationally expensive and are unsuitable for real-time implementation except for very simple cases [Barraquand *et al.*, 1992]. Online obstacle avoidance approaches on the other hand, are substantially faster and well-suited for real-time applications. Online obstacle avoidance can be achieved by employing the popular artificial potential field method [Khatib, 1986; Rimon and Koditschek, 1992], where the robot is guided by potential fields that exert repulsive forces away from the obstacles and an attractive force toward the desired position. More recently, the extra freedom of the coordinate transformation in the feedback linearisation method has been exploited to solve the autonomous obstacle avoidance problem in [Fujimoto *et al.*, 1998]. Biologically motivated, non-model-based methods have also been considered, including fuzzy logic [Dassanayake *et al.*, 2001; Mdebe *et al.*, 2003], neural networks [Yang and Meng, 2001], and genetic algorithms [Toogood *et al.*, 1995; Gill and Zomaya, 1998]. Although attempts to incorporate Lyapunov-like formalisms into such frameworks have been made in such works as [Jin *et al.*, 1995; Forti and Nistri, 2003], stability and convergence properties of these methods remain, in general, difficult to analyze.

Another important consideration in the controlling of robot manipulators is the physical constraints. Ignoring these will cause saturation as well as sustaining physical damage when a joint position or joint rate is commanded beyond its physical bounds. Time-scaling [Hollerbach, 1983; Sugie *et al.*, 2003] is a standard technique employed to avoid rate saturation along pre-defined trajectories. An alternative approach is the "Windup Feedback Scheme" [Litt *et al.*, 1996; Mutambara, 1998], where, whenever a joint position or joint rate is saturated, the unmet control demands are redistributed among the remaining unsaturated joints. There have also been numerous studies of obstacle avoidance and/or physical constraints for redundant manipulators

[Wikman and Newman, 1992; Chan and Dubey, 1995; Choi and Kim, 2000; Chen and Liu, 2002]. The applicability of these methods, however, is restricted to redundant manipulators only. For general manipulators, there are few works that address both obstacle constraints and physical limits in an integrated framework. Sugie et al. [2003] uses a two step process involving feedback linearisation (with an extra degree of freedom) to address the online obstacle avoidance problem along with a coupled time-scaling adjustment for bounded joint rate control.

This paper addresses the problem of autonomous, or online, obstacle avoidance for general robot manipulators subject to physical constraints on the robot's joint positions and joint rates. The obstacles are assumed to be fixed and stationary, and we only consider set-point regulation in this paper. The controller design is based on the PBC framework, with modifications made to the control Lyapunov function (clf) such that the constraints are strictly satisfied for all time. The structure of the modified clf resembles those used in the artificial potential field method. We differ from the earlier developments by directly integrating the constraint equations into the clf to derive a unified control law that achieves autonomous obstacle avoidance, respects joint position and joint rate limits, and achieves local stabilisation of the set-point. The modification of the clf can be thought of as a form of energy shaping, both the kinetic and potential energy terms in the classical storage function obtained in PBC. The approach suffers from the same limitations of the artificial potential field approach in regard to the possible presence of local minima in the clf.

The paper is organised as follows. Section 2 provides an overview of the classical PBC method. Section 3 details the main results whilst concluding statements are contained in Section 4.

2 Classical passivity-based control for robotic manipulators

In this section we present a brief recap of the classical theory of PBC of robot manipulators for the set-point regulation problem.

Consider a rigid and fully-actuated n-link robot manipulator with no external forces, that is, no end-effector contacts with the environment, and no external disturbances. The dynamics of such systems is described by the Euler-Lagrange equation [Ortega *et al.*, 1998]

$$\mathcal{D}(q)\ddot{q} + \mathcal{C}(q,\dot{q})\dot{q} + g(q) + \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) = \tau \tag{1}$$

where we use the following notation

$q \in \mathbb{R}^n$	generalised joint coordinates,
$\mathcal{D}(q) \in \mathbb{R}^{n\times n}$	generalised inertia matrix,
$\mathcal{C}(q,\dot{q}) \in \mathbb{R}^{n\times n}$	Coriolis-centrifugal matrix,
$g(q) \in \mathbb{R}^n$	gravitational torques,
$\mathcal{F}(\dot{q})$	Rayleigh dissipation function,
$\tau \in \mathbb{R}^n$	applied input torques

For all serial manipulators

$$\mathcal{D}(q)^T = \mathcal{D}(q) > 0, \quad \forall q \in \mathbb{R}^n, \tag{2}$$

and

$$\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) \geq 0, \qquad \text{and} \qquad \frac{\partial \mathcal{F}}{\partial \dot{q}}(0) = 0. \tag{3}$$

Let the vector $[\tilde{q}, \dot{q}]^T$ define the system state, where

$$\tilde{q} = q - q_d \tag{4}$$

represents the error between the actual and the desired link position. Select the following positive definite function as the candidate clf

$$\mathcal{L}(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^T\mathcal{D}(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P \tilde{q}, \tag{5}$$

which is derived from the kinetic energy of the system along with the "shaped" potential energy. The control gain matrix $K_P \in \mathbb{R}^{n\times n}$ is constant, diagonal, and positive definite. Note that the potential energy has been shaped such that the set-point $[q_d, 0]^T$ is now the equilibrium of the system.

Differentiating (5) with respect to time yields

$$\dot{\mathcal{L}} = \dot{q}^T\mathcal{D}(q)\ddot{q} + \frac{1}{2}\dot{q}^T\dot{\mathcal{D}}(q)\dot{q} + \tilde{q}^T K_P \dot{q} \tag{6}$$

Substituting the system dynamics (1) into (6) gives

$$\dot{\mathcal{L}} = \frac{1}{2}\dot{q}^T\left[\dot{\mathcal{D}}(q) - 2\mathcal{C}(q,\dot{q})\right]\dot{q} - \dot{q}^T\frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) + \dot{q}^T\left[\tau - g(q) + K_P\tilde{q}\right] \tag{7}$$

The first term on the right hand side (RHS) of (7) is null due to the passivity properties of mechanical systems [Ortega *et al.*, 1998]. The second term is negative semi-definite (or dissipative) due to the properties of the Rayleigh dissipation function, see (3). To stabilise the system, the following input torque is chosen, which renders $\dot{\mathcal{L}}(\tilde{q},\dot{q})$ negative semi-definite

$$\tau = g(q) - K_P\tilde{q} - K_D\dot{q} \tag{8}$$

where the control gain matrix $K_D \in \mathbb{R}^{n\times n}$ is constant, diagonal, and positive definite. The resulting closed-loop dynamics is given by

$$\mathcal{D}(q)\ddot{q} = -K_P\tilde{q} - K_D\dot{q} - \mathcal{C}(q,\dot{q})\dot{q} - \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) \tag{9}$$

which renders

$$\dot{\mathcal{L}} = -\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) - \dot{q}^T K_D \dot{q} \leq 0, \quad \forall \tilde{q}, \dot{q} \in \mathbb{R}^n. \tag{10}$$

Application of Lyapunov's direct method [Khalil, 2002] to (5) and (10) guarantees convergence to the invariant set characterised by $\mathcal{S} := \{(\tilde{q}, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n \mid \dot{q} = 0\}$, and application of LaSalle's Invariance Principle [Khalil, 2002] ensures that the only forward invariant subset of this set under the closed-loop dynamics (9) is the desired equilibrium point $[q - q_d, \dot{q}]^T = [0, 0]^T$. Detailed proof of these results can be found in such robotics text as [Sciavicco and Siciliano, 2003].

3 Constrained passivity-based control

This section proposes a modification to the 'classical' PBC design for robot manipulators to incorporate obstacle constraints as well as physical constraints on joint positions and joint rates.

In the following derivation, obstacle constraints and joint position constraints are represented mathematically by one-sided inequalities expressed in terms of the joint positions. Each constraint is represented by a separate constraint function.

Assumption 3.1. *For each obstacle or joint position constraint, there exists a differentiable function $h_i(q)$ and a constant $\Delta_i \in \mathbb{R}$ such that*

$$h_i(q) \geq \Delta_i \tag{11}$$

characterises the accessible workspace for that constraint.

Remark 3.2. *Non-smooth transitions such as edges and corners of an obstacle or singularities in joint positions can be accommodated by approximating the non-smooth constraint with a differentiable constraint. In practice, the function $h_i(q)$ is only required to be differentiable on the set $h_i(q) > \Delta_i$. It is acceptable to work with constraint functions $h_i(q)$ that are non-differentiable on the constraint boundary itself.*

Rate constraints can be accommodated as long as they can be expressed as quadratic functions of joint rates.

Assumption 3.3. *For each rate constraint there exists a smoothly varying positive semi-definite matrix $Q_j(q) \geq 0$ and a smooth function $\Omega_j(q)$ such that the rate constraint can be expressed as*

$$\frac{1}{2}\dot{q}^T Q_j(q)\dot{q} \leq \Omega_j(q).$$

Remark 3.4. *The simplest rate constraints are where each and every individual joint rate is bounded as follows*

$$|\dot{q}_j(t)| \leq B_j, \quad t \geq 0, \quad j = 1, ..., n, \tag{12}$$

where $B_j > 0$ is a constant. In this case, choosing $\Omega_j = B_j^2/2$, the rate constraints are expressed as

$$\frac{1}{2}\dot{q}_j(t)^2 = \frac{1}{2}\dot{q}^T e_j e_j^T \dot{q} = \frac{1}{2}\dot{q}^T Q_j \dot{q} \leq \Omega_j, \quad t \geq 0,$$

for $j = 1, ..., n$. Here e_j denotes the unit vector in the j'th direction and the matrix $Q_j = e_j e_j^T \geq 0$ is positive semi-definite. A limitation of Assumption 3.3 is that the rate constraints have to be symmetric about the origin. Thus, a rate constraint $-a < \dot{q}_j(t) < b$ where $a, b \in \mathbb{R}_+$, $a \neq b$ cannot be achieved.

Denote the number of configuration constraints, that is, obstacle and joint position constraints, by N and the number of rate constraints by M. We introduce some notation to simplify the following derivation

$$\Phi(q) = \prod_{i=1}^{N} \phi_i(q), \quad \phi_i(q) = h_i(q) - \Delta_i, \tag{13}$$

for $i = 1, \ldots, N$, and

$$\Psi(q, \dot{q}) = \prod_{j=1}^{M} \psi_j(q, \dot{q}), \quad \psi_j(q, \dot{q}) = \Omega_j(q) - \frac{1}{2}\dot{q}^T Q_j(q)\dot{q}, \tag{14}$$

for $j = 1, \ldots, M$. The admissible constraint set for the problem is the set of states defined by

$$\begin{aligned} \mathcal{S} &= \{(q, \dot{q}) \mid \Phi(q) > 0 \text{ and } \Psi(q, \dot{q}) > 0\} \\ &= \{(q, \dot{q}) \mid \Phi(q)\Psi(q, \dot{q}) > 0\}, \end{aligned} \tag{15}$$

whose boundary is given by

$$\partial\mathcal{S} = \{(q, \dot{q}) \mid \Phi(q)\Psi(q, \dot{q}) = 0\} \tag{16}$$

The control problem considered is that of stabilisation to the target set-point $[q_d, 0]$. It is desired to have control that behaves as do the PBC designs when distant from the constraints and is modified to ensure that the constraints are always respected. The underlying idea of the approach is similar to that of the artificial potential field method. We differ from the earlier developments in that we directly integrate the constraint equations into the clf and use this clf to derive a unified control law that fully respects the system's dynamics. To ensure a well posed problem we make the following final assumption.

Assumption 3.5. *The initial condition $(q_0, \dot{q}_0) \in \mathcal{S}$. The desired link position $q_d \in \mathcal{S}$, and q_d and q_0 lie in the same connected component of $\mathcal{S}$.*

Consider the candidate clf

$$V(\tilde{q}, \dot{q}) = \frac{\mathcal{L}(\tilde{q}, \dot{q})}{\Phi\Psi} = \frac{1}{2\Phi\Psi}\left[\dot{q}^T \mathcal{D}(q)\dot{q} + \tilde{q}^T K_P \tilde{q}\right], \tag{17}$$

where the function $\mathcal{L}(\tilde{q}, \dot{q})$ is as defined earlier in (5). The constraint functions Φ and Ψ are identically zero on the constraint boundary $\partial\mathcal{S}$. Consequently, $V(\tilde{q}, \dot{q})$ is asymptotically infinite on $\partial\mathcal{S}$. The proposed candidate clf is similar to the barrier functions used in optimisation methods and the underlying idea is closely linked to the artificial potential field method. The advantage of the approach taken is that the function $V(\tilde{q}, \dot{q})$ can be thought of as a shaped energy function for the constrained system.

Theorem 3.6. *Consider the dynamics (1) for a serial manipulator. Given configuration and rate constraints satisfying Assumptions 3.1 and 3.3, and functions Ψ and Φ as defined by (13) and (14). Define $\mathcal{L}(\tilde{q}, \dot{q})$ according to (5), and define*

$$a_\phi : \mathbb{R}^n \to \mathbb{R}^n, \quad a_\phi(q) := \sum_{s=1}^{N} \left(\prod_{i \neq s}^{N} \phi_i(q) \right) \frac{\partial \phi_s}{\partial q} \tag{18}$$

$$a_\psi : \mathbb{R}^{2n} \to \mathbb{R}^n, \quad a_\psi(q, \dot{q}) := \sum_{s=1}^{M} \left(\prod_{j \neq s}^{N} \psi_j(q, \dot{q}) \right) \left[\frac{\partial \Omega_s}{\partial q} - \frac{1}{2}\dot{q}^T \dot{Q}_j \dot{q} \right] \tag{19}$$

$$P : \mathbb{R}^{2n} \to \mathbb{R}^{n \times n}, \quad P(q, \dot{q}) := \sum_{s=1}^{M} \left(\prod_{j \neq s}^{N} \psi_j(q, \dot{q}) \right) Q_s \tag{20}$$

Choose the torque input to be

$$\tau(\tilde{q}, \dot{q}) = g(q) + \mathcal{D}(q) \Big[\Phi\Psi\mathcal{D}(q) + \mathcal{L}\Phi P \Big]^{-1} \Big[-\Phi\Psi \{K_P \tilde{q} + K_D \dot{q}\} + \mathcal{L}\{\Psi a_\phi + \Phi a_\psi\} + \mathcal{L}\Phi P \mathcal{D}^{-1}(q) \Big\{ \mathcal{C}\dot{q} + \frac{\partial \mathcal{F}}{\partial \dot{q}}(\dot{q}) \Big\} \Big], \tag{21}$$

where the control gain matrices $K_P, K_D \in \mathbb{R}^{n \times n}$ are constant, diagonal, and positive definite. Then for any initial condition $[q_0, \dot{q}_0]^T$ and desired link position q_d satisfying Assumption 3.5, all trajectories of the closed-loop system remain inside the admissible constraint set $\mathcal{S}$ for all time, and converge to the invariant set characterised by the following equality

$$\Phi(q)\Psi(q,0)K_P\tilde{q} = \mathcal{L}(\tilde{q}, 0)\left[\Psi(q,0)a_\phi(q) + \Phi(q)a_\psi(q,0)\right], \tag{22}$$

which contains the set-point $[q_d, 0]^T$.

To simplify the notation, in the following development, functions are written without their arguments unless wherever confusion may arise.

Proof. Consider the clf given by (17). Differentiating with respect to time yields

$$\dot{V} = \frac{\dot{\mathcal{L}}\Phi\Psi - \mathcal{L}(\dot{\Phi}\Psi + \Phi\dot{\Psi})}{(\Phi\Psi)^2}. \tag{23}$$

Taking the time derivatives of Φ and Ψ gives

$$\dot{\Phi} = \langle a_\phi, \dot{q} \rangle$$
$$\dot{\Psi} = \langle a_\psi, \dot{q} \rangle - \langle P\mathcal{D}^{-1} \left[\tau - \mathcal{C}\dot{q} - g(q) - \frac{\partial \mathcal{F}}{\partial \dot{q}} \right], \dot{q} \rangle$$

respectively, where $\langle \cdot, \cdot \rangle$ denotes the inner product. Substituting $\dot{\Phi}$, $\dot{\Psi}$, and (7) into the expression for $\dot{V}$ yields

$$\dot{V} = \frac{\Phi\Psi}{(\Phi\Psi)^2} \Big\{ \frac{1}{2}\dot{q}^T \left[\dot{\mathcal{D}} - 2\mathcal{C} \right] \dot{q} - \dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}} + \dot{q}^T \left[\tau - g(q) + K_P \tilde{q} \right] \Big\} - \frac{\mathcal{L}}{(\Phi\Psi)^2} \Big\{ \langle a_\phi, \dot{q} \rangle \Psi + \Phi \langle a_\psi, \dot{q} \rangle - \Phi \langle P\mathcal{D}^{-1} \left[\tau - \mathcal{C}\dot{q} - g(q) - \frac{\partial \mathcal{F}}{\partial \dot{q}} \right], \dot{q} \rangle \Big\}. \tag{24}$$

Recalling that $\dot{q}^T[\dot{\mathcal{D}} - 2\mathcal{C}]\dot{q} = 0, \forall q, \dot{q} \in \mathbb{R}^n$, and $\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}} \geq 0$, and collecting like terms together, one obtains

$$\dot{V} = \frac{\dot{q}^T}{(\Phi\Psi)^2} \Big\{ \left[\Phi\Psi\mathcal{D} + \mathcal{L}\Phi P \right] \mathcal{D}^{-1}\tau + \Phi\Psi \left[K_P\tilde{q} - g(q) \right] - \mathcal{L}\left[\Psi a_\phi + \Phi a_\psi \right] - \mathcal{L}\Phi P\mathcal{D}^{-1} \left[\mathcal{C}\dot{q} + g(q) + \dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}} \right] \Big\} - \frac{1}{\Phi\Psi}\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}}. \tag{25}$$

Note that the matrices $[\Phi\Psi\mathcal{D} + \mathcal{L}\Phi P]$ and $\mathcal{D}$ are both positive definite and thus have well-conditioned inverses for all $\tilde{q}, \dot{q} \in \mathcal{S}$. As a consequence, the proposed feedback control (21) is well-defined for all $\tilde{q}, \dot{q} \in \mathcal{S}$. Substituting (21) into (25) renders

$$\dot{V} = \frac{-1}{\Psi\Phi}\dot{q}^T \frac{\partial \mathcal{F}}{\partial \dot{q}} - \frac{\dot{q}^T K_D \dot{q}}{\Phi\Psi} \leq 0. \tag{26}$$

It follows from (26) that

$$V(t) \leq V(0). \tag{27}$$

As the desired joint position q_d lies properly inside $\mathcal{S}$ from Assumption 3.5, it follows that $\mathcal{L}$ is strictly positive on the constraint boundary $\partial\mathcal{S}$. From (13) and (14), the functions Φ and Ψ are identically zero on $\partial\mathcal{S}$. Thus, the clf V is unbounded (to positive infinity) on $\partial\mathcal{S}$. However, (27) guarantees that V remains upper-bounded. Consequently, the closed-loop trajectories remain inside the admissible constraint set $\mathcal{S}$ for all time.

Application of Lyapunov's direct method [Khalil, 2002] to (17) and (26) guarantees that $\dot{q} \to 0$. From (26), it follows that $\dot{V} = 0$ only if $\dot{q} = 0 = \ddot{q}$. By examining (1) and (21), it is straightforward to verify that at the equilibrium set $\dot{q} = 0 = \ddot{q}$, the closed-loop dynamics is given by

$$\begin{aligned} g(q) = g(q) & \\ &+ \mathcal{D}(q)\Big[\Phi(q)\Psi(q,0)\mathcal{D}(q) + \mathcal{L}(\tilde{q},0)\Phi(q)P(q,\dot{q})\Big]^{-1} \\ &\qquad \Big[-\Phi(q)\Psi(q,0)K_P\tilde{q} \\ &\qquad\quad +\mathcal{L}(\tilde{q},0)\left\{\Psi(q,0)a_\phi(q) + \Phi(q)a_\psi(q,0)\right\}\Big] \end{aligned}$$

Cancelling $g(q)$ and pre-multiplying both sides by $[\Phi(q)\Psi(q,0)\mathcal{D} + \mathcal{L}(\tilde{q},0)\Phi(q)P(q,0)]\,\mathcal{D}^{-1}$ results in the following equality

$$\Phi(q)\Psi(q,0)K_p\tilde{q} = \mathcal{L}(\tilde{q},0)\left[\Psi(q,0)a_\phi(q) + \Phi(q)a_\psi(q,0)\right] \tag{28}$$

Application of Lasalle's Invariance Principle [Khalil, 2002] yields the conclusion that all closed-loop trajectories converge to the forward invariant set defined by (28). Furthermore, as $\mathcal{L}(q_d, 0) = 0$, it follows from (28) that the set-point $[q_d, 0]^T$ lies inside this invariant set. □

It is possible in certain applications that the bracketed term on the RHS of (28) is uniformly zero for all $q \in \mathcal{S}$

$$\Psi(q,0)a_\phi(q) + \Phi(q)a_\psi(q,0) = 0. \tag{29}$$

One instance when (29) holds naturally is when the only active constraints are joint rate constraints for which the parameters Q_j and Ω_j, see (14), are constants. In such cases, (28) simplifies to

$$\Phi(q)\Psi(q,0)K_p\tilde{q} = 0. \tag{30}$$

Since $\Phi(q)\Psi(q,0) > 0$ for all $q \in \mathcal{S}$, the equality (30) is true if and only if $\tilde{q} = 0$. It follows from Lasalle's Invariance Principle that the set-point $[q_d, 0]^T$ is globally asymptotically stable in such cases.

Another case that is of practical interest is when the joint rate constraints are expressed as a bound on the kinetic energy of the system,

$$\Psi(q,\dot{q}) = \Omega - \frac{1}{2}\dot{q}^T\mathcal{D}(q)\dot{q}, \tag{31}$$

where $\Omega > 0$ is a constant. In this particular case there is no direct bound on any single joint velocity, however, the overall kinetic energy of the system is upper bounded by the constant Ω.

Corollary 3.7. *Consider the dynamics (1) for a serial manipulator. Given general configuration constraints $\Phi(q)$ satisfying Assumption 3.1, a single rate constraint $\Psi(q,\dot{q})$ of the form (31), and the admissible constraint set $\mathcal{S}$ as defined by (15). Define $\mathcal{L}(q,\dot{q})$ according to (5) and $a_\phi(q)$ according to (18). Choose the torque input to be*

$$\tau(\tilde{q},\dot{q}) = g(q) - \frac{\Psi}{\Phi(\Psi+\mathcal{L})}\left(\Phi\left[K_P\tilde{q} + K_D\dot{q}\right] + \mathcal{L}a_\phi\right), \tag{32}$$

where the control gain matrices $K_P, K_D \in \mathbb{R}^{n\times n}$ are constant, diagonal, and positive definite. Then for any initial condition $(q_0, \dot{q}_0)$ and desired link position q_d satisfying Assumption 3.5, all trajectories of the closed-loop system remain inside the admissible constraint set $\mathcal{S}$ for all time, and converge to the invariant set characterised by the following equality

$$K_P\tilde{q} = -\frac{\mathcal{L}(\tilde{q},0)}{\Phi(q,0)}a_\phi(q), \tag{33}$$

which contains the set point $[q_d, 0]^T$.

Proof. The proof is analogous to the proof of Theorem 3.6. The key difference is that the passivity properties of the system can now be exploited in the time derivative of Ψ as follows

$$\begin{aligned} \dot{\Psi} &= -\frac{1}{2}\dot{q}^T\dot{\mathcal{D}}\dot{q} - \dot{q}^T\mathcal{D}\ddot{q} \\ &= -\frac{1}{2}\dot{q}^T\left[\dot{\mathcal{D}} - 2C(q,\dot{q})\right]\dot{q} + \dot{q}^T\frac{\partial\mathcal{F}}{\partial\dot{q}} - \dot{q}^T\left[\tau - g(q)\right] \\ &= \dot{q}^T\frac{\partial\mathcal{F}}{\partial\dot{q}} - \dot{q}^T\left[\tau - g(q)\right] \end{aligned}$$

Substituting the above expression into (23) leads to the result. Note that the term $\dot{q}^T\frac{\partial\mathcal{F}}{\partial\dot{q}}$ coming from the time derivative of Ψ can be left as a general dissipation term in this case, (it needed to be explicitly cancelled in the proof of Theorem 3.6), resulting in

$$\dot{V} = -\frac{(\Psi+\mathcal{L})}{\Psi^2\Phi}\dot{q}^T\frac{\partial\mathcal{F}}{\partial\dot{q}} - \frac{\dot{q}^TK_D\dot{q}}{\Phi\Psi} \leq 0$$

□

The advantage of Corollary 3.7 is that the control law (32) is a modified PD-control. This type of control is desirable as a robust low level stabilisation technique. The rate constraint expressed as a bound on the kinetic energy is physically intuitive. The final complexity of the control law is dependent on the complexity of the configuration constraint function Φ, or more precisely, its partial derivative a_ϕ.

Remark 3.8. *Given a kinetic energy bound on the robot then any single joint could, in theory, have a velocity* $\dot{q}_j(t)$ *up to the bound given by*

$$|\dot{q}_j(t)| \leq \sqrt{\Omega/I_j},$$

where I_j *is the minimum inertia configuration for that joint* j*. The normal action of the energy bound will constrain the joint rates that correspond to large values of kinetic energy. If there is a large amount of kinetic energy in a single joint then the action of the energy bound will be to naturally redistribute this energy among all joints of the robot, at the same time as reducing the overall kinetic energy. The situation that is most dangerous is when there are sensitive low inertia links with rate constraints on the end of heavier arms with high inertia. In this case it is necessary to individually bound the rates of the low inertia links.*

The function $V(\tilde{q}, \dot{q})$ defined by (17) is only one of the many control Lyapunov functions whose derivative along the trajectories of system (1) can be rendered negative semi-definite by the above constrained control design procedure. A class of such functions of practical interest is

$$V(\tilde{q}, \dot{q}) = \frac{\mathcal{L}(\tilde{q}, \dot{q})}{\alpha(\Phi\Psi)} = \frac{1}{2\alpha(\Phi\Psi)} \left[\dot{q}^T \mathcal{D}(q)\dot{q} + \tilde{q}^T K_P \tilde{q}\right],$$

where $\alpha(.)$ is a non-negative function and $\alpha(0) = 0$. If it is desired to have no effect from the barrier function outside of a neighborhood of size δ of the boundary of the admissible constraint set, then it is simply a case of choosing $\alpha(.)$ to be a monotonic non-decreasing function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\alpha(0) = 0$, $\alpha(x) = 1$ for all $x > \delta$, and such that α is smooth on $x > 0$. An example of an analytic barrier function with adjustable boundary effect is

$$V = \frac{\mathcal{L}(\tilde{q}, \dot{q})}{\tanh(\sigma\Phi\Psi)},$$

where $\sigma > 0$ is a constant. Choosing σ large will limit the effect of the barrier function to the immediate vicinity of the barrier itself, and vice versa.

Remark 3.9. *The proposed control law does not bound the demanded torque input. It is physically impossible to have arbitrary configuration, rate, and torque bounds. It is simple to construct a counter-example by choosing initial conditions close to a configuration constraint with non-zero velocity. The torque required to stop the robot before the constraint is reached can be infinitely large.*

In general, however, if the initial conditions are not 'too close' to the boundary and the constraint functions Φ *and* Ψ *are not 'too aggressive' then it is expected that the behaviour of the closed-loop system will share the nice energy minimising properties of PBC designs.*

4 Conclusions

In this paper we have addressed the problem of online obstacle avoidance for robot manipulators subject to physical constraints on joint positions and joint rates. These control objectives are achieved simultaneously by combining the ideas of passivity-based control and artificial potential field method. The key advantages of the proposed control lie in its simplicity and its basis in energy-based stabilisation, leading to simple and effective stabilising control that respects configuration and rate constraints. Global asymptotic stability of the set-points cannot be achieved in the presence of arbitrary configuration constraints due to the possible presence of local minima in the clf. Rate constraints where each individual joint rate is bounded by a constant do not introduce local minima in the clf and for such cases the closed-loop system is globally asymptotically stable.

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