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Dynamic Output Feedback Control of Single-Link Flexible Manipulators

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Abstract

This paper presents an efficient technique to design dynamic feedback control scheme for single-link flexible manipulators. A linear model can be derived for the robotic system using the assumed-mode method. Conventional techniques such as pole-placement or LQR require physical measurements of all system states, posing a stringent requirement for its implementation. To overcome this problem, a low-order state functional observer is proposed here for reconstruction of the state feedback control action. The observer design involves solving an optimisation problem with the objective to generate a feedback gain that is as close as possible to that of the required feedback controller. A condition for robust stability of the closed-loop system under the observer-based control scheme is given. The attractive features of the proposed technique are that the resulted functional state observer is of a very low order and it requires only sensor measurements of only the output- the tip position of the arm.

1 Introduction

There has been an increasing interest in the design and control of lightweight robots over the years thanks to many salient advantages over the conventional rigid robots. Lightweight manipulators are however prone to deflection and elastic vibration due to their vibratory modes and low damping factors. Accurate position control of flexible arms remains therefore an interesting problem. Several control schemes have been proposed for vibration suppression and performance enhancement in flexible arm control. These control strategies are based on a number of control approaches including optimal control [Cannon and Schmitz, 1984; Krishnan and Vidyasagar, 1988], adaptive control [Nemir *et al.*, 1988; Clarke *et al.*, 1987], and variable structure systems [Nathan and Singh, 1989; Qian and Ma, 1992; Thomas and Bandyopadhyay, 1997] methods. In flexible arm control, it is known that stability gain margin of the closed-loop system can be increased by using low-order compensators instead of high-order design [Cannon and Schmitz, 1984]. The Hankel norm minimisation technique is used to obtain a reduced order model for the system [Krishnan and Vidyasagar, 1988]. Most of the cases use feedback control that requires either the availability of the complete state vector or a state-estimation scheme. In this paper, in order to reduce the dimension of the observer-controller system, a low-order linear state function observer is proposed to reconstruct the required control law [Trinh and Ha, 2000], using a parameter optimisation process. The optimisation objective is to generate a matrix that is maximally close to the given feedback gain of the required feedback controller. A condition is derived to guarantee stability of the closed-loop system under the proposed observer-based control scheme. A step-by step design algorithm is given. Simulation results of a single-link flexible arm are provided to illustrate the design procedure and potential of the proposed technique.

2 Single-link flexible beam model

The flexible robot arm is in general an infinite dimensional system. The number of modes to be retained depends on the limit of energy and the maximum bandwidth of the actuator and sensors. Dynamic modelling of single-link flexible arms has been addressed by many researchers (see eg., [Krishnan and Vidyasagar, 1988], [Qian and Ma, 1992], [Hastings and Books, 1986]). The position of a point P on the arm, shown in Figure 1, can be represented as:

$$y(x,t) = x\theta(t) + w(x,t)\,, \tag{1}$$

where x is the position distance from the hub, θ is the hub angle, and $w(x,t)$ is the small elastic deflection from the arm neutral axis.

As described in [Hastings and Books, 1986], a partial differential equation (PDE) can be obtained from the system energy equation by applying Hamilton's principle. A series of natural vibration modes of the system can then be derived from solving the PDE characteristic equation. Using the assumed-mode method the deflection may be expressed as

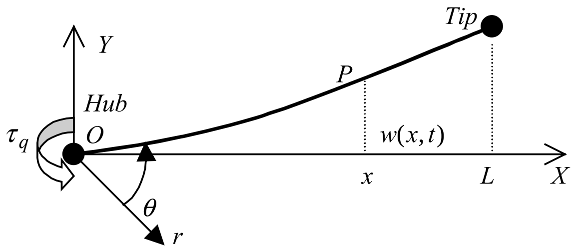

Figure 1. The arm deflection at (*x,t*).

$$w(x,t)=\sum_{0}^{\infty}\phi_i(x)q_i(t)\,, \quad (2)$$

where $q_i(t)$ is the generalised coordinate of the system and $\phi_i(x)$ is the eigen-function of the *i*-th mode.

By using Lagrange equation, a series of decoupled ordinary differential equations of the form

$$(\ddot{q}_i+2\zeta_i\dot{q}_i+\omega_i q_i)=\frac{1}{I_T}\phi_i'(0)\tau_q;\ i=0,2,...,\infty\,, \quad (3)$$

where ω_i is the *i*-th vibration mode natural frequency, ζ_i is the damping factor, I_T is the total of inertia of the hub and the arm, and τ_q is the actuation torque applied at the hub.

Practically, the series (3) can be truncated at some finite number N according to the actuator/sensor bandwidth [Cannon and Schmitz, 1984]. Thus, including the rigid body mode ($\omega_0=0$) the system dimension becomes $n=2(N+1)$. The system can then be described by

$$\begin{aligned}\dot{X}&=A_0X+B_0u\\ y_{TP}&=C_0X,\end{aligned} \quad (5)$$

where $X=[q_0\ \dot{q}_0\ q_1\ \dot{q}_1 \ldots q_{N-1}\ \dot{q}_{N-1}\ q_N\ \dot{q}_N]^T\in R^n$ is the state vector, $u=\tau_q\in R$ is the control signal, and y_{TP} is the tip position measurement that can obtained by using a light-emitting diode fixed at the arm tip and a position sensor mounted at the hub,

$$A_0=\begin{bmatrix}0 & 1 & & & & \\ 0 & -a & & & & \\ & & 0 & 1 & & \\ & & -\omega_1^2 & -2\zeta_1\omega_1 & & \\ & & & & \ddots & \\ & & & & 0 & 1\\ & & & & -\omega_N^2 & -2\zeta_N\omega_N\end{bmatrix},\ B_0=\frac{1}{I_T}\begin{bmatrix}0\\1\\0\\\phi_1'(0)\\\vdots\\0\\\phi_N'(0)\end{bmatrix},$$

$$C_0=\begin{bmatrix}L & 0 & \phi_1(L) & 0 & \ldots & \phi_N(L) & 0\end{bmatrix}, \quad (6)$$

and L is the arm length. The coordinates $q_i(t)$ ($i=0,1,2,...,N$) can be obtained from the measurement equation [Qian and Ma, 1992]:

$$\begin{bmatrix}y_\theta\\ y_{SG_1}\\ .\\ y_{SG_{N-1}}\\ y_{TP}\end{bmatrix}=\begin{bmatrix}1 & \phi_1'(0) & . & \phi_{N-1}'(0) & \phi_N'(0)\\ 0 & \frac{\delta}{2}\phi_1''(L_1) & . & \frac{\delta}{2}\phi_{N-1}''(L_1) & \frac{\delta}{2}\phi_N''(L_1)\\ . & . & . & & .\\ 0 & \frac{\delta}{2}\phi_1''(L_{N-1}) & . & \frac{\delta}{2}\phi_{N-1}''(L_{N-1}) & \frac{\delta}{2}\phi_N''(L_{N-1})\\ L & \phi_1(L) & . & \phi_{N-1}(L) & \phi_N(L)\end{bmatrix}\begin{bmatrix}q_0\\ q_1\\ .\\ q_{N-1}\\ q_N\end{bmatrix}, \quad (7)$$

where δ is the arm thickness, y_θ is joint angle measurement obtained from an encoder and y_{SG_i} ($i=1,2,...,N-1$) are measurements from the strain gauges SG*i* located at distances L_i from the hub [Qian and Ma, 1992], or from a linear CCD array mounted inside the camera with the position of the pixels in the CCD array indicating the arm deflection [Krishnan and Vidyasagar, 1988]. This, however, leads to a high implementation cost if the number of vibration modes retained, *N*, is sufficiently large. Unlike the approaches, in this paper the control action is reconstructed using *only information of the tip position measurement*, y_{TP}. Let us first transform system (5) into a canonical form by introducing $X=T_cz$ with the transformation defined by

$$T_c=[C_0^T\quad \ker(C_0)]\,, \quad (8)$$

where $\ker(C_0)$ is any ortho-normal basis for the null space of C_0, obtained from the singular value decomposition. The state-space model (5) is now rewritten in the form

$$\begin{aligned}\dot{z}&=Az+Bu\\ y_{TP}&=Cz,\end{aligned} \quad (9)$$

where $z=T_c^{-1}X\in R^n$ is the new state vector, and

$$\begin{aligned}&A=T_c^{-1}A_0T_c\ ,\ B=T_c^{-1}B_0,\\ &C=[C_1\ \ 0],\ \ C_1=C_0C_0^T\in R.\end{aligned} \quad (10)$$

Let us assume that a linear state feedback control law

$$u=F_0X \quad (11)$$

is obtained to assign desired closed-loop eigenvalues or to satisfy a specified performance objective for the system (5). This can be done using the well-documented pole-placement or optimal control method. The required control law then takes the form

$$u=F_0T_cz=Fz\,. \quad (12)$$

The problem is to design a low-order functional observer to estimate the feedback control law (12) using information of the tip position y_{TP}.

3 Observer Design

Towards the generation of the control signal (12), let us first decompose the feedback gain matrix *F* as follows

$$F=KT+WC\,, \quad (13)$$

where $K\in R^{1\times p}$, $T\in R^{p\times n}$, and $W\in R$ are constant matrices to be determined, and p is the observer order lying in the interval $[1,n\text{-}1)$. The feedback control signal can then be expressed as

$$u(t) = Fz(t) = (KT + WC)z(t) = Kv(t) + Wy_{TP}(t) \qquad (14)$$

where

$$v(t) = Tz(t) \in R^{p} \qquad (15)$$

is the observer state. Let us now consider the following p-th order observer dynamics

$$\dot{v}(t) = Ev(t) + TBu(t) + Gy_{TP}(t) , \qquad (16)$$

where $E \in R^{p\times p}$ is a stable matrix to be selected and $G \in R^{p\times 1}$ is a constant matrix to be determined. Let $e(t)$ be defined as the error between the observer state and its estimate, $Tz(t)$, i.e. as

$$e(t) = v(t) - Tz(t) . \qquad (17)$$

Taking derivative of (17) and using (9) yield

$$\begin{aligned}\dot{e}(t) &= \dot{v}(t) - T\dot{z}(t) = Ev + TBu + Gy_{TP} - TAz - TBu \\ &= E(v(t) - Tz(t)) + (GC - TA + ET)z.\end{aligned} \qquad (18)$$

Given a stable matrix E, if matrices G and T are determined such that

$$GC - TA + ET = 0 , \qquad (19)$$

the observer error dynamics then become

$$\dot{e}(t) = Ee(t) . \qquad (20)$$

Accordingly, (16) can act as a linear functional observer for the system (9) under the control law (12), provided that matrix E is stable and equations (13) and (19) are satisfied. Exact solutions to (13) and (19) can be found in [Trinh and Ha, 2000] under the satisfaction of some condition on the lower bound of the observer order. This paper seeks to overcome this limitation by proposing an alternative procedure based on a parameter optimisation process. Let us first partition matrices A, T and vector F as follow:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \; T = \begin{bmatrix} T_1 & T_2 \end{bmatrix}, \; F = \begin{bmatrix} F_1 & F_2 \end{bmatrix}, \qquad (21)$$

where

$$A_{11} \in R, \; A_{12} \in R^{1\times(n-1)}, \; A_{21} \in R^{(n-1)\times 1}, \; A_{22} \in R^{(n-1)\times(n-1)},$$
$$T_1 \in R^{p\times 1}, \; T_2 \in R^{p\times(n-1)}, \; F_1 \in R, \text{ and } F_2 \in R^{1\times(n-1)}. \qquad (22)$$

By incorporating (10) and (21) into equations (13) and (19), after some rearranging, the following equations can be obtained:

$$F_1 = KT_1 + WC_1 , \qquad (23)$$

$$F_2 = KT_2 , \qquad (24)$$

$$GC_1 - (T_1 A_{11} + T_2 A_{21}) + ET_1 = 0 , \qquad (25)$$

$$ET_2 - T_1 A_{12} - T_2 A_{22} = 0 . \qquad (26)$$

As matrix E is selected according to the desired dynamics for the observer to be constructed, there are five unknown matrices (namely K, W, G, T_1 and T_2) in equations (23)-(26) to be solved for. As C_1 in (10) is invertible, matrices W and G can be derived from K, T_1 and T_2 as:

$$W = (F_1 - KT_1)C_1^{-1} , \qquad (27)$$

$$G = (T_1 A_{11} + T_2 A_{21} - ET_1)C_1^{-1} . \qquad (28)$$

Given E, let us now attempt to solve for K, T_1 and T_2 from equations (24) and (26).

Remark 1: Matrix E is selected such that the observer response at least 3-5 times faster than the system response determined by $eig(A + BF)$. Furthermore, given T_1 the Lyapunov equation (26) has a unique solution for T_2 if matrices E and A_{22} do not have common eigenvalues [Luenberger, 1971].

Instead of trying to find an exact solution for matrices K, T_1 and T_2, our proposed approach here is to find a solution such that the resulting control feedback signal is as close as possible to the given control law (12) by minimising the following matrix norm:

$$\rho = \| F_2 - KT_2 \| . \qquad (29)$$

Remark 2: For the above minimisation, matrix K may be chosen as

$$K = F_2 T_2^{+} , \qquad (30)$$

where T_2^{+} is the Moore-Penrose pseudo-inverse of T_2.

Examination of equations (24) and (26) reveals that this minimisation problem may be solved by given E and searching for matrix T_1 such that the solution to the Lyapunov equation (26), i.e. T_2, will minimise (29). In order to find T_1 such that KT_2 is as lose as possible respectively to F_2, a *parameter optimisation* technique will be used. All of the elements of T_1 are now considered as optimisation parameters of the following optimisation problem:

$$\begin{cases} \text{Minimise} & \rho(T_1) = \| F_2 - F_2 T_2^{+} T_2 \| \\ \text{subject to} & ET_2 - T_1 A_{12} - T_2 A_{22} = 0. \end{cases} \qquad (31)$$

Consequently, matrices K, T_1 and T_2 can be obtained and hence, matrices W and G can be computed from equations (27) and (28), respectively.

4 Stability analysis

In the following, a stability condition will be derived for the closed-loop system using the proposed observer-based feedback.

Lemma 1

Consider a linear system

$$\dot{X}(t) = JX(t) + \Delta JX(t) , \qquad (32)$$

where $X(t)$ is the state vector, matrices J and ΔJ are respectively known and constant but unknown. Let the nominal system (i.e. $\Delta J = 0$) be stable, then the perturbed system (32) is asymptotically stable if the following condition is satisfied:

$$\| \Delta J \| < \alpha \equiv \frac{1}{\| (sI - J)^{-1} \|_\infty}, \qquad (32)$$

where I is the unity matrix and $\| . \|_\infty$ denotes the H_∞ norm.

Theorem 1

Consider the linear system (5) that is asymptotically stable under the feedback control law (11). Let matrix E of dimension p ($1 \le p < (n-1)$) be selected to have desired eigenvalues that are different from those of A_{22} defined in (10) and (22). If there exists matrix T such that the following condition is satisfied

$$\mu = \| B_0 \Delta F T_c^{-1} \| = < \alpha \equiv \frac{1}{\| (sI_n - J)^{-1} \|_\infty}, \qquad (33)$$

where

$$\Delta F = [0 \quad KT_2 - F_2], \; J = A_0 + B_0 F_0 . \qquad (34)$$

and matrices T_c and K are given respectively in (8) and (30), then systems (16) can be used as low-order linear functional observers to generate the control law (14), that is as close as possible the control law (11), and the closed-loop system remains asymptotically stable.

Proof

The development of the linear functional observer (16) has been presented above. Matrix E of dimension p is selected according to Remark 1, with p taking the value from the lowest in the interval $1 \le p < (n-r)$. If there exists matrix T, derived from the parameter optimisation problem (31), then K, W, and G are determined from T_1 and T_2 in accordance with (30), (27) and (28), respectively. As equation (24) is not exactly solved, stability of the closed-loop system may not be guaranteed. In fact, by using the proposed functional observer (16), with relevant matrices obtained from the optimisation process (31), and the feedback control law (14), the system state equation becomes

$$\dot{X}(t) = A_0 X(t) + B_0 (Kv(t) + W y_{TP}(t)) .$$

As E is selected such that the observer desired dynamics are fast enough in comparison with the system dynamics, $e(t) \to 0$ according to (20), and hence $v(t) \to Tz(t)$. Thus, the closed-loop system dynamics are given by

$$\dot{X}(t) = A_0 X(t) + B_0 (KT + WC) z(t) .$$

Due to the inexact solution to (24),

$$KT + WC = [F_1 \;\; KT_2] = F + \Delta F ,$$

where $\Delta F = [0 \quad KT_2 - F_2] = [0 \quad F_2 T_2^+ T_2 - F_2]$ is the difference between the new feedback gain ($[F_1 \quad KT_2]$) and the original feedback gain ($F = [F_1 \quad F_2]$), which has been *minimised* in the optimisation process. We obtain therefore

$$\dot{X}(t) = A_0 X(t) + B_0 (F + \Delta F) T_c^{-1} X(t)$$
$$= (A_0 + B_0 F_0 + B_0 \Delta F T_c^{-1}) X(t) = (J + \Delta J) X(t).$$

As the nominal part ($\dot{X}(t) = JX(t)$) is stable, and $\Delta J = B_0 \Delta F T_c^{-1}$, Lemma 1 is now applied to derive the stability condition (33) for the closed-loop system. This concludes the proof.

In summary, instead of using the law (11), the control signal

$$u(t) = Kv(t) + W y_{TP}(t) \qquad (35)$$

can be constructed for the system (5), where $v(t)$ is the state of the p-th order state functional observer:

$$\dot{v}(t) = Ev(t) + TT_c^{-1} B_0 u(t) + G y_{TP}(t) . \qquad (36)$$

5 Design Algorithm

Based on the above development, a design algorithm for the proposed control scheme is given in the following steps:

Step1: Determine a suitable state feedback gain matrix F_0 by using any existing control technique.

Step 2: Obtain matrices T_c, A, B, C_1 and F following (8), (10), and (12). Partition matrices A and F according to (21).

Step 3: Compute $\alpha \equiv \frac{1}{\| (sI - J)^{-1} \|_\infty}$. A method given in [Francis, 1987] can be used for this step.

Set $j = 0$.

Step 4: Set the order of the observer (16) as $p = 1 + j$.

Step 5: Select E in accordance with Remark 1, solve the optimisation problem (31) for T, and then derive K according to (30).

Step 6: Check condition (33), if satisfied, go to step 6; else set $j = j + 1$ and go to Step 4.

Step 7: Compute matrices W and G respectively from equations (27) and (28).

Remark 3: In the above design algorithm, a lowest order ($p = 1$) is first assigned for the observer (16). The search for a control law that is as close as possible to the desired feedback law is obtained upon the satisfaction of the stability condition for the overall observer-based system. Otherwise, the observer order can be gradually increased until this condition is met. The procedure is therefore expected to result in a linear functional observer of a low order.

6 Simulation results

This section shows simulation results of the single-link flexible beam, reported in [Krishnan and Vidyasagar, 1988]. The beam length is L=1.0m, the total moment of inertia is I_T=8.29.10^{-2}kgm^2. Table 1 gives the system parameters for the first five flexible modes with damping factors $\zeta_i = 0.0015$; $i = 1,2,...,5$. Let us first assume that the number of retaining modes is N=2 (n=6) [Thomas and Bandyopadhyay, 1997]. The above design algorithm is applied as follows.

Step 1: Using the LQR design

$$F_0\text{=-lqr2}(A_0\text{+ 1.*eye}(n), B_0, \text{eye}(n) , 100) , \qquad (37)$$

we obtain F_0=[-0.4735 -0.3933 -0.7719 -0.1655 2.1878

0.1711]. The closed-loop system poles can be found as eig(A_0+ B_0F_0)= 1.0e+002 *{ -0.0263 ± 1.3174i, -0.0297 ± 0.5586i, -0.0236 ± 0.0035i}.

Table 1. Model parameters for the first five modes

Mode	Natural frequency, ω_i	$\phi_i(L)$	$\phi_i'(0)$
1	55.89 rad/sec	-0.931	2.886
2	131.75 rad/sec	-1.027	-2.345
3	313.81 rad/sec	1.169	-0.910
4	603.67 rad/sec	-11.187	-0.454
5	993.9 rad/sec	1.190	-0.272

Step 2: Calculating T_c, A, B, C_1 and F in accordance with (8), (10), and (12):

T_c =[1.0000 -0.8110 0 0 0 0
0 0 0.6716 0 0.7409 0
-0.9310 -0.3929 0.5489 0 -0.4976 0
0 0 0 1.0000 0 0
-1.0270 -0.4335 -0.4976 0 0.4511 0
0 0 0 0 0 1.0000],

A =1.0e+4*[0 0 0.0000 -0.0000 0.0000 -0.0000
0 0 -0.0001 -0.0000 -0.0001 -0.0000
0 0 -0.0000 0.0001 -0.0000 -0.0000
0.2908 0.1227 -0.1715 -0.0000 0.1554 0
0 0 0.0000 -0.0000 0.0000 0.0000
1.7827 0.7524 0.8637 0 -0.7830 -0.0000],

B^T =[-0.0000 -0.0000 8.1017 34.8130 8.9371 -28.2871], C_1 =2.9215, and F=[-2.0017 -0.2610 -1.7765 -0.1655 1.0796 0.1711].

Step 3: The spectrum of $|sI-(A+BF)|$ is shown in Figure 2. The value of α is found to be α =0.0399.

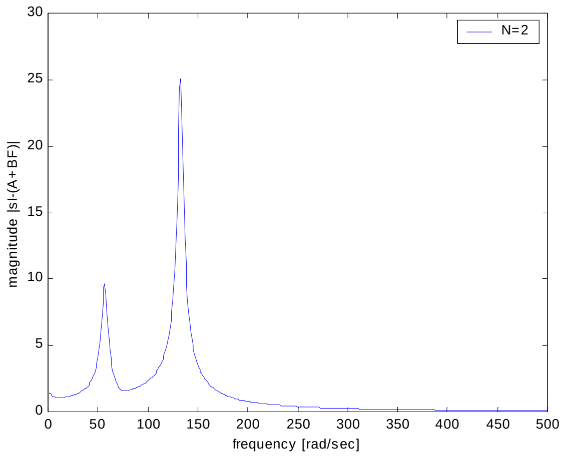

Figure 2. Spectrum of $|sI-(A+BF)|$ for N=2.

Step 4-5: Set p=1. Choose E=-2. Solving the optimisation process (31) gives T_1=1.0500, T_2=[-0.8500 -0.3527 0.0006 -0.3880 0.0001], and K =0.4306.

Step 6: Since μ =0.00096 < α, condition (33) is satisfied. The eigenvalues of the closed-loop system under the observer-based feedback control can be found as eig(A_0+ $B_0[F_1 \;\; KT_2]T_c^{-1}$)= 1.0e+02*{ -0.0020 ± 1.3184i, -0.0137 ± 0.0168i, -0.0007 ± 0.5565i}.

Step 7: Calculating W= -0.8399 and G = 2.0975.

Thus, the feedback control law for the flexible arm model with two retaining vibration modes can be reconstructed using a first order observer and information of the tip position only. Figure 3 shows the step response of the tip position and the control torque, using the proposed observer-based control scheme and the full-state feedback control (37). Note that increasing the observer order, eg. p=2, will result in a better reconstruction of the feedback control.

Let us now apply the technique for the case N=5 (n=12) [Krishnan and Vidyasagar, 1988]. In Step 1 the full-state feedback control law (37) by the LQR technique gives F_0=[-0.4735 -0.3933 -0.7568 -0.1655 2.0865 0.1712 4.0660 0.2351 2.8259 0.1402 0.3341 0.0163]. The system desired closed-loop eigenstructure is eig(A_0+ B_0F_0)=1.0e+002*{ -0.0152 ± 9.9390i, -0.0129 ± 6.0367i, -0.0176 ± 3.1381i, -0.0263 ± 1.3174i, -0.0297 ± 0.5586i, -0.0236 ± 0.0035i}. Matrices T_c, A, B, C_1 and F are calculated in Step 2 according to (8), (10), and (12). The spectrum of $|sI-(A+BF)|$ is shown in Figure 4 where value of α is found to be α =0.0031 (Step 3). After unsuccessful trials with p=1, we repeat Steps 4-7 with p=2.

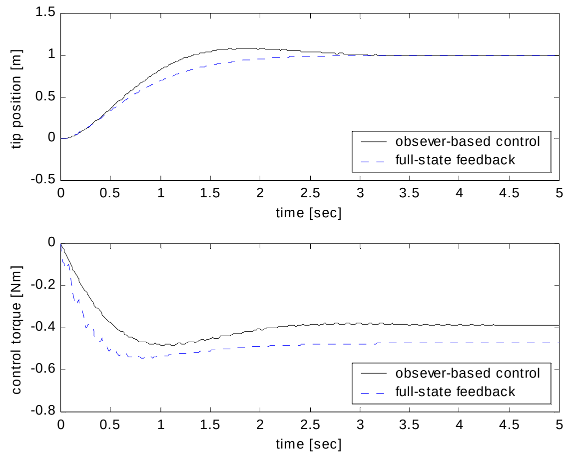

Figure 3. Tip position and control torque responses (N=2).

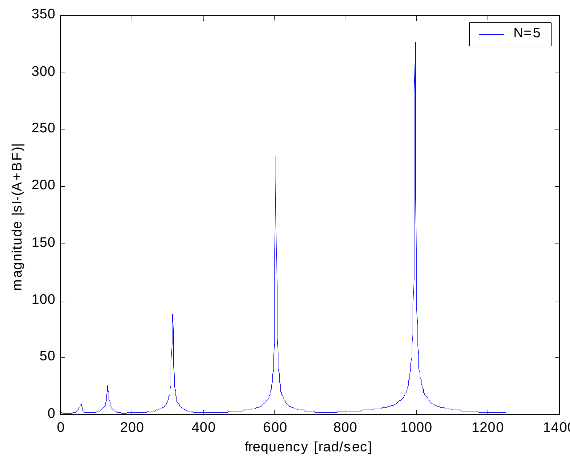

Figure 4. Spectrum of $|sI-(A+BF)|$ for N=5.

Selecting E=[-5 0; 0 -8], the optimisation (31) is solved to give T_1=[1.0750; 0.9750], T_2, and K=[4.1877 -4.5303].

Condition (33) is satisfied with μ =0.0023 < α. The poles of the closed-loop system under the observer-based feedback control are obtained as eig($A_0 + B_0[F_1\ KT_2]T_c^{-1}$) = 1.0e+02*{ -0.0149 ± 9.9390i, -0.0090 ± 6.0368i, -0.0047 ± 3.1381i, -0.0019 ± 1.3177i, -0.0170 ± 0.0158i, -0.0015 ± 0.5581i}. Finally, W and G are calculated as W= -0.2175 and G = [4.9347; 7.7014].

In summary, the feedback control law (37) for the flexible arm model with five retaining vibration modes can be reconstructed using a second-order observer and information of the tip position only. Figure 5 depicts the step response of the tip position and the control torque, using the proposed observer-based control scheme and the full-state feedback control (37).

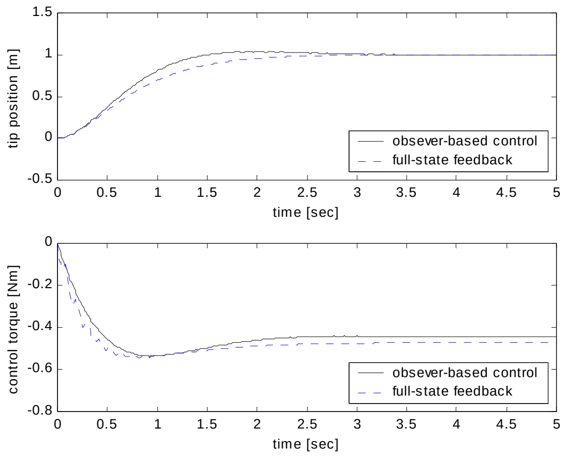

Figure 5. Tip position and control torque responses (N=5).

The results obtained above demonstrate the efficacy and feasibility of the proposed observer-based control technique for the control of flexible manipulators. The main contributions of the control scheme is that it does not require information of the mode coordinates and their time derivatives in order to dampen these vibrations in the control of the tip position, and that the resulting functional observer is of a very low order.

7 Conclusion

We have presented an efficient technique for the control of a single-link flexible manipulator using a low-order linear functional observer. Based on the assumed mode method, a linear model for the system is obtained with a finite number of vibration modes. A feedback controller is first designed to place the closed-loop system poles or to achieve an optimal performance index. A low-order state functional observer is then constructed to estimate the state feedback control action. The observer design involves solving an optimisation problem with the objective to generate a feedback gain that is as close as possible to that of the required feedback controller. A condition for robust stability of the closed-loop system under the observer-based control scheme is developed. A step-by-step design procedure is presented. The attractive features of the proposed technique are that the resulting functional state observer is of a very low order and it requires only the tip position measurement. The proposed control scheme is illustrated through the control of a flexible beam with five retaining vibration modes. Simulation results demonstrate the validity of the proposed technique.

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References

[Cannon and Schmitz, 1984] R.H. Cannon, Jr., and E. Schmitz, "Initial experiments on the end-point control of flexible one-link robot," *Int. J. Robotics Research*, vol. 3, pp. 62-75, 1984.

[Clarke *et al.*, 1987] D.W. Clarke, C. Mohtadi, and P.S. Tuffs, "Generalised predictive control- Part I: The basic algorithm," *Automatica*, vol. 23, no. 2, pp. 137-148, 1987.

[Francis, 1987] B.A. Francis, *A Course in Control Theory*. Springer-Verlag, Berlin, 1987.

[Hastings and Books, 1986] G.G. Hastings and W.J. Books, "Variable structure control of a robotic arm with flexible links," in *Proc. 1986 IEEE Int. Conf. Robotics and Automation*, pp. 1024-1029, 1986.

[Krishnan and Vidyasagar, 1988] H. Krishnan and M. Vidyasagar, "Control of a single-flexible beam using a Hankel-norm based reduced order model," in *Proc. 1988 IEEE Int. Conf. Robotics and Automation*, pp. 9-14, 1988.

[Luenberger, 1971] D.G. Luenbeger, An introduction to observers. *IEEE Trans. Automat. Contr.,* 16, 596-602, 1971.

[Nathan and Singh, 1989] P.J. Nathan and S.N. Singh, "Variable structure control of a robotic arm with flexible links," in *Proc. 1989 IEEE Int. Conf. Robotics and Automation*, pp. 882-887, 1989.

[Nemir *et al.*, 1988] D.C. Nemir, A.J. Koivo, and R.L. Kashyap, "Pseudolinks and the self-tuning control of a non-rigid link mechanism," *IEEE Trans. Syst., Man, and Cybern.,* vol. 18, no. 1, pp. 40-48, 1988.

[Qian and Ma, 1992] W.T. Qian and C.C.H. Ma, "A new controller design for a flexible one-link manipulator," *IEEE Trans. Automat. Contr.,* vol. 37, no. 1, pp. 132-137, 1992.

[Thomas and Bandyopadhyay, 1997] S. Thomas and B. Bandyopadhyay, "Comments on 'A new controller design for a flexible one-link manipulator'," *IEEE Trans. Automat. Contr.,* vol. 42, no. 3, pp. 425-429, 1997.

[Trinh and Ha, 2000] H. Trinh, and Q. Ha, "Design of linear functional observers for linear systems with unknown inputs". *Int. J. Systems Science*, vol. 31, pp. 741-749. 2000.