Friction Torques Estimation and Compensation for Robot Arms

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Abstract

This paper considers the estimation and compensation of the unknown dynamic friction forces for the motion control of robot manipulators. Using Tustin's friction model, we develop a novel estimation routine to obtain the estimates of those unknown parameters. The parameters in mass matrix and Coriolis and Centrifugal matrix are also estimated. The estimate results can then be used in a PD-plus-gravity-plus-friction control scheme to eliminate the error in tracking control.

Key words: robot control, PD control, tracking control, parameter estimation, friction compensation.

1 Introduction

In robot arm motion control, the major causes of positioning errors are due to the unknown gravity force and friction force of the robots. Although a high position control gain may be applied to reduce the errors, it is not always feasible as the high gains may excite the unmodeled high frequency mode of robots. In force control, since the contacting force between a robot and its environment may compose of gravity force, friction force as well as controlled force, the modeling and compensation of the first two portions become critical to ensure good performance. Our previous work on the estimation and compensation of the gravity force and static friction has been reported in [Liu, 1998] and [Quach and Liu, 2000]. In this study we focus on the identification of dynamic frictions.

By nature, friction is a complicated combination of all force components distributed along the contact surfaces between two slide or rotate substances. It is linear in velocity at high speed motion but shows effective nonlinearities when the velocity is low. Due to this complexity its precise modeling and estimation are both difficult tasks. For the general case of robot arm parameter estimation, significant researches have been reported in, e.g., [An *et al.,* 1985,], [Armstrong, 1988], [Canudas *et al.,* 1989,], [Canudas and Aubin, 1992], [Nasri and Bolmsjo, 1997] and [Kozlowski, 1998]. A sequential hybrid estimation algorithm, in which estimation was performed by standard recursive estimation algorithms while the robot model was formulated in continuous time, has been proposed to estimate arm parameters for each link in [Canudas and Aubin, 1992]. In [Nasri and Bolmsjo, 1997], a candidate function based statistic scheme was reported. An approach subject to Newton-Euler description was given in [An *et al.,* 1985,]. However, due to the structural complexity of robot dynamics and a large number of estimated parameters involved, those estimation schemes require tedious computation and very rich stimulating signals to meet the condition of persisting excitation [Sastry and Bodson, 1989].

In this paper, a new approach is proposed to estimate all parameters defined in the Tustin's friction mathematic model, which gives a fairly complete description on different fritions, For a robot with all rotational joints, it firstly estimates the viscous friction, kinetic friction at a high velocity. At the same time, the robot parameters such as those in the mass matrix, Coriolis and centrifugal matrix, and gravity torque vector are also estimated. Secondly, the static friction is estimated at the steady state using step input at any valid position. Note that this differ from our previous paper [Quach and Liu, 2000] where the robot moved to its right-up position so that the gravity force is zero and the steady state error is equal to its static friction. Finally, it estimates the empirical term, which represents the decline speed to kinetic friction at low velocity. Once all estimates are obtained, they are used to implement a PD-plus-gravity-plus-friction tracking control scheme to reduce the tracking error.

2 Robot Model and Friction Model

The dynamic equation of a robot with n joints is given as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau, \qquad (1)$$

where $q = q(t) \in \Re^n$ is the joint variable vector; $\tau \in \Re^n$ is the applied torque vector; $M(q) \in \Re^{n\times n}$ is the mass matrix; $C(q,\dot{q}) \in \Re^{n\times n}$ is the Coriolis and Centrifugal matrix; $G(q) \in \Re^n$ is the gravity torque vector; and $F(\dot{q}) \in \Re^n$, given by $F(\dot{q}) = [f_1(\dot{q}_1), \cdots, f_n(\dot{q}_n)]^T$, is the friction torque vector.

In PD control, the applied torque vector is

$$\tau = k_p(q_d - q) + k_v(\dot{q}_d - \dot{q}), \tag{2}$$

where q_d denotes the desired position vector of the robot arm, k_p is the control position gain matrix, and k_v is the control derivative gain matrix.

In (1), friction $f_i(\dot{q}_i)$ is a complex nonlinear force having significant effects on robot arm dynamics. It depends on many factors such as temperature as well as velocity history (a change in friction lags the corresponding change in velocity). At low velocity, the stick-slip oscillation may occur due to negative viscous friction which also includes the so-called Stribeck effect. The Stribeck effect is a friction phenomenon that arises from the use of fluid lubrication. It decreases friction when the velocity is increasing but still very close to $\dot{q} = 0$. In this regard, Tustin was the first one to make use of a model with negative viscous friction in the analysis of feedback control. For the machanism of Tustin's model see [Armstrong, 1991] in which a detailed analysis on many friction models can be found. Due to its close approximation, Tusin's model is used in this paper. To describe the transition from $\dot{q} \to 0^+$ to $\dot{q} \to 0^-$, the model has been slightly modified. Shown in figure 1, using Tustin's model, the $f_i(\dot{q}_i)$ can be modelled as

$$f_i(\dot{q}_i) = f_{vi}\dot{q}_i + sgn(\dot{q}_i)\left[f_{ki} + (f_{si} - f_{ki})e^{-\frac{|\dot{q}_i|}{k_i}}\right], \tag{3}$$

where f_{vi}, f_{ki}, and f_{si} are the positive viscous friction coefficient, kinetic friction coefficient, and static friction coefficient of joint i respectively with $f_{si} > f_{ki}$; and k_i is a small positive constant giving the transient velocity from nonlinear friction to kinetic friction. For a well-designed robot arm, particular a direct-drive arm, the friction will be small. Equation (3) is only a rough model of the friction. A more comprehensive model may include a term of the form $f_c(q,\dot{q})$, which includes the linear dependence of the friction on the displacement due to the Dahl effect [Schilling, 1990]. Measurement have proven that the viscous friction is not exactly proportional to the velocity [Prüfer and Wahl, 1994].

3 Friction Parameter Estimation

3.1 Review of Previous Work

The gravity estimation and compensation algorithm can be found in [Liu, 1998]. We extended this work to include the estimation and compensation of the static force

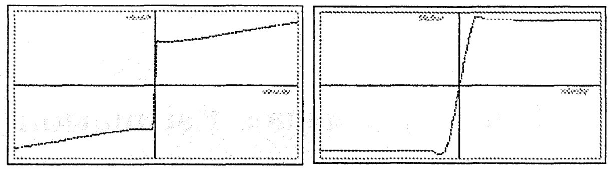

Figure 1: (a) friction vs. velocity, (b) friction at very low velocity

in a 3-step control scheme in [Quach and Liu, 2000]. The 3-step set-point control modifies its control torques by varying the set-points so that the robot reaches the initial (desired) set-point with zero error at most three steps. In fact, the first two steps of this scheme is used to gather information to estimate the gravity torque and static friction torque, which are the major causes of errors in the steady state. Then the estimate values are added to the applied torque and drive the robot arm to the desired set-point without errors.

3.2 Friction Parameter Estimation

Since the friction model in (3) is nonlinear, it is very difficult to factorise it to a liner-in-parameter form. We suggest that the paramter estimation is taken in three steps. First the viscous friction and kinetic friction are estimated at high velocity, next the static friction is estimated at steady-state using a step input, and finally the empirical parameter k_i of joint i is estimated at low velocity.

Step 1: *Estimate all linear parameters at high velocity.* At high velocity, $e^{\frac{-|\dot{q}_i|}{k_i}} \approx 0$, (3) becomes

$$f_i \approx f_{vi}\dot{q} + sgn(\dot{q}_i)f_{ki}. \tag{4}$$

(4) together with the elements in $M(q)$, $C(q,\dot{q})$, and $G(q)$ have linear-in-parameter property. They can be factorised into the form $W^T\Theta$, where W is the regressor matrix, and Θ is the parameter vector to be estimated. It is possible to apply standard recursive least square algorithm to estimate the parameters.

Step 2: *Estimate the static friction using step input.* To estimate static friction, we move the robot arms to any valid position using step input. According to [Quach and Liu, 2000], at steady state, we have

$$F_s + G = k_p(q_d - q),$$

where $F_s = [f_{s1}, f_{s2}, \cdots, f_{sn}]^T$ is the static friction vector. The vector G has been estimated in the previous step, then

$$\hat{F}_s = k_p(q_d - q) - \hat{G}. \tag{5}$$

If all joints are exactly right up, the gravity torques are zero. (5) becomes

$$\hat{F}_s = k_p(q_d - q). \tag{6}$$

Step 3: *Estimate the empirical constant k at low velocity.* At low velocity, since all the parameters except k_i are estimated and known. We have

$$y = sgn(\dot{q})(\hat{f}_s - \hat{f}_k)e^{\frac{-|\dot{q}|}{k}},$$

or

$$k = \frac{-|\dot{q}|}{\ln \frac{y}{sgn(\dot{q})(\hat{f}_s - \hat{f}_k)}}, \tag{7}$$

with

$$0 < \frac{y}{sgn(\dot{q})(\hat{f}_s - \hat{f}_k)} < e, \tag{8}$$

and

$$y = \tau - \hat{M}(q)\ddot{q} - \hat{C}(q,\dot{q})\dot{q} - \hat{f}_v\dot{q} - sgn(\dot{q})\hat{f}_k - \hat{G},$$

where the hat ($\hat{}$) denotes the estimate value of the parameters, and f_v, f_k, k are the vector corresponding to the viscous friction, kinetic friction, the empirical parameter vector respectively, e is the base of the natural logarithm. (8) is necessary, because $k > 0$. From (7), since the parameter k has a non-linear contribution to the total friction force, it makes difficult to estimate this parameter using traditional recursive least square algorithm. One of the solution is to obtain the value of k in each step during the estimation period at low velocity. Then the sum of the square of these values are used to calculate the quadratic mean using the following equation:

$$k = \sqrt{\frac{\sum_{j=1}^{n} k^2(j)}{n}}. \tag{9}$$

Note that we need to avoid to apply a very low speed to estimate the parameters, since at very low speed, the stick-slip oscillation during the low speed motion occurs due to the negative viscous friction. This has been observed by many investigators.

4 Case Study

4.1 Estimation and Compensation for A Two Link Plannar Arm

For a two-link plannar arm, the mass matrix $M(q)$ is

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix},$$

and the Coriolis and centrifugal matrix is

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 0 \end{bmatrix}.$$

Let

$$\begin{aligned} m_{111} &= I_{l1} + m_{l1}{l_1^c}^2 + {k_{r1}}^2 I_{m1} + I_{l2} \\ &\quad + m_{l2}({l_1}^2 + {l_2^c}^2) + I_{m2} + m_{m2}{l_1}^2, \end{aligned}$$

$$m_{112} = m_{l2}l_1 l_2^c,$$

$$m_{121} = I_{l2} + m_{l2}{l_2^c}^2 + k_{r2}I_{m2},$$

then the elements of the two matrices are computed to be

$$m_{11} = m_{111} + 2m_{112}cos(q_2), \tag{10}$$

$$m_{12} = m_{21} = m_{121} + m_{112}cos(q_2), \tag{11}$$

$$m_{22} = I_{l2} + m_{l2}{l_2^c}^2 + {k_{r2}}^2 I_{m2}, \tag{12}$$

$$c_{11} = -m_{112}\dot{q}_2 sin(q_2), \tag{13}$$

$$c_{12} = -m_{112}(\dot{q}_1 + \dot{q}_2)sin(q_2), \tag{14}$$

$$c_{21} = m_{112}\dot{q}_1 sin(q_2), \tag{15}$$

where I_{li}, I_{mi} are the moment of inertia of the link i and motor i respectively; m_{li} and m_{mi} are the mass of link i and motor i respectively; l_i and l_i^c are the length and center of link i respectively; and k_{ri} is the gear ratio of link i.

The gravity torque vector $G(q)$ is

$$G(q) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \overline{g}\begin{bmatrix} cos(q_1) & cos(q_1+q_2) \\ 0 & cos(q_1+q_2) \end{bmatrix}\begin{bmatrix} g_{11} \\ g_{12} \end{bmatrix}, \tag{16}$$

where g_1 and g_2 are the gravity torques contribution to joint 1 and joint 2 respectively, and $\overline{g} = 9.80665m/sec$ is the gravity acceleration constant. And $g_{11} = m_{l1}l_1^c + (m_{m2} + m_{l2})l_1$, $g_{12} = m_{l2}l_2^c$. Substituting (10) to (16), (4), and (2) into (1), we obtain

$$\begin{aligned} &m_{111}\ddot{q}_1 + m_{112}\left[(2\ddot{q}_1 + \ddot{q}_2)cos(q_2) - (2\dot{q}_1 + \dot{q}_2)\dot{q}_2 \right. \\ &\left. sin(q_2)\right] + m_{121}\ddot{q}_2 + g_1 + f_{v1}\dot{q}_1 + sgn(\dot{q}_1)f_{k1} = \tau_1, \end{aligned} \tag{17}$$

$$\begin{aligned} &m_{121}\ddot{q}_1 + m_{112}\left[\ddot{q}_1 cos(q_2) + \dot{q}_1^2 sin(q_2)\right] + m_{22}\ddot{q}_2 \\ &\quad + g_2 + f_{v2}\dot{q}_2 + sgn(\dot{q}_2)f_{k2} = \tau_2, \end{aligned} \tag{18}$$

The equations (17) and (18) can be factorised as the form of

$$y = W^T\Theta \tag{19}$$

where the parameter vector is

$$\Theta = [m_{111}, m_{112}, m_{121}, m_{22}, g_{11}, g_{12}, f_{v1}, f_{v2}, f_{k1}, f_{k2},]^T, \tag{20}$$

the elements of the regressor matrix are

$$w_{11} = w_{32} = \ddot{q}_1, \quad w_{31} = w_{42} = \ddot{q}_2,$$

$$w_{12} = w_{41} = w_{52} = w_{72} = w_{81} = w_{92} = w_{10,1} = 0,$$

$$w_{21} = (2\ddot{q}_1 + \ddot{q}_2)cos(q_2) - (2\dot{q}_1 + \dot{q}_2)\dot{q}_2 \sin(q_2),$$

$$w_{22} = \ddot{q}_1 cos(q_2) + \dot{q}_1^2 sin(q_2),$$

$$w_{51} = \overline{g}cos(q_1), \quad w_{61} = w_{62} = \overline{g}cos(q_1 + q_2),$$

$$w_{71} = \dot{q}_1, \quad w_{82} = \dot{q}_2, \quad w_{91} = sgn(\dot{q}_1), \quad w_{10,2} = sgn(\dot{q}_2),$$

and the elements of the observer vector y are

$$y_1 = \tau_1, \quad y_2 = \tau_2.$$

By applying least square algorithm, all elements in Θ can be estimated.

The static friction is estimated using (5). The empirical parameter k can be estimated at low velocity using (7) and (9).

4.2 Simulation Results

The above computations were applied to a simulation program, in which the two-link robot parameters were selected to be: $m_{l1} = 4.0kg$, $m_{l2} = 2.0kg$, $l_{l1} = 0.5m$, $l_{l2} = 0.25m$, $l_1^c = 0.25m$, $l_2^c = 0.15m$, $I_{l1} = 1.0kg.m^2$, $I_{l2} = 0.8kg.m^2$, and the moments of inertia and masses of the motors were assumed to be very small compared with those of links and could be neglected. We assumed that the arm was direct-drive one. The home position of the robot was selected to be $q_1 = 90^o$, and $q_2 = 0^o$.

We estimated the friction coefficients in the positive velocity direction only. For the negative velocity direction, the same scheme could be applied. The friction torque parameters used were: $f_{v1} = 0.049N.m$, $f_{v2} = 0.077N.m$, $f_{k1} = 0.08N.m$, $f_{k2} = 0.12N.m$, $f_{s1} = 0.09N.m$, $f_{s2} = 0.132N.m$, $k_1 = 0.1rad/s$, $k_2 = 0.11rad/s$. The control gains are: $k_{p1} = 58N.m/rad$, $k_{p2} = 40N.m/rad$, $k_{v1} = 25N.m.s/rad$, and $k_{v2} = 10N.m.s/rad$. Figure 2 shows the setpoints we used in the simulation.

Step 1: *Linear parameters estimation.* At high velocity, the set points were

$$q_{1d}(t) = 1.1t, \quad q_{2d}(t) = 0.9t.$$

The results are shown in figure 3 to figure 10. The thick lines are the actual values, and the thin lines are the estimated values.

Step 2: *Static frictions estimation.* At this stage, first we moved the joints to

$$q_{1d} = 1.4radian, \quad q_{2d} = -0.5radian,$$

then we moved them to their home position:

$$q_{1d} = 1.571radian, \quad q_{2d} = 0radian.$$

By doing this, we ensured that the static frictions were estimated in the positive velocity direction. Then (5) was applied to estimate the static friction. The results are shown in figure 11.

Step 3: *Empirical constant k estimation.* At low velocity, the set points were

$$q_{1d}(t) = 0.03t + 1.301, \quad q_{2d}(t) = 0.02t - 0.18,$$

and (9) was applied to estimate k. The results are shown in figure 12 and 13. We can see that the estimated values are very close to their actual values.

5 PD-Plus-Gravity-Plus-Friction Control

To utilise the estimation, the estimated results were then used to implement a PD-plus- gravity-plus-friction (PDGF) control, whose control scheme is

$$\tau = k_p(q_d - q) + k_v(\dot{q}_d - \dot{q}) + \hat{G} + \hat{F},$$

instead of (2) in PD control. We apply the PDGF tracking control to the same model above with

$$q_{1d} = 2.5sin(0.5t), \quad q_{2d} = 1.5sin(0.8t).$$

The results are shown in figure 15. Figure 14 shows the tracking control using pure PD controller. We can see that the PDGF gives a better performance; it reduces the tracking errors significantly.

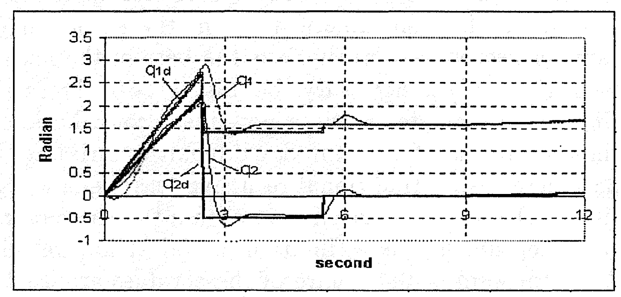

Figure 2: Desired positions used to estimate the parameters

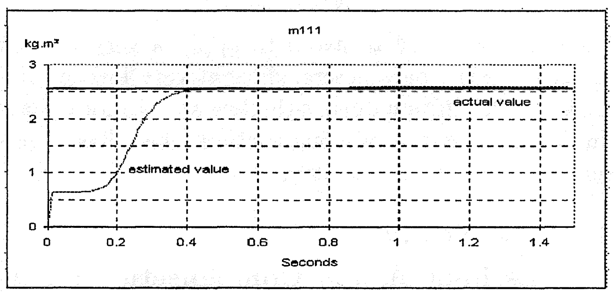

Figure 3: Estimation of m_{111}

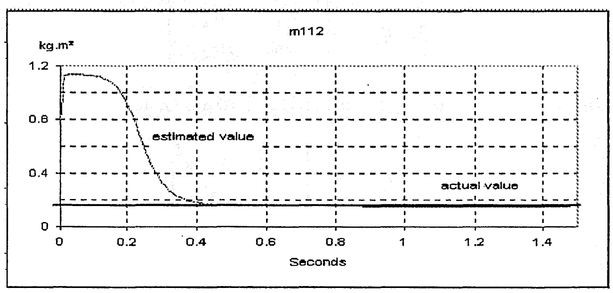

Figure 4: Estimation of m_{112}

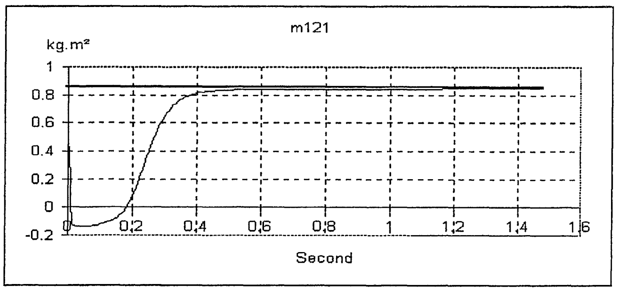

Figure 5: Estimation of m_{121}

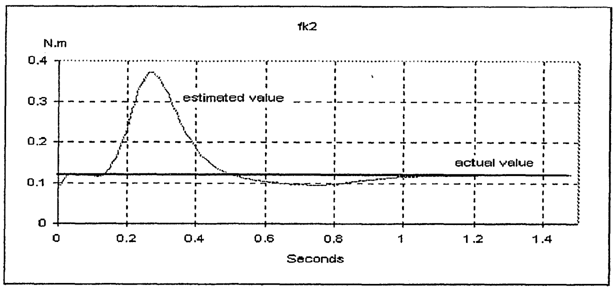

Figure 10: Estimation of f_{k2}

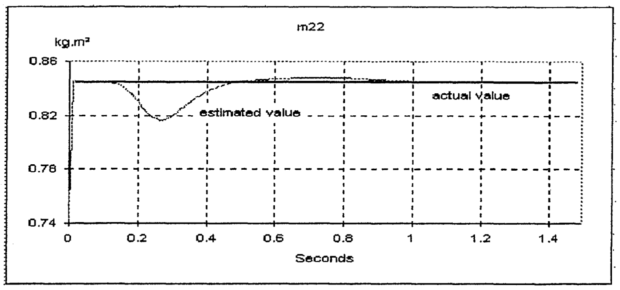

Figure 6: Estimation of m_{22}

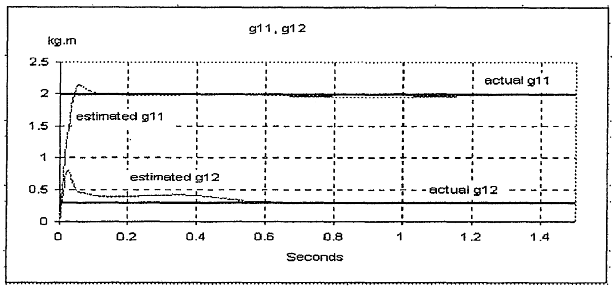

Figure 11: Estimation of g_{11} and g_{12}

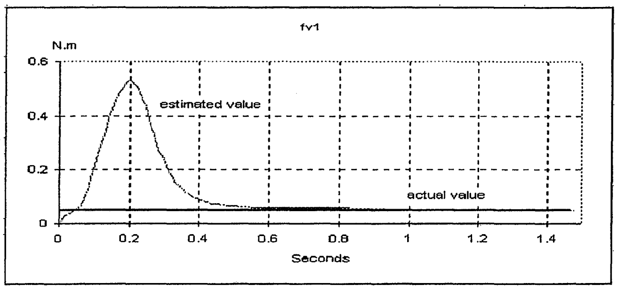

Figure 7: Estimation of f_{v1}

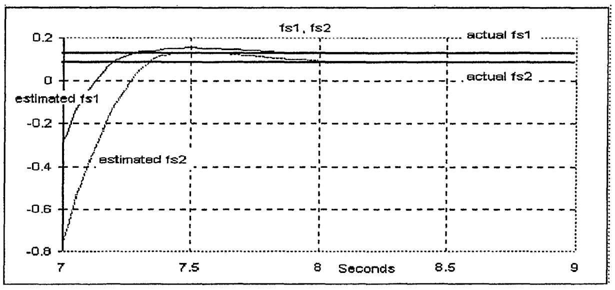

Figure 12: Estimation of f_{s1} and f_{s2}

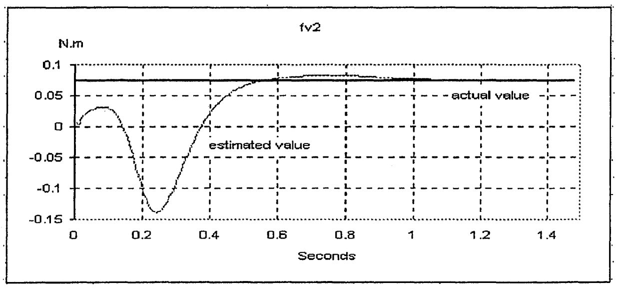

Figure 8: Estimation of f_{v2}

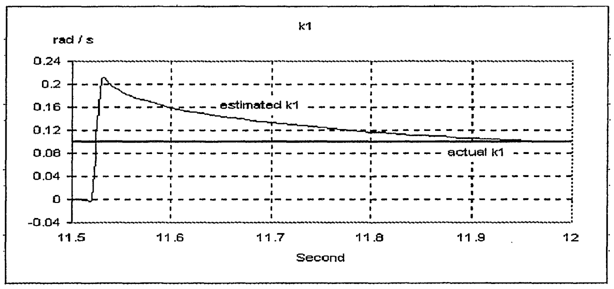

Figure 13: Estimation of k_1

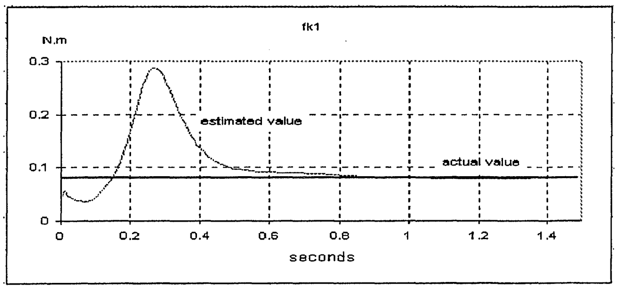

Figure 9: Estimation of f_{k1}

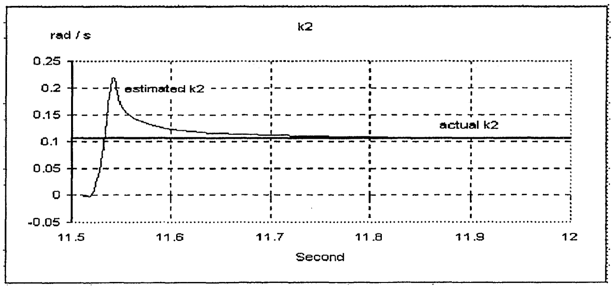

Figure 14: Estimation of k_2

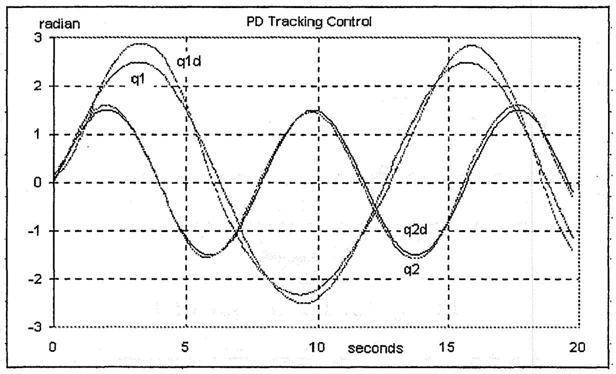

Figure 15: PD tracking control

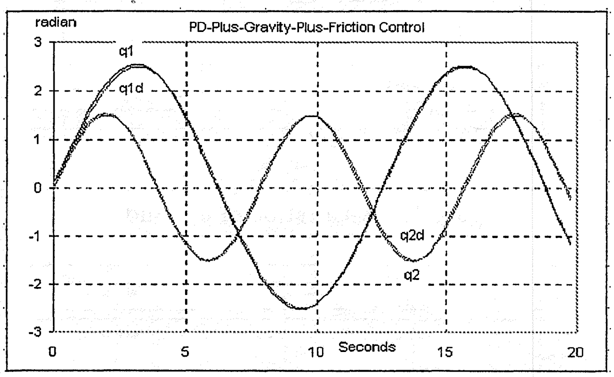

Figure 16: PDGF tracking control

6 Conclusion

A friction torque estimation scheme has been presented. It shows that the friction parameters can be easily obtained by estimating its linear model first at high velocity. Then the static friction can be estimated using step input. Finally, the empirical parameter k is estimated at low velocity, where we can simplify the equation and apply the quadratic mean method. The results show that the estimated values are close to their actual values. To verify the effectiveness of the algorithm, we use the results in a PD-plus-gravity-plus-friction (PDGF) tracking control scheme to compare with a pure PD tracking control, which shows much improved performance.

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