Controlling formations of multiple mobile robots with inter-robot collision avoidance

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Abstract

In this paper, we investigate the problem of inter-robot collision avoidance in multiple mobile robot formation control. Two methodologies are utilized, namely Virtual Robot tracking by [Jongusuk and Mita, 2001] and *l*-*l* control by [Desai *et al.*, 1998] to establish formation and avoid collision among robots. We point out that the framework in Virtual Robot tracking is potentially subject to collision among robots. This drawback is overcome in our design by incorporating a different reactive scheme in the incident possibility of collision. To prove the advantages of our framework, we demonstrate in simulation the case of three robots moving in formation and avoiding inter-robot collisions.

1 Introduction

The issue of control and coordination for multiple mobile robots have revolved around two major tasks; first, the robot platoon must maintain desired shapes such as a line, a column or a ring formation. The motivation is that multiple robots are capable of performing many applications that single robots cannot. Examples of these applications include box pushing [Lewis and Tan, 1998], load transportion [Johnson and Bay, 1995] and capturing/enclosing an invader [Yamaguchi, 1999].Second, the robots have to simultaneously avoid collisions between themselves and with obstacles in the environment.

Essentially, there are three approaches in controlling multiple mobile robots formation, namely: leader-following, behavior-based and virtual-structure, see [Nguyen *et al.*, 2004] and references therein. While the virtual structure approach utilizes centralized controllers, the leader-following and behavioral approaches often apply decentralized controllers using local information. To deal with collision avoidance, some researchers used optimal motion planing [Reeds and Shepp, 1990; Kavraki *et al.*, 1996], which can be very computationally expensive, while others used some type of feedback control with reactive schemes [Desai *et al.*, 1998; Bicho and Monteiro, 2003; Ögren and Leonard, 2003]. These feedback controls come with formal proofs of satisfactory system performance and formation acquisition. One advantage of these schemes is that they can be applied to small, heterogeneous robots with limited communication range.

In the context of leader-following control, the problem of collision between robots in transient phase is important, although has not been explicitly addressed. Jongusuk and Mita [2001] have introduced an interesting idea for tracking control of multiple mobile robots using virtual robots (VR) combined with *l*-*l* control, by [Desai *et al.*, 1998], in a obstacle-free environment. However, the VR control method does not necessarily guarantee acceptable collision avoidance among robots in some cases. In this paper, a remedy for the problems associated with their method is proposed. A reactive control switching scheme is utilized to avoid collisions among robots. This reactive control switching scheme applies different parameters to *l*-*l* control to lead the robots to safe positions for formation achievement with minimum number of control switchings.

The rest of the paper is organized as follows: in section 2 we present the VR control framework and discussions on its weaknesses. Section 3 shows our control design and section 4 discusses how it accounts for the VR control problems. Simulation results are presented in section 5 and finally in sections 6, a conclusion is drawn together with dicussion on future research directions.

2 Tracking Control of Multiple Mobile Robots

2.1 Problem Formulation

The VR problem formulation is adopted, a platoon of unicycle mobile robot is considered, whose kinematic model of each robot is given by

$$\dot{q}_i = B_i u_i = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} u_i, i = 1, 2, \ldots, n; \qquad (1)$$

where $q_i = [x_i, y_i, \theta_i]^T$ is the state vector, (x_i, y_i) is the position in global frame and θ_i is the orientation; and $u_i = [v_i, \omega_i]^T$ is the control input, with v_i being the translational velocity and w_i is the angular velocity, of robot i. In addition, the robots satisfy non-holonomic velocity constraints, which encompass pure-rolling and non-slipping conditions,

$$\begin{array}{lll} \text{non} - \text{slipping} & : & \dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0, \qquad (2) \\ \text{pure rolling} & : & \dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i = v_i\ . \qquad (3) \end{array}$$

Assumptions

(i) Robots are of the same model and satisfy non-slipping and pure-rolling constraints.

(ii) The workspace is flat and contains no obstacle.

(iii) The reference robot follows a smooth trajectory and maintains positive velocity.

(iv) Each follower robot is indexed by a distinctive priority number and aware of others' indexes.

(v) Each robot can extract necessary information via its communication equipment.

Problem statement

Giving initial positions and orientations of the follower robots and the motion of the reference robot, the objective is to design for follower i such that as $t \to \infty$,

1. *Formation is established.*
2. *No collision among robot i and any robot j.*
3. *A overall motion that satisfies the limitation of communication range.*

2.2 Virtual Robot Tracking

The concept of VR is used to avoid collisions between the follower robots and the reference robot. The virtual robot is a hypothetical robot being placed such that it has r-l clearances from the follower and the same orientation. In this case, l defines longitudinal clearance and r defines clearance along the wheel axis. Denote $q_r = [x_r, y_r, \theta_r]$ the reference robot's state vector, $q_i = [x_v i, y_v i, \theta_v i]$ the VR of follower i, $i = 1, 2, \ldots, n$

The relationship between VR and the follower robot is as follows,

$$\begin{aligned} x_{vi} &= x_i - r\sin\theta_i + l\cos\theta i \\ y_{vi} &= y_i + r\cos\theta_i + l\sin\theta i \qquad (4) \\ \theta_{vi} &= \theta_i \quad . \end{aligned}$$

The kinematic model of VR is then,

$$\begin{aligned} \dot{q}_{vi} &= \begin{bmatrix} \cos\theta_i & -r\cos\theta_i - l\sin\theta_i \\ \sin\theta_i & -r\sin\theta_i + l\cos\theta_i \\ 0 & 1 \end{bmatrix} u_i \\ &= \left[\begin{array}{cc} \multicolumn{2}{c}{\mathbf{B_{vi}}} \\ \hline 0 & 1 \end{array}\right] u_i \quad . \qquad (5) \end{aligned}$$

The idea of VR tracking is to use VR to track the reference robot, then the follower will approach the desired position in the formation as its VR approaches the reference robot. In Figure 1, it is shown that the VR approach the reference robot in a internal shape x_e and y_e. Details about the controller can be found in [Jongusuk and Mita, 2001]. Note that $\mathbf{B_{vi}}$ must be non-singular or l must be different from zero, which means a line formation cannot be achieved.

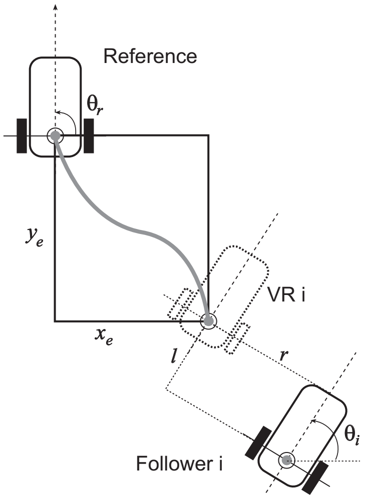

Figure 1: VR tracking model

2.3 *l*-*l* Control

This control concept was introduced in [Desai *et al.*, 1998] for establishing multiple mobile robot formation. The aim of this control is to maintain the desired lengths, l^d_{13} and l^d_{23} of a robot (robot number 3) from its two leader robots (robot number 1 and 2 in Figure 2).

The kinematic equations for robot 3 are given as follows,

$$\begin{aligned} \dot{l}_{13} &= v_3\cos\gamma_1 - v_1\cos\psi_{13} + D\omega_3\sin\gamma_1 \\ \dot{l}_{23} &= v_3\cos\gamma_2 - v_2\cos\psi_{23} + D\omega_3\sin\gamma_2 \qquad (6) \\ \dot{\theta} &= \omega_3 \quad . \end{aligned}$$

where $\gamma_i = \theta_i + \psi_{i3} - \theta_3, (i = 1, 2)$.

Details about the control can be found in [Desai *et al.*, 1998]. Also note the singularity case when $\sin(\gamma_1 - \gamma_2)$, the control law is undefined.

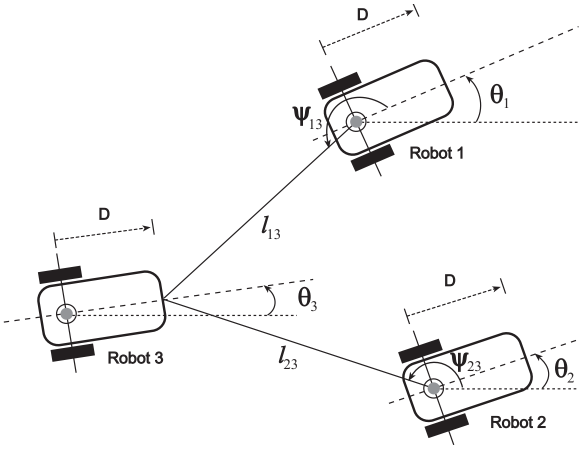

Figure 2: Notation for *l-l* control model

2.4 Controller Design

Detecting Collision

In Figure 3, the solid circle, which covers the whole robot centering at the control point, has radius D and d is the required clearances between robots. Let (x_i, y_i) and (x_j, y_j) denote the control points of robot i and j, then the distance between robot i and j is:

$$\rho_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\ . \tag{7}$$

One has therefore,

$$\rho_{ij} \quad > 2(D+d) \quad \rightarrow \text{safe} \tag{8}$$

$$\rho_{ij} \quad \leq 2(D+d) \quad \rightarrow \text{collision} \tag{9}$$

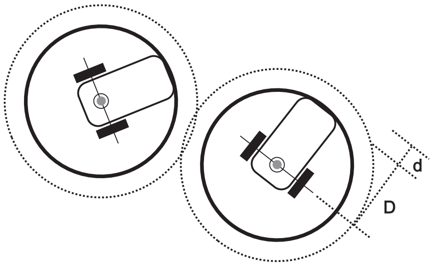

Figure 3: Collision Avoidance Model

Control Algorithm

Jongusuk and Mita [2001] use VR tracking for formation establishment and collision avoidance between the follower robots and the reference robot; and *l-l* control for collision avoidance among the follower robots. Essentially, when collisions are detected, the follower robots with lower priority should switch to *l-l* control to avoid collision with those robots having higher priorities, while the latter do not have to change their control laws. The algorithm is outlined in Figure 4.

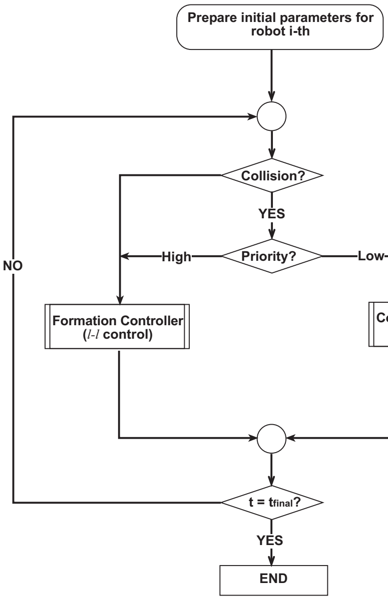

Figure 4: Algorithm flow chart

With regards to how to choose desired lengths for *l-l* control, Jongusuk and Mita defines two situations, when the target is inside the accessible area of low priority robot, the shaded areas in Figure 5 ;and when its outside that area. TG_1 and TG_2 are the targets of follower 1 and 2, respectively; and P_x is where follower 2 will be led to using *l-l* control with parameters l_{13}^d and l_{23}^d. Depending on each situation, the design of l_{13}^d and l_{23}^d, which is equivalent to design of P_x, will be chosen accordingly.

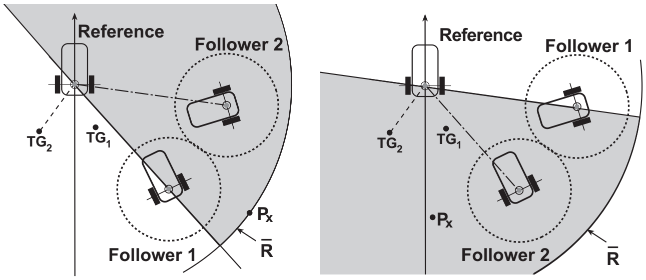

Figure 5: [Left] Target TG$_2$ is outside accessible area
[Right] Target TG$_2$ is inside accessible area

2.5 Discussion

When a VR tracks the reference robot, the solution exponentially converges to an region bounded by x_e and y_e, shown in Figure 1. However, this does not necessarily guarantee that a follower robot will not collide with the reference robot. It is observed that during the tracking process, the VR rotates and the follower goes inside the internal region x_e and y_e, or initially the follower robot is inside the internal region, as shown in Figure 6, then it may collide with the reference robot.

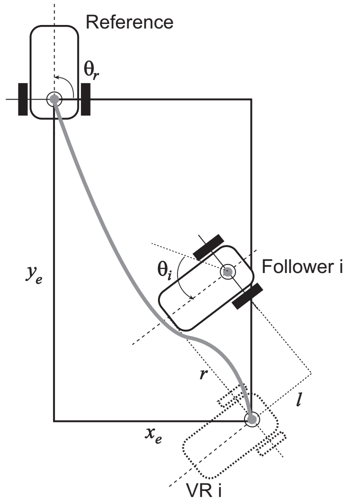

Figure 6: Potential collision with VR

Another problem arises with the design of P_x when collision happens. Since P_x can only be within the accessible area and TG_2 may be outside of that area, it is not guaranteed that when switching back to VR control, follower 2 can track TG_2 without colliding with follower 1 and this can happen repeatedly. For example, when TG_2 is very near the line connecting the reference robot and TG_1, since follower 2 cannot directly "go around" follower 1 to the target, probably many collisions will occur before follower 2 reach its desired position.

To deal with these problems, we propose reactive another scheme combining VR and *l*-*l* control that can guarantee collision avoidance between robots. Firstly, this scheme first apply *l*-*l* control to deal with collisions between the follower robots and the reference robot. Secondly, in case of collisions among the follower robots, it will minimize the number of collisions by using *l*-*l* control intelligently.

3 Proposed Control Framework Design

The control framework here is designed for the case of three robots; one reference robot and two follower robots. In section 2.5, it has been pointed out that VR tracking does not guarantee collision avoidance between follower robots and the reference robot, thus another mechanism to ensure collision avoidance is needed and *l*-*l* control will be used for this purpose. The idea is that should collision occur between any follower robot *i* and the reference robot according to the collision detection criteria proposed by [Jongusuk and Mita, 2001], *l*-*l* control will be used to drive the follower robot to diverge but heads to target position so that collision will most likely not happen after switching back to VR tracking.

In the control framework, four main cases will be considered as follows. In case 1, there is a potential collision between a follower forbot and the reference robot. In case 2, there is a potential collision between the two followers and the distance between the high priority follower and the reference is sufficient for the lower priority robot to go between. Case 3 covers similar situations in case 2 except that the distance between the high priority follower and the reference is insufficient. Lastly, case 4 includes situations when the low priority follower has potential collisions with both high priority follower and the reference robot.

To deal with these cases, the VR of the reference robot (VRR) is introduced as a VR with -*r* and -*l* clearances from the reference robot, where *r*-*l* are the desire clearances of follower *i*. This virtual robot will be at the desired position of follower *i* in the formation. The detail control for each case will be described in the following sections.

Case 1

The follower robot will switch to *l*-*l* control in order to go to P_x, as illustrated in Figure 7, by using two leaders: the reference robot and the VRR, with l_{13}^d and l_{23}^d designed as follows,

$$\begin{array}{ll} l_{13}^d & = 2(D+d) + D + \delta \\ l_{23}^d & = \left|\sqrt{r^2+l^2} - 2(D+d) - D\right| + \delta \ . \end{array} \qquad (10)$$

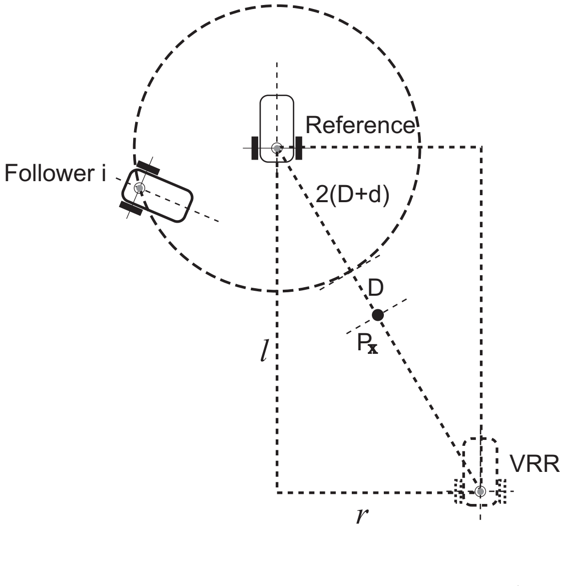

Figure 7: Using *l*-*l* control in collision between follower *i* and the reference robot

Using *l*-*l* control with the above parameters will drive follower i closer to the desired position while going around the safety boundary of the reference robot, which is a circle whose diameter is $2(D+d)$ centering at the reference robot's control point. *l*-*l* control is used to drive follower i not directly to its desired position because driving follower i to go near the safety boundary of the reference robot will make it less likely to collide with the other robot.

The reason that P is $2(D+d)+D$ away from the reference robot rather than $2(D+d)$ is when using *l*-*l* control, the distance from the front castor of the third robot, or follower i, to the control point of its leader, or the reference robot, is considered instead of the distances between their control points, or ρ_{ri}. Thus in order to ensure ρ_{ri} will be sufficient for collision avoidance, we have to increase l_{13}^d by the distance from the third robot's control point to its front castor, which is D. In practice, one can not drive follower i to P because P lies on the line connecting the reference robot and its VR, where the control is undefined due to singularity. Another reason is that we need to ensure the distance between follower i and the reference robot to be stricly greater than $2(D+d)+D$ to thoroughly avoid collision. For these reasons, a small positive amount δ is deliberately augmented to both l_{13}^d and l_{23}^d.

Case 2

There is a potential collision between the two followers and $\rho_{r1} > 4(D+d)$, as depicted in Figure 8. Follower 2 (lower priority) will have to apply *l*-*l* control to avoid collision. In this case, leader 1 is the follower 1 and leader 2 is the VR of the reference robot. l_{13}^d is the same as in (10) and l_{23}^d is similar to the one in (10) except that $\sqrt{r^2+l^2}$ is replaced by distance between VRR and follower 1, or ρ_{rr1}. Therefore,

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= |\rho_{rr1}-2(D+d)-D|+\delta \ . \end{aligned} \tag{11}$$

However, there are situations where if (11) is applied, then it is most likely that follower 2 will collide with both the reference and follower 1 in attempting to go to the desired position, as depicted in Figure 9. This usually happens when target TG$_2$ is in opposite half plane divided by the line connecting follower 1 and the reference robot, and the distance between the reference robot and follower 1 is less than or equal to $4(D+d)$. Hence the need for special treatments in such situations.

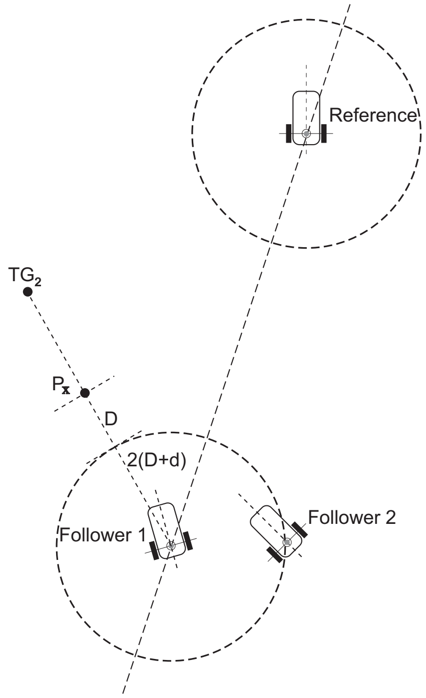

Figure 8: Using *l*-*l* control in collision between followers

Case 3

There is a potential collision between follower robot 2 and follower robot 1, and $\rho_{r1} \le 4(D+d)$. *l*-*l* control will be used to drive follower 2 to "go behind" follower 1. The specific method is shown in Figure 9, where follower 2 is driven to P_x and the line connecting P_x and follower 1 is always perpendicular to the line connecting follower 1 and follower 2. This is accomplished by setting leader 1 in *l*-*l* control to be follower 1, and leader 2 to be a "virtual robot" placed at P_x, with l_{13}^d and l_{23}^d are as follows,

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= \delta \ . \end{aligned} \tag{12}$$

Effectively, follower 2 will revolve around follower 1 safety region clockwise, or counter-clockwise when follower 2 is on the left side of TG$_2$ in Figure 9. This control is applied until follower 2 escapes collision with follower 1, and TG$_2$ and follower 2 are on the same side with respect to the line connecting follower 1 and the reference robot. This also means follower 2 have been driven to go outside of the accessible area in Figure 5. Then we can apply VR tracking again to drive follower 2 to its target TG$_2$. This will guarantee that follower 2 can always go to the desired position regardless of the positions of the reference robot and follower 1.

Case 4

Follower 2 should apply *l*-*l* control to go to a safe position. That position should be at least $2(D+d)+D$ away from other robots. δ is augumented to l_{13}^d and l_{23}^d for the same reason as explained above. Therefore,

$$\begin{aligned} l_{13}^d &= 2(D+d)+D+\delta \\ l_{23}^d &= 2(D+d)+D+\delta \ . \end{aligned} \tag{13}$$

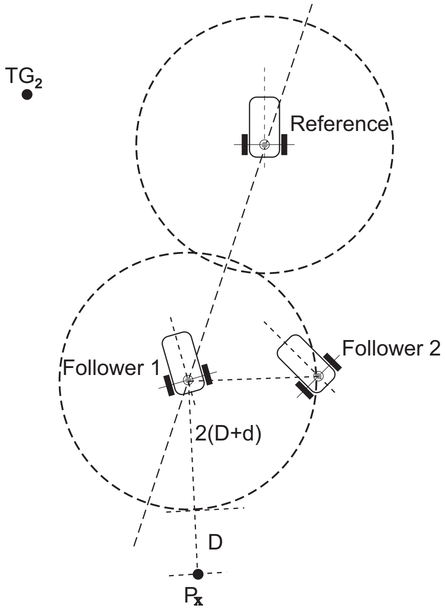

Figure 9: Using *l-l* control to drive follower 2 take rear route around follower 1

4 Discussion

Using this design framework, the drawbacks with the original design by [Jongusuk and Mita, 2001] have been overcome and a collision-free movement for a group of three robots is achieved. The framework have combined VR tracking control and *l-l* control to avoid collision between robots, at the same time driving robots to their desired positions with minimum effort. When follower 2 has to take the rear route around follower 1, it attempts maintains the minimum distance with follower 1, which also means that it has maintains minimal communication range with the reference robot while avoiding collision with follower 1. This should sufficiently satisfy the limited communication range restriction.

This framework can be extended to the multiple robot case, where the platoon of robot can be divided into multiple three-robot groups. Then each of the three-robot groups can be treated as an individual unit, with control points being its the reference robot. The safety boundary can be extended from the reference robot so that it covers all three robots in the group; and the same or another control framework can be applied to avoid collision between groups in a obstacle-free environment. To incorporate obstacle avoidance, we can use similar strategy found in [Desai *et al.*, 1998] using *l-l* control, where one of the distances is the distance to the obstacle.

5 Simulation result

In this section, we show the implementation of our controls and control framework in simulation for the three robot case. The simulation is implemented in Matlab and Simulink. The aim of this simulation is to validate if collision is detected and avoided properly using our approach. We assume that there is no parameter variations or external disturbances. Initial parameter are set as follows,

- Common : $D = 3,\ d = 1,\ \delta = 0.1$
- Reference : $q_r(0) = [100, 0, 0]^T\ ,\ u_r = [2.5, 0]^T$
- Follower 1 : $q_1(0) = [90, -10, 0]^T\ ,\ (r, l) = (10, 10),$
- Follower 2 : $q_2(0) = [90, -30, 0]^T\ ,\ (r, l) = (-10, 1)$

Assume that follower robot 2 has the lowest priority and the reference robot has the highest one.

Figure 10 depicts how follower 2 detects collision with follower 1 and try to go behind follower 1. In this figure, R stands for the reference robot, F1 stands for follower robot 1 and F2 stands for follower robot 2. Figure 11 shows the distances between follower 2 and respectively the reference robot and follower 1 during that time, where l_{13} is the distance between the reference robot and follower 2 and l_{23} is the distance between follower 1 and follower 2. First, follower 2 had apparently an incidence of collision with follower 1 near $x = 100$ and $y = -20$ and at around the 2.5^{th} second, when attempting to go to the desired position. It then decided that it must take the rear route because follower 1 and the reference are too close, by applying the method described in Figure 9. After taking the rear route, follower 2 went to the same side as its desired position with respect to the line connecting follower 1 and the reference robot. It applied VR tracking control again, resulting in another collision possibility with follower 1 near $x = 100$ and $y = -20$ and at the 6^{th} second. At this moment, it applied the method described in Figure 8 to avoid collision. After that, when switching back to VR control, it had an incidental collision with reference robot near $x = 130$ and $y = 0$ and at the 14^{th} second. This time it applied the method described in Figure 7 to go around the reference and finally, follower 2 approached the desired position.

6 Conclusion

In this paper, we have presented a new approach for controlling multiple mobile robots (three-robot case) in formation using leader-following strategy while ensuring collision-free movement. We introduced a new framework using the combination of VR tracking control and *l-l* control. This framework is proven to overcome previous shortages including potential collision with the reference robot and too many collisions among the follower robots. We have illustrated some of our framework's advantages in simulation.

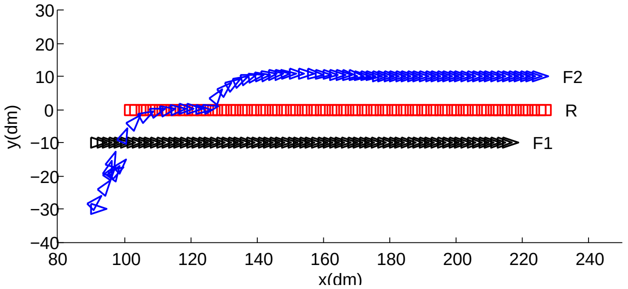

Figure 10: Follower 2 avoid collision with follower 1 and the reference robot.

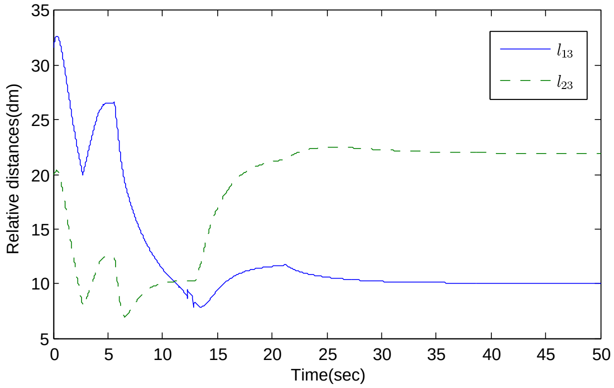

Figure 11: Distances from follower 2 to the referenre robot and follower 1.

We plan to extend our framework to multiple robots and incorporation of collision avoidance as we have mentioned in our discussion. This is possible since the use of *l*-*l* control for collision avoidance have been demonstrated in [Desai *et al.*, 1998]. Graph theory will also be useful in designing formation change for obstacle avoidance; for instance, change to column formation when moving through narrow passage way. Work is under way towards testing our framework's adaptation capability to such formation change. We are also interested in incorporating some treatment for parameter perturbations and external disturbances. Inspired by Ögren and Leonard [2003], who dealt with parameter uncertainties indirectly by defining a uncertainty region around each robot, we plan to use Variable Structure Systems (VSS) approach as a treatment to the problem of parameter uncertainties and disturbances, making use of the well-recognised property of robustness of the VSS approach. These topics are the subject of our future study.

Acknowledgments

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