Proc. 2001 Australian Conference on Robotics and Automation
Sydney, 14-15 November 2001

Dynamics of the New UWA Robot

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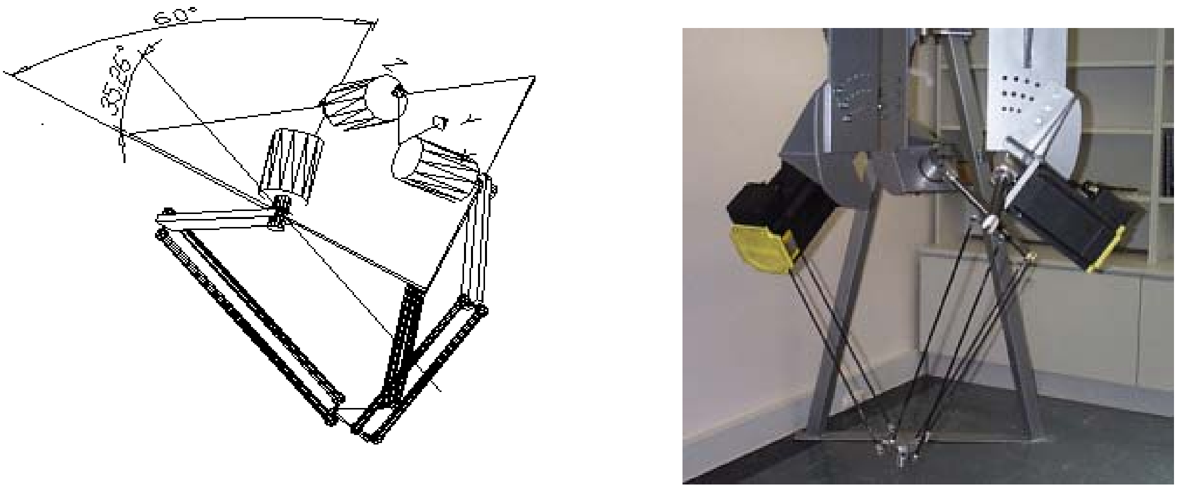
Abstract

This contribution presents the dynamics modelling method suited to solving both the inverse and direct problems for parallel robots. The method is based on the application of Hamilton's canonical equations in carefully chosen dependent co-ordinates, called the *extended space.* The appropriateness and effectiveness of the method is presented by solving the dynamics of the New University of Western Australia Robot - NUWAR. NUWAR, described in detail during 1999 ACRA in Brisbane, is a variation of the well-known Delta parallel robot, having a maximum workspace volume. It is shown that the proposed method is numerically more efficient that the more traditional acceleration-based ones (e.g. Lagrange and Newton-Euler). Hamilton's equations do not require accelerations as input data to the inverse dynamics algorithm. This results in the better accuracy of the algorithm. The solution of the forward problem of dynamics was obtained without any difficulties by transforming the original system of differential-algebraic equations of index 2 to ordinary differential equations and using a standard stepping algorithm with Baumgardte stabilisation.

1 Introduction

Manipulators of parallel robots have the end effector moved around the workspace by a number of serial kinematic chains in parallel. This is a distinguishing feature of parallel robots when compared to the traditional, serial ones. Parallel manipulators possess a number of advantages when compared to traditional serial arms. They offer generally much higher rigidity and smaller mobile mass than their serial counterparts. These features allow much faster and more precise manipulations. NUWAR (Fig. 1, [Miller, 1999]) is capable of achieving end-effector accelerations of 600 m/s^2!

Fast and precise robot manipulation requires control algorithms that make the best use of the information extracted from the dynamics analysis of the robot (Feedforward, Computed Torque [An *et al.*, 1988], Resolved Acceleration [Craig, 1988], Model-Reference Adaptive Control [Luh *et al.*, 1988] and the references cited therein). Common to all these control approaches, except the last, is the problem linked to the solution of the inverse dynamics problem for the manipulators, and the challenge of doing so in real time.



a) b)

Figure 1. New University of Western Australia Robot – NUWAR a) layout; b) prototype

This has necessitated an adapted numerical representation and a high computing efficiency. The NUWAR parallel robot's [Miller, 1999] intrinsic nature is representative of most of the complexities pertinent to the difficulties commonly encountered in the dynamic modelling of parallel structures. These include problematic issues such as the complicated, spatial kinematic structure, which includes many passive joints, the dominance of inertial forces over the frictional and gravitational components, rapidly changing dynamics, which places restrictions on the length of the sampling interval and hence limits controller sophistication (i.e. the available inverse dynamics calculation time), etc.

2 Co-ordinate Choice for the Description of a Parallel Robot - Concept of the *Extended Space*

A number of approaches have been proposed suited to the task of developing parallel robot dynamics models, e.g. [Luh and Zheng, 1985, Nakamura and Ghodoussi, 1989, Lin and Song, 1990, Miller and Clavel, 1992, Zhang and Song, 1993, Baiges and Duffy, 1996, Etemandi *et al.* 1997, Wang and Gosselin, 1998]. These methods assume the manipulator to be a system of rigid bodies connected by ideal kinematic pairs with an absence of friction. Most often the closed kinematic chains are "temporarily cut" (artificially separated), and well-known efficient algorithms utilised for the solution of the inverse problem of dynamics for the resulting tree-structure. Further in the development, the cuts are removed by the introduction of closure conditions (holonomic constraints). This permits transforming of the results obtained for the tree-structure into those of the original closed-loop mechanism.

In many exceptions with more complex spatial kinematic structures (such as the NUWAR) such approaches fail, when the intended purpose of the development is that of real time control. The reason of failure is a large number of passive degrees of freedom, which in the tree-structure approach require description: one differential equation of motion for each passive degree of freedom, resulting in too large a number of equations for real time applications.

When applying energy-based methods for non-redundant mechanisms, there exists a strong temptation to select a minimal set of (independent) co-ordinates in articulation space. Since NUWAR has three degrees of freedom, one is naturally inclined to select three generalised co-ordinates, for example angles in articulated joints (θ_i , i=1,.., 3 – no. of DOF) and then evaluate a set of three Lagrange equations of the second kind for these co-ordinates. Such equations would be the formulae for the unknown control torques.

However, due to the complexity of the geometrical model, the evaluation of the Lagrangian (or Hamiltonian) and especially its derivatives (which would have to include also the derivatives of the solutions of equations of the model of geometry) with three co-ordinates only, is found to be extremely involved and tedious. Another way of formulating equations of motion in independent co-ordinates is based on the concept of projection to the space tangent to constraints [Blajer, 1992]. The variation of this method, leading to the formulation of Kane's equations has been recently applied in [Tsai, 2000] to Gough-Stewart platform. I believe that, because of the usual complexity of parallel robot geometry models, the formulation in dependent co-ordinates leads to more efficient algorithms.

My approach is to choose smartly such co-ordinates describing the manipulator that would be suitable for evaluation of the Lagrangian of the robot and simultaneously their number would be smaller than the number of kinematic pairs of the fifth class (equivalent single-degree-of-freedom joints).

Of principal interest in the case of the non-redundant parallel robot analysis and control are the behaviours of the travelling plate and the actuated joints. Therefore, for the three-degree-of-freedom robot description I propose the use of all co-ordinates belonging to the sum of the task and joint spaces of the robot:

$\{q_i\}=\{x, y, z, \theta_1, \theta_2, \theta_3\}$, i=1,..,6.

where {x, y, z} describe the position of the centre of the travelling plate and $\{\theta_1, \theta_2, \theta_3\}$ - the angles in the actuated joints. For such set of co-ordinates $\{q_i\}$ we propose the name *extended space*, see also [Miller, 1992, Miller, 1994].

The number of co-ordinates in extended space is much smaller than the number of relative co-ordinates. In the case of three-degree-of-freedom NUWAR robot, which possesses 21 rotational joints, the number of co-ordinates in extended space is six, and the number of relative co-ordinates is thereby 15.

Derivation of the Lagrange function for NUWAR in the extended space is relatively straightforward. The same applies to the calculation of constraint derivatives – manipulator geometrical model equations. The constraint derivatives are of much simpler form than the alternative, which is the use of the inverse Jacobian of the robot.

3 Application of Hamilton's Canonical Equations to NUWAR Dynamics Modelling

Generalised momenta **p** can be presented in the following form:

$$\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{q}}} = \frac{\partial(\frac{1}{2}\dot{\mathbf{q}}^T \cdot \mathbf{M} \cdot \dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}} = \mathbf{M} \cdot \dot{\mathbf{q}} \qquad (1)$$

where L is the Lagrangian, **q** – the vector of dependent co-ordinates and **M –** is the mass matrix. Similarly:

$$\frac{\partial H}{\partial \mathbf{q}} = \frac{\partial(\mathbf{p}^T\dot{\mathbf{q}} - L)}{\partial \mathbf{q}} = -\frac{\partial L}{\partial \mathbf{q}} = -L_{\mathbf{q}}$$

$$\Rightarrow$$

$$-L_{\mathbf{q}} = \mathbf{Q}_{ex} - \Phi_{\mathbf{q}}^T\lambda - \dot{\mathbf{p}} \qquad (2)$$

$$\Rightarrow$$

$$\dot{\mathbf{p}} = L_{\mathbf{q}} + \mathbf{Q}_{ex} - \Phi_{\mathbf{q}}^T\lambda$$

where $\mathbf{Q}_{ex}$ is the vector of external forces, $\Phi_{\mathbf{q}}$ – Jacobian of the constraint equations and λ – the vector of unknown Lagrange multipliers.

Equations 1 and 2 form the following canonical equations:

$$\mathbf{p} = \mathbf{M} \cdot \dot{\mathbf{q}} \qquad (3)$$

$$\dot{\mathbf{p}} = L_{\mathbf{q}} + \mathbf{Q}_{ex} - \Phi_q{}^T \cdot \lambda \qquad (4)$$

Equations 3, 4 and the model of robot's geometry were used to derive the equations of motion for NUWAR. **M**, $L_{\mathbf{q}}$ and $\Phi_{\mathbf{q}}$ were evaluated using Mathematica [Wolfram, 1999].

The final form of the equations governing the motion of NUWAR is as follows:

$$\mathbf{p} = \mathbf{M} \cdot \dot{\mathbf{q}} \quad , \quad \mathbf{q_i}=\{x, y, z, \theta_1, \theta_2, \theta_3\}, \quad i=1,..,6. \qquad (5)$$

$$\dot{\mathbf{p}} = L_{\mathbf{q}} - \Phi_q{}^T \cdot \lambda \, , \quad \mathbf{q_i}=\{x, y, z\}, \quad i=1,..,3. \qquad (6)$$

$$\dot{\mathbf{p}} = L_{\mathbf{q}} + \mathbf{Q}_{ex} - \Phi_q{}^T \cdot \lambda \, , \quad \mathbf{q_i}=\{\theta_1, \theta_2, \theta_3\}, \quad i=4,..,6. \qquad (7)$$

$$\Phi_i = 0 \quad , \quad i=1,..,3. \qquad (8)$$

There are no $\mathbf{Q}_{ex}$ terms in the first three $\dot{\mathbf{p}}$ equations because they correspond to the x, y and z co-ordinates. The only external forces acting on NUWAR are torques corresponding to the θ_1, θ_2, θ_3 co-ordinates.

4 Solution of the Inverse Problem of Dynamics

In the case of the inverse problem of dynamics – trajectory known, external forces (desired motor torques) unknown – three steps are required to arrive at the solution of Equations 5-8:

1. In the first step the generalised momenta are evaluated in directions x, y, z, θ_1, θ_2, θ_3 from equations 5 for the sample instant *n*. Only generalised positions and velocities are required as input. Accelerations are not needed.
   For the trajectory start point of the initial conditions of zero velocities, these terms can safely be initiated to zero.
2. In the second step one can substitute the numerically (using e.g. a two-point formula) calculated derivatives of momenta $\dot{p}_x, \dot{p}_y, \dot{p}_z$ into equations 6 for the directions x, y, z belonging to the task space of the robot. These three equations for any given position (on the desired trajectory) constitute a set of three linear equations from which the required multipliers λ_j, j=1,2,3 can be easily obtained.
3. The equations 7, after the substitution of the derivatives of generalised momenta in directions θ_i belonging to the joint space of the robot, and the multipliers λ_i, are the explicit formulae for torques Q_{ex_i}, i = 4,5,6.

An important feature of the model obtained by the Hamilton-based approach is that the solution of inverse problem does not require acceleration information (measurements or estimates) as an input state to the model. Only the positions x, y, z, θ_1, θ_2, θ_3 and the corresponding velocities are necessary. Instead of accelerations one has to provide derivatives of momenta obtained numerically. This approach removes usually long terms of the form $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}})$ from the equations and simplifies calculations. This advantage can save time in calculation of kinematics and simultaneously improve model accuracy. The presence of accelerations in dynamic models of Newton-Euler and Lagrange based approaches is inherent. The application of Hamilton's canonical equations eliminates this requirement. Our calculations showed that the inverse dynamics model based on the method of Hamilton required 28% fewer floating point operations than the more traditional Lagrange based one.

5 Solution of the Forward Problem of Dynamics

In the case of the forward problem of dynamics the dynamical equations 5-7 and constraint equations 8 form the system of differential-algebraic equations (DAEs). Usually, solving DAEs is considerably more difficult than integrating systems of ordinary differential equations (ODE), see e.g. [Bayo and Garcia de Jalon, 1994, Blajer, 1998] and references cited therein (the second reference is particularly worth recommending because it contains a large bibliography of Russian language multibody systems literature). Even though a number of multibody systems researchers suggest that obtaining solutions to DAEs can cause serious trouble, I easily obtained accurate simulation results using the simplest method, based on transforming DAEs into the system of ordinary differential equations (ODEs) by differentiating equations of geometrical constraints and using a standard fourth-fifth order stepping procedure (ODE45 in [Matlab, 1999]) with Baumgardte stabilisation [Baumgardte, 1972].

The method was verified by feeding the solution for time histories of control torques obtained from the inverse dynamics algorithm into the forward dynamics algorithm. The trajectory initially used as the input to the inverse dynamics algorithm was reproduced almost exactly. This validates the appropriateness and accuracy of methods used. The results of computer simulations are presented in Section 6.

It should be noted here that the formulation of equations of motion based on Hamilton's equations possesses an inherent advantage over acceleration-based formulations. The system of Hamilton's equations with multipliers is of index two – acceleration based formulations result in systems of DAEs of index three (for discussion see [Blajer, 1998] and [Brenan *et al.* 1989])

6 Results of Computer Simulations

Computer simulations were performed for the prototype of NUWAR [Miller, 1999]:

- Length, mass and moment of inertia of the arm - 0.26m, 0.977 kg, 0.01822 kgm^2
- length and mass of the forearm - 0.48m, 0.0296 kg
- radial distance of each motor from the centreline - 0.194m
- displacement in radial direction of the midpoint of each pair of spherical joints on the travelling plate - 0.03m
- spacing of the forearms - 0.05m
- Travelling plate mass - 0.2807 kg
- Parallelogram-control arm joint mass - 0.0099 kg

A vast range of trajectory types was generated. The geometric shapes included straight line, ellipse, sheared ellipse and clothoide. The time-motion programs used were parabolic ("bang-bang,"), cycloidal ("sine on ramp") and fifth order polynomial [LePage, 1999]. The trajectories served as input to inverse kinematics and inverse dynamics problems. Obtained motor torque time histories were used as input to forward dynamics calculations. Equation derivation was done with Mathematica [Wolfram, 1999]. Numerical calculations and animation of results were performed with Matlab [Matlab, 1999].

Here we present results for a typical, symmetrical, elliptical pick-and-place trajectory:

Start point -> [0, -0.2, -0.9] [m]

Mid point -> [0, 0, -0.5] [m]

End point -> -> [0, +0.2, -0.9] [m] and back to the start point, followed by the travelling plate according to the "3-4-5 polynomial" motion program. This results in continuous accelerations and finite jerks. The maximum accelerations of the travelling plate were 294.82 m/s^2.

Figure 2 shows time histories of motor torques required to execute our trajectory – the solution of the inverse dynamics problem. The maximum torques required are 35.68 Nm, which is close to the maximum motor torque (41.76 Nm), so that the trajectory under discussion is close to the limit of the capabilities of the manipulator. It is worth noting that the maximum motor torques required by NUWAR are slightly less than those required by the Delta to follow the same trajectory (36.30 Nm).

Figure 3 presents the comparison of the trajectory calculated with a forward dynamics algorithm described in Section 5, using the motor torque histories of Figure 2, with the original trajectory used as the input of inverse dynamics. The two trajectories match exactly proving the appropriateness and accuracy of methods proposed in this paper.

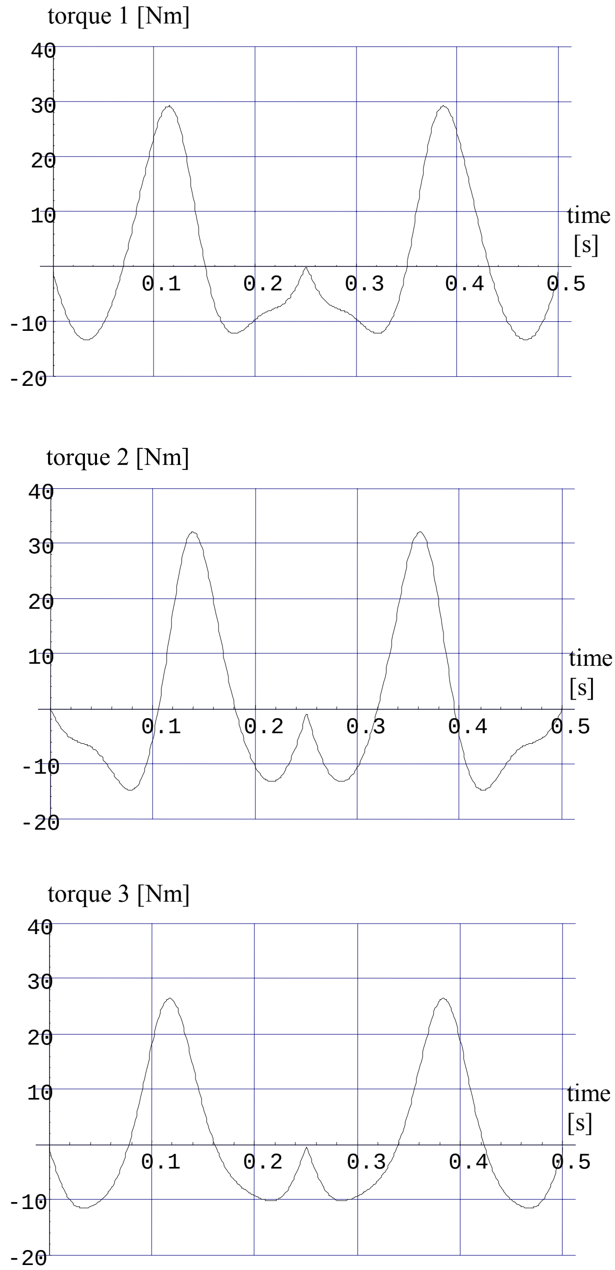


Figure 2. Results of inverse dynamics calculations – time histories of motor torques required to produce the elliptical trajectory described above.

8 Conclusions and Discussion

The efficient method of solving dynamics of parallel robots was developed and applied to the New UWA Robot. The method was based on Hamilton's canonical equations in extended space. The use of co-ordinates of the extended space (the union of the task and joint spaces) results in much fewer equations than in the case of using relative co-ordinates, and in a simple Lagrangian.

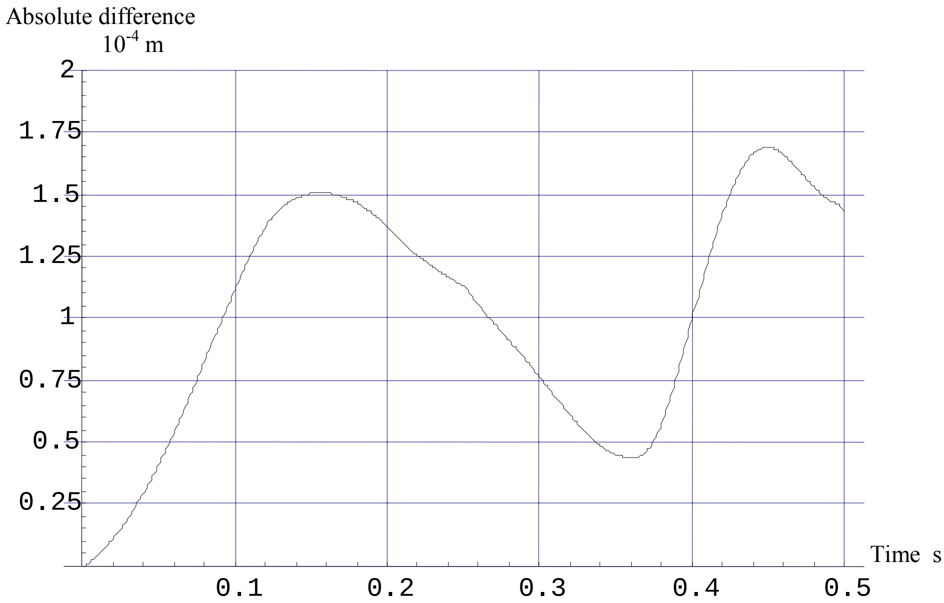


Figure 3. Results of forward dynamics calculations – the difference between the trajectory serving as input to inverse dynamics and the result of forward dynamics calculations using torque histories of Figure 2.

The use of Hamilton's equations is advantageous in application to the inverse problem of dynamics. The method is numerically more efficient than the more traditional Lagrange and Newton-Euler ones because Hamilton's equations do not require accelerations as input data to the inverse dynamics algorithm and terms of the form $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}})$ involving accelerations are absent. This results in the better accuracy of the algorithm. Due to the extremely high accelerations, the NUWAR control system band-width needs to be quite high. The experience with Delta robot [Miller and Clavel, 1992] suggests that the control signal should be updated every, approximately, one millisecond. The calculation time of the presented inverse dynamics algorithm is considerably less than the model-based control system requirement.

The solution of the forward problem of dynamics was obtained without any difficulties by transforming the original system of differential-algebraic equations to ordinary differential equations and using a standard stepping algorithm with Baumgardte stabilisation. The inherent advantage of using Hamilton's equations for the direct problem of dynamics is that, together with algebraic equations of constraints, they form the system of differential–algebraic equations (DAE's) of index 2, not of index 3 as is the case in the acceleration-based formulations. Numerical integration of index 2 systems is known to be more stable than solution of index 3 systems.

Acknowledgements

This work was funded by the Faculty of Mathematical and Engineering Sciences of The University of Western Australia Research Launching Grant and The Australian Research Council.

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