Theory and Experiment of Decentralized Nonlinear and Adaptive Control of Robot Arms

Ming Liu
Department of Electrical and Computer Systems Engineering
Monash University, VIC 3800
Australia
m.liu@eng.monash.edu.au

Abstract

After summarizing a set of theoretical results on decentralized nonlinear and adaptive control schemes for robot arm tracking proposed in [Liu, 1997] and [Liu, 1999], this paper presents some experimental results obtained by running the schemes on a real robot. The results verified the effectiveness of the schemes by showing the superior robustness and tracking performance.

1 Introduction

For tracking control of robot manipulators, although many excellent centralized control algorithms, (see [Craig, 1988] and [Slotine and Li, 1987] for example), have been developed, the majority of the robot systems are still controlled by simple linear decentralized controls such as PD and PID schemes due to the simplicity of algorithms and controller structures. Thus, to develop advanced centralized control algorithms has been a challenging task for last twenty years.

Even though the decentralized control for linear and linear dominant systems have been well developed (see, e.g., [Sezer and Siljak, 1981], [Siljak, 1978], [Ioannou, 1986], [Gavel and Hsia, 1987] and [Shi and Singh, 1992] for references), the decentralized control for nonlinear systems is still a premature area. In the case of robot tracking, the major difficult is caused by the nonlinear couplings and quadratic nonlinearity in the dynamics. The early efforts on robot decentralized adaptive control have been reported in [Seraji, 1988], [Gavel and Hsia, 1987], [Fu and Fu, 1988] and [Khorrami and Ozguner, 1988]. However, they suffered from various drawbacks such as undesirable interconnection structure which involves control actions [Gavel and Hsia, 1987], very large controller gains [Fu and Fu, 1988] or restriction on motion velocity [Seraji, 1989]. All approaches resulted in local stability of tracking errors.

In our recent work [Liu, 1997] and [Liu, 1999], the decentralized control has been studied using passivity property of robot systems. The local ultimate stability conditions of PD control was obtained using either the Lyapnonv analysis [Liu, 1999] or input-output analysis [Liu, 1998]. It showed that the stability condition on the PD gain can be interpreted as a nonlinear version of small gain theorem [Desoer and Vidyasagar, 1975]. By reasoning that the local stability was caused by the quadratic nonlinearity inherited from the centrifugal and Coriolis force, a cubic feedback was added to PD control, which improved the local stability of PD control to global stability. Furthermore, applying the linear-in-parameter property of robots' equations of motion, an additional adaptive control was introduced to reduce the tracking errors. The simulation study verified the effectiveness of the algorithms.

After giving a summary on our theoretical results, this paper presents the experimental outcome obtained recently by running our algorithms on a PUMA 560 robot. To explore the effects of the quadratic nonlineary of robot arm, rather fast reference trajectories and rather slow sampling rate were used. The results showed that compared with traditional PD control, the nonlinear control and adaptive control proposed could improve system's robustness and reduce the tracking error up to 45% in our test-bed.

2 Motion Equations

We consider the equations of motion of rigid robot arms with n DOFs:

$$D(q)\ddot{q} + h(q,\dot{q}) + g(q) = \tau - f(\dot{q}). \qquad (1)$$

In (1), $q = q(t) \in \Re^n$ is the robot arm joint angle vector; $\tau \in \Re^n$, the input torque vector; $D(q) = D^T(q) \in \Re^{n\times n}$, the positive definite inertia matrix; $h(q,\dot{q}) = (I_n \otimes \dot{q}^T)H_v(q)\dot{q} = H_m(q,\dot{q})\dot{q} \in \Re^n$, where $H_v(q) = [H_1^T(q) \ \cdots H_n^T(q)]^T$ is an $n^2 \times n$ matrix, the centrifugal and Coriolis torque; $g(q) \in \Re^n$, the gravity torque; and $f(\dot{q}) \in \Re^n$, the friction torque, which consists of viscous, Coulomb and stiction frictions and is

bounded by $\|f(\dot{q})\| \leq f_v\|\dot{q}\| + f_c$, where $f_v, f_c > 0$ are constants.

It is well known that in (1), $(\tau - f) \to \dot{q}$ is a passive mapping ([Arimoto and Miyazaki, 1984] and [Slotine and Li, 1987]), i.e., matrix

$$S = \dot{D}/2 - H_m = -S^T \tag{2}$$

is skew-symmetrical.

Consider the tracking problem we denote the reference trajectories as $q_d \in C^2$ and the position tracking error vector as $e = q_d - q$. Further, by introducing a generalized tracking error vector r [Slotine and Li, 1987]:

$$r = \dot{e} + \Phi e \tag{3}$$

where $\Phi = diag\{\phi_1, \cdots, \phi_n\} > 0$ is a constant matrix, (1) can be rewritten as

$$D(q)\dot{r} + H_m(q,\dot{q})r = \xi - \tau \tag{4}$$

$$\begin{aligned} \xi &= D(q)(\ddot{q}_d + \Phi\dot{e}) + H_m(q,\dot{q})(\dot{q}_d + \Phi e) \\ &\quad +g(q) + f(\dot{q}). \end{aligned} \tag{5}$$

Thus, for a class of q_d and Φ given, equations (3), (4) and (5) define the tracking error dynamics of (1) in which input torque τ is to be designed.

The feedback system (3), (4) and (5) has the following properties: i). Mapping $(\xi - \tau) \mapsto r$ defined by (4) is passive [Liu, 1999]. ii). Under an assumption that all joints of the robots considered are revolute, there exist some positive constants μ_{max}, c_H and c_g such that $\|D(q)\| \leq \mu_{max}$, $\|H_v(q)\| \leq c_H$, and $\|g(q)\| \leq c_g$. Further, it can be shown (see [Liu, 1999]) that the cascade mapping $r \mapsto (e, \dot{e}) \mapsto \xi$ defined by (3) and (5) satisfies

$$\|\xi\|_{T\infty} \leq \beta_1 + \beta_2\|r\|_{T\infty} + \beta_3\|r\|^2_{T\infty} \tag{6}$$

where $\|\cdot\|_{T\infty}$ is the truncated L_∞ norm and β_1, β_2 and $\beta_3 > 0$ are some constants given by

$$\begin{aligned} \beta_1 &= c_H\|\dot{q}_d\|^2_{T\infty} + f_v\|\dot{q}_d\|_{T\infty} + \mu_{max}\|\ddot{q}_d\|_{T\infty} \\ &\quad +(2c_H\|\dot{q}_d\|_{T\infty} + \mu_{max}\|\Phi\|_\infty + f_v)\|\Phi\|_\infty\|e_0\|_\infty \\ &\quad +c_H\|\Phi\|^2_\infty\|e_0\|^2_\infty + c_g + f_c \\ \beta_2 &= 2c_H\|\dot{q}_d\|_{T\infty} + \mu_{max}\|\Phi\|_\infty + 2c_H\|\Phi\|_\infty\|e_0\|_\infty \\ &\quad +f_v \\ \beta_3 &= c_H. \end{aligned} \tag{7}$$

3 Decentralized Feedback Control

To formulate the decentralized tracking error dynamics, we split the overall error dynamics (4) and (5) into n joint-torque-based subsystems. The i-th subsystem is given by the i-th component of (4):

$$\sum_{j=1}^{n} d_{ij}(q)\dot{r}_j + \dot{q}^T H_i(q)r = -\tau_i + \xi_i, \quad i = 1, 2, \cdots n \tag{8}$$

where $d_{ij}(q)$ is the i-j-th entry of $D(q)$ and ξ_i, the i-th component of (5), is given by

$$\begin{aligned} \xi_i &= \sum_{j=1}^{n} d_{ij}(q)(\ddot{q}_{dj} + \phi_j\dot{e}_j) + \dot{q}^T H_i(q)(\dot{q}_d + \Phi e) \\ &\quad +g_i(q) + f_i(\dot{q}). \end{aligned} \tag{9}$$

Given error subsystem (8) and interconnection (9), control task is to design a class of *local feedback and global feedforward* laws $\tau_i = \tau_i(r_i, q_d, \dot{q}_d, \ddot{q}_d)$ so that r is as small as possible and all internal signals remain bounded.

3.1 PD Linear Control

The most popular decentralized control is the linear PD control given by

$$\tau_i = k_i r = k_i e_i + k_i\phi_i\dot{e}_i. \tag{10}$$

Due to its computational and structural simplicity, (10) has been broadly adopted by robot arms. For set-point control, it is well known that together with a gravity force compensation it ensures the global and asymptotical stability of positioning errors [Arimoto and Miyazaki, 1984]. In tracking, the local ultimate boundedness of tracking errors has been studied in [Kawamura *et al.*, 1988] and [Liu, 1997] etc. Using our formulations, the L_∞ stability can be stated by the following theorem:

Theorem 1 *Consider the error dynamics (4) and (5) controlled by (10). If $r_0 = r(0)$ and k_i satisfy*

$$\|r_0\|_\infty < \delta_2, \quad k_{min} > n(\beta_2 + 2\sqrt{\beta_1\beta_3}) \tag{11}$$

then the tracking error r converges to a residual set

$$\Omega_{fp} = \{r : \|r\|_\infty \leq \delta_1\}$$

where

$$\delta_1 = \frac{1}{2n\beta_3}\{(k_{min} - n\beta_2) - \sqrt{(k_{min} - n\beta_2)^2 - 4n^2\beta_1\beta_3}\} \tag{12}$$

$$\delta_2 = \frac{1}{2n\beta_3}\{(k_{min} - n\beta_2) + \sqrt{(k_{min} - n\beta_2)^2 - 4n^2\beta_1\beta_3}\} \tag{13}$$

$k_{min} = \min_{i\in n}\{k_i\}$, and β_1, β_2 and β_3 are given by (7).

Proof: See [Liu, 1997].

Theorem 1 can be proven using either the Lyapunov approach [Liu, 1997] or input-output approach [Liu, 1998]. From input-output point of view, (11) can be interpreted as a nonlinear version of small gain theorem [Desoer and Vidyasagar, 1975]. Further examination on (12) and (13) indicates that its local (rather than global) stability is caused by $\beta_2 = c_H$, the strength the centrifugal and Coriolis force. It is also easy to see that if $k_i \to \infty$ the closed loop system becomes globally asymptotically stable. However, as large k_i will stimulate the un-modeled high frequency modes of robots, it has to be avoided in applications.

3.2 Nonlinear Decentralized Control

To improve the stability of PD control, we proposed the following decentralized PD-plus-nonlinear (PD+NL) control law in [Liu, 1997]:

$$\tau_i = (k_i + \nu_i)r_i + \rho_i r_i^3 \tag{14}$$

which leads to the following *global* stability theorem:

Theorem 2 *Consider tracking error dynamics (3), (4) and (5), in which the input torque is given by (14). If k_i, ν_i and ρ_i in (14) satisfy the conditions*

$$k_{min} > 0, \quad \nu_{min} > 0, \quad \rho_{min} > n\beta_3/2 \tag{15}$$

then the tracking error r will globally converge to a residual set

$$\Omega_{fc} = \{r : \|r\|^2 \leq d_{fc}\} \tag{16}$$

with a rate not slower than $exp(-\eta_c t)$, where $\|\cdot\|$ is the Euclidean norm,

$$d_{fc} = \frac{n^2}{4k_{min}} \left(\frac{(2\beta_2 + \beta_3)^2}{2(2\rho_{min} - n\beta_3)} + \frac{\beta_1^2}{\nu_{min}} \right) \frac{\lambda_{max}(D)}{\lambda_{min}(D)} \tag{17}$$

$\lambda_{max(min)}(\cdot)$ is the maximum (minimum) eigenvalue of $(\cdot)$, $k_{min} = \min_{i\in n}\{k_i\}$, $\rho_{min} = \min_{i\in n}\{\rho_i\}$ and $\nu_{min} = \min_{i\in n}\{\nu_i\}$.

Compared with the stability condition of PD control, (15) simply requires k_i and ν_i to be positive. Further, (15) is totally independent of β_1 and β_2 (the sizes $D(q)$, $g(q)$, $f(\dot{q})$ and q_d (see (7))). This implies that the cubic feedback gain is solely determined by c_H (the size of $H_v(q)$). As far as (15) holds the global ultimate stability can be guaranteed no matter how large the sizes $D(q)$, $g(q)$ and $f(\dot{q})$ are (even though smaller k_{min} may result in large tracking errors). This highlights the significance of the cubic feedback; it not only achieves global stability but also improves the robustness of the closed loop system.

Further, if the PD gains can be enlarged to satisfy certain conditions the following corollary holds:

Corollary 1 *Under the same conditions of Theorem 1, if (15) holds but in which k_{min} is enlarged to satisfy*

$$k_{min} > n(2\beta_2 + \beta_3)/2 \tag{18}$$

then the conclusion of Theorem 1 applies except that d_{fc} in (16) becomes

$$d_{fc} = \frac{n^2\beta_1^2}{\nu_{min}(2k_{min} - n(2\beta_2 + \beta_3))} \frac{\lambda_{max}(D)}{\lambda_{min}(D)}. \tag{19}$$

Proof: See [Liu, 1996].

Equations (17) and (19) indicate how the PD control and cubic control affect the stability; the cubic control is to achieve the global stability and the PD control is to ensure the bounded tracking errors.

3.3 Nonlinear-plus-Adaptive Control

Having achieved global stability, we present a solution to further reduce the tracking errors by applying the decentralized adaptive control.

We denote $\bar{q} \equiv (q, \dot{q})$, $\bar{q}_i \equiv (q_i, \dot{q}_i)$ and $\bar{q}_d \equiv (q_d, \dot{q}_d, \ddot{q}_d)$. Applying the linear-in-parameter property of the equation of motion, ξ_i can be rewritten as $\xi_i = w_i(\bar{q}_i, \bar{q}_d)\theta_i + \xi_{ai}$, where w_i is a regression row vector and $\theta_i = [\theta_{i1}, \cdots, \theta_{ik_i}]^T \in \Re^{k_i}$ the unknown constant parameter vector. Subsequently, (8) and (4) become, respectively,

$$\sum_{j=1}^{n} d_{ij}\dot{r} + \dot{q}^T H_i(q)r = -\tau_i + w_i(\bar{q}_i, \bar{q}_d)\theta_i + \xi_{ai} \tag{20}$$

$$D(q)\dot{r} + H_m(q, \dot{q})r = -\tau + W(\bar{q}, \bar{q}_d)\theta + \xi_a \tag{21}$$

where $\theta = [\theta_i^T, \cdots, \theta_n^T]^T \in \Re^{\sum_{i=1}^n k_i}$, and $W(\bar{q}, \bar{q}_d)\theta + \xi_a = \xi$. $W(\bar{q}, \bar{q}_d) = diag\{w_1(\bar{q}_1, \bar{q}_d), \cdots, w_n(\bar{q}_n, \bar{q}_d)\} \in \Re^{n\times\sum_{i=1}^n k_i}$. In view of (6), we can deduce that there exist some constants β'_i, which satisfy $0 < \beta'_i \leq \beta_i$, $i = 1, 2, 3$, such that

$$\|\xi_a\|_{T\infty} \leq \beta'_1 + \beta'_2\|r\|_{T\infty} + \beta'_3\|r\|^2_{T\infty}. \tag{22}$$

As $w_i(\bar{q}_i, \bar{q}_d)$ can be obtained by kinematics and dynamics study, they are supposed to be known. We then can introduce the following decentralized PD-plus-nonliear-plus-adaptive (PD+NL+AD) control:

$$\tau_i = (k_i + \nu_i)r_i + \rho_i r_i^3 + w_i(\bar{q}_i, \bar{q}_d)\hat{\theta}_i, \quad i \in n \tag{23}$$

where $\hat{\theta}_i$ is an estimate of θ_i. Let $\varphi_i = \theta_i - \hat{\theta}_i$ be the parameter estimation error vector, the adaptaion law is given by

$$\dot{\varphi}_i = -\dot{\hat{\theta}}_i = -\Gamma_i w_i^T(\bar{q}_i, \bar{q}_d)r_i + \sigma_i\Gamma_i\hat{\theta}_i \tag{24}$$

where $\sigma_i > 0$ is a constant and

$$\Gamma_i = diag\{\gamma_{i1}, \cdots, \gamma_{ik_i}\} \in \Re^{k_i\times k_i} \tag{25}$$

is a positive definite constant matrix. Let $\varphi = [\varphi_1^T, \cdots, \varphi_n^T]^T$, the following theorem applies:

Theorem 3 *Under the assumptions that $w_i(\bar{q}_i, \bar{q}_d), i \in n$, are known, consider system (21) and (22) and its input torque (23) to (24). If the controller parameters in (23) satisfy that*

$$k_{min} > 0, \quad \nu_{min} > 0, \quad \rho_{min} > n\beta'_3/2 \tag{26}$$

the tracking error r and parameter estimation error φ of the overall system will globally converge to a residual set

$$\Omega_{fa} = \{(r, \varphi) : \|r\|^2 + \|\varphi\|^2 \leq 2\eta_a/(min\{\lambda_{min}(D), \gamma_{min}\}\zeta_a)\} \tag{27}$$

at a rate at least as fast as $exp(-\eta_a t)$, where

$$\zeta_a = min\left\{\frac{2k_{min}}{\lambda_{max}(D)}, \quad \sigma_{min}\gamma_{min}\right\} \tag{28}$$

$$\eta_a = \frac{(n(2\beta_2' + \beta_3'))^2}{8(2\rho_{min} - n\beta_3')} + \frac{n\beta_1'}{4\nu_{min}} + \frac{\sigma_{max}^2}{2\sigma_{min}}\|\theta\|_\infty^2. \tag{29}$$

In above, $\sigma_{max(min)} = \max(\min)_{i\in n}\{\sigma_i\}$, $\gamma_{min} = \min_{i\in n, j\in\sum k_i}\{\gamma_{ij}\}$ is the minimum element of matrix Γ_i and β_1', β_2' and β_3' are given by (22).

Proof: See [Liu, 1999]

In Theorem 3, the PD gain k_{min} and ν_{min} are only supposed to be positive. If we make k_{min} in (26) even more positive then the following corollary holds:

Corollary 2 *Under the same conditions given in Theorem 3, if (26) holds but in which k_{min} satisfies*

$$k_{min} > n(2\beta_2' + \beta_3')/2$$

then same convergence feature of tracking error and parameter estimation error given in Theorem 3 apply but ζ_a and η_a in (27) become

$$\zeta_a = min\left\{\frac{2k_{min} - n(2\beta_2' + \beta_3')}{\lambda_{max}(D)}, \quad \sigma_{min}\gamma_{min}\right\} \tag{30}$$

$$\eta_a = \frac{n\beta_1'}{4\nu_{min}} + \frac{\sigma_{max}^2}{2\sigma_{min}}\|\theta\|_\infty^2. \tag{31}$$

Proof: See [Liu, 1996]

Theorem 3 shows that adaptation parameters have no affect on the global stability of the algorithm. Rather, they only affect the size of the residual set of tracking errors as shown by (30) and (31).

In general, a quantitative comparison of tracking error size of PD+NL+AD, given by (28) (29) and (30) (31), and that of PD+NL, given by (17) and (19), needs the specifications on the difference between β_i and β_i' and the size of θ for a particular robot, which is quite complicated. However, the simulations have shown reduced tracking error of PD+NL+AD. In the following section we will show that this is also true for a real robot.

4 Experimental Results

In this section we present experimental results to clarify the performances of the algorithms.

4.1 Test-bed

The test-bed is a PUMA 560 robot, controlled by a PC-based controller developed in our laboratory. Because robot joints are driven by DC motors via gearing transmissions it is convenient to model the system on the motor shaft side. Let τ_m and q_m be the input torque vector and angle vector on motor shafts, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_6\}$ be the gearing ratio matrix in which $\lambda_i = q_i : q_{mi} < 1$, $J_m = diag\{J_1, \cdots, J_6\}$ be motor inertial matrix, $f_m(\dot{q}_m)$ be the equivalent friction on motor shaft and $\tau_c = [\tau_{c1}, \cdots, \tau_{c6}]^T$ be torque command vector from the PC-based controller. We then have $\tau_m = K_c\tau_c$, where $K_c = diag\{k_{c1}, \cdots, k_{cn}\}$ is the current loop gain matrix. Thus, on the motor shaft side, the equations of motion becomes

$$\bar{D}(q)\ddot{q}_m + \bar{H}(q,\dot{q})\dot{q}_m + \bar{g}(q) + \bar{f}_m(\dot{q}_m) = K_c\tau_c \tag{32}$$

where $\bar{D}(q) = J_m + \Lambda D(q)\Lambda$, $\bar{H}(q,\dot{q}) = \Lambda H_m(q,\dot{q})\Lambda$ and $\bar{g}(q) = \Lambda g(q)$. It can be easily verified that the passivity property (2) applies to (32).

To simplify the task and not lose generality, we only run robot's shoulder (as join1) and elbow (as joint 2) and frozen all other joints. Let $q_1 = q_2 = 0$ be the up-right post of the robot arm, the items in (32) are given by

$$\begin{aligned}
\bar{D}(q) &= \begin{bmatrix} J_1 + \lambda_1^2(\bar{\theta}_1 + 2m_2 l_1 l_{c2}\cos q_2) & \lambda_1\lambda_2(\bar{\theta}_3 + \bar{\theta}_4\cos q_2) \\ \lambda_1\lambda_2(\bar{\theta}_3 + \bar{\theta}_4\cos q_2) & J_2 + \lambda_2^2\bar{\theta}_3 \end{bmatrix} \\
\bar{H}(q,\dot{q}) &= -\bar{\theta}_4\sin q_2 \begin{bmatrix} \lambda_2\dot{q}_2 & \lambda_1\dot{q}_1 + \lambda_2\dot{q}_2 \\ \lambda_1\dot{q}_1 & 0 \end{bmatrix} \\
\bar{g}(q) &= \begin{bmatrix} \lambda_1(\bar{\theta}_2\cos q_1 + m_2 g l_{c2}\cos(q_1 + q_2)) \\ \lambda_2 m_2 g l_{c2}\cos(q_1 + q_2) \end{bmatrix}
\end{aligned}$$

respectively, where m_i, l_i and l_{ci} are the mass, length, and mass center coordinate of link i respectively; g is the acceleration of gravity and $\bar{\theta}_i$ are unknown parameters given by $\bar{\theta}_1 = m_1 l_{c2}^2 + m_2(l_1^2 + l_{c2}^2) + I_1 + I_2$, $\bar{\theta}_2 = g(m_1 l_{c1} + m_2 l_1)$, $\bar{\theta}_3 = m_2 l_{c2}^2 + I_2$ and $\bar{\theta}_4 = m_2 l_1 l_{c2}$, in which I_i is the inertial tensor of link i. The gearing ratios are, respectively, $\lambda_1 = 1 : 62.611$ and $\lambda_2 = 1 : 107.815$, which significantly reduces the coupling effects among different joints at slow motion.

4.2 Open-structure Controller

Since user's own algorithms could not be run on PUMA's original LSI-11/VAL controller, a PC-based controller has been developed. The platform is a 100MHz Pentium running the QNX multi-tasking real time operating system. Bypassing the original LSI-11 micro-processor-based host controller of PUMA, the PC was interfaced to six digital joint servos via the original DRV-11 bus. The DRV-11 bus protocol was kept for the communication between the PC host and joints. In experiment, motor current control mode was used therefore the signals sent from the PC to joint servos were motor torque commands. A GUI was also developed utilizing QNX Photon Application Builder (PhAB) which could be used for calibration, control algorithms selection, parameter modification, reading/writing joint encoders and on-line

display of joint trajectories. The real-time control code of a variety of algorithms was written using WATCOM C/C++. For further details, see [Liu *et al.*, 1998].

4.3 Controller Design

Let q_{dm} be the motor position reference trajectory vector and $e_m = q_{dm} - q_m$ be the positioning error vector. The control algorithms were represented on motor shaft side, i.e., PD+NL and PD+NL+AD were, respectively, $\tau_{ci} = \left((k_{mi} + \nu_{mi})r_{mi} + \rho_{mi} r_{mi}^3\right)/k_{ci}$ and $\tau_{ci} = \left((k_{mi} + \nu_{mi})r_{mi} + \rho_{mi} r_{mi}^3 + w_i\theta_i\right)/k_{ci}$ in which $r_{mi} = \dot{e}_{mi} + \phi_{mi} e_{mi}$ and τ_{ci} is the torque command for joint i. In sequel, we also denote σ_i and γ_{ij} given in (24) and (25) as σ_{mi} and γ_{mij}.

The interconnection acting on the motor shaft side, denoted as $\bar{\xi}$, was given by

$$\begin{aligned}\bar{\xi} &= \bar{D}(q)(\ddot{q}_{dm} + \Phi_m \dot{e}_m) + \bar{H}_m(q,\dot{q})(\dot{q}_{dm} + \Phi_m e_m) \\ &\quad + \bar{g}(q) + \bar{f}(\dot{q}_m) \\ &= W\theta + \bar{\xi}_a = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \bar{\xi}_a\end{aligned}$$

where $w_1 = [\ddot{q}_{dm1} + \phi_{m1}\dot{e}_{m1} \quad \cos q_1 \quad \ddot{q}_{dm2}]$ and $w_2 = [\ddot{q}_{dm2} + \phi_{m2}\dot{e}_{m2} \quad \ddot{q}_{dm1}\cos q_2 \quad \ddot{q}_{dm1}]$ were regression vectors for two subsystems, respectively. The estimated constant parameters were

$$\theta_1 = [\theta_{11} \quad \theta_{12} \quad \theta_{13}]^T = [J_1 + \lambda_1^2\bar{\theta}_1 \quad \lambda_2\bar{\theta}_2 \quad \lambda_1\lambda_2\bar{\theta}_3]^T$$
$$\theta_2 = [\theta_{21} \quad \theta_{22} \quad \theta_{23}]^T = [J_2 + \lambda_2^2\bar{\theta}_3 \quad \lambda_1\lambda_2\bar{\theta}_4 \quad \lambda_1\lambda_2\bar{\theta}_3]^T$$

in which $\bar{\theta}_i$ are given in Subsection 4.1. Even though the friction term $\bar{f}(\dot{q}_m)$ of PUAM has been investigated in many occasions (see, e.g., [Liu, 1991]), we treated it as a uncertainty belonging to $\bar{\xi}_a$.

4.4 Performance Evaluation

In the test, the same reference trajectories were used for all schemes. Describing in the arm link side, they were $q_{d1}(t) = 0.73\sin 1.25t$ (rad) and $q_{d2}(t) = -1.17\sin 1.89t$ (rad). We first run the PD control and tuned its gains k_{mi} and ϕ_{mi} $(i = 1, 2)$ up until the tracking errors reached their minimums without exciting robot's high frequency modes. With the well tuned gains we kept running the PD control for a while and thereafter switched to PD+NL or PD+NL+AD control for the performance comparisons. The controller parameters used in three schemes are shown in Tables 1 and 2. In PD+NL+AD, the estimation gains $\gamma_{mij} = 0.1$ for $j = 1, 2$.

Table 1. Joint 1 (shoulder) controller parameters

shoulder	$k_{m1} + \nu_{m1}$	ϕ_{m1}	ρ_{m1}	γ_{m1j}	σ_{m1}
PD	0.85	0.5	-	-	-
PD+NL	0.85	0.5	1e-6	-	-
PD+NL+AD	0.85	0.5	1e-6	0.1	0.4

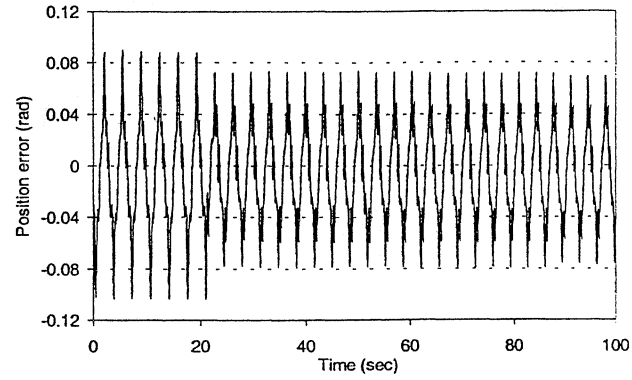

Figure 1: Position tracking errors of joint 1 (shoulder) under PD and PD+NL control.

Table 2. Joint 2 (elbow) controller parameters

elbow	$k_{m2} + \nu_{m2}$	ϕ_{m2}	ρ_{m2}	γ_{m2j}	σ_{m2}
PD	0.65	0.4	-	-	-
PD+NL	0.65	0.4	1e-6	-	-
PD+NL+AD	0.65	0.4	1e-6	0.1	1.0

Figure 1 shows the position tracking error of robot's shoulder under PD and PD+NL control while Figure 2 shows the error of elbow motion. As stated, the PD control was switched to PD+NL control at $t = 20$ second. With a fairly small gains $\rho_{mi} = 1e-6$ for the cubic feedback term shown in Tables 1 and 2, a considerable reduction on the position tracking error were obtained by PD+NL scheme for both joints. Further, Figures 3 and 4 show the profiles of position tracking errors for the shoulder and elbow under the control of PD and PD+NL+AD schemes, respectively. Initially, the arm was running by PD. At $t = 20$ second PD+NL+AD control took it over and position tracking errors reduced exponentially by about 50% for both joints.

The profiles of the $\hat{\theta}_1/k_{c1}$ and $\hat{\theta}_2/k_{c2}$ are shown in Figures 5 and 6, respectively. They all converge to bounded regions. The bounded tracking errors and parameter estimates indeed resulted in bounded control torques for both joints, as shown in Figures 7 and 8, respectively.

5 Conclusions

After giving a summary of the decentralized nonlinear and adaptive control schemes proposed in [Liu, 1999] and [Liu, 1997], this paper presented experimental results obtained by running the schemes on a PUMA 560 robot system. The results verified the effectiveness of the schemes by showing their improved tracking performances superior to the traditional PD control. The significance of the scheme lies on the fact that they can be applied to most robots without changing their hardware configurations.

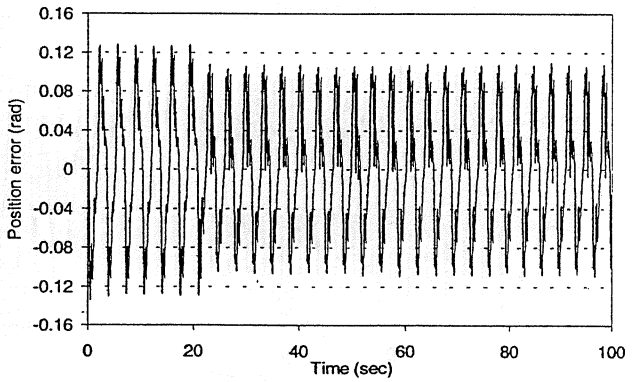

Figure 2: Position tracking errors of joint 2 (elbow) under PD and PD+NL control.

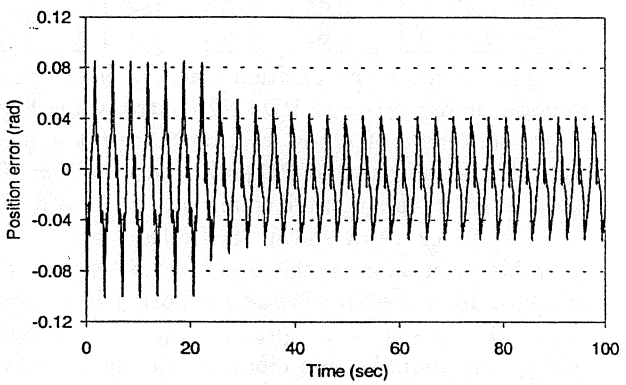

Figure 3: Position tracking errors of joint 1 (shoulder) under PD and PD+NL+AD control.

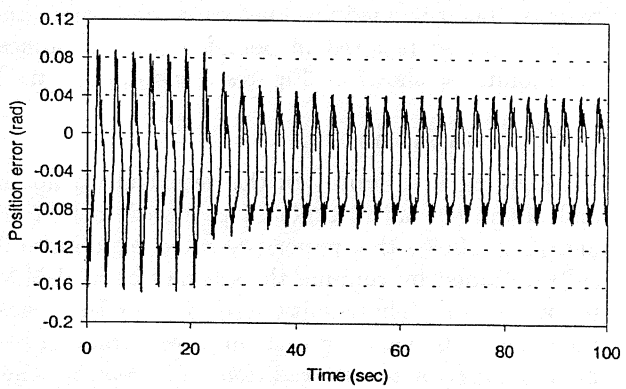

Figure 4: Position tracking errors of joint 2 (elbow) under PD and PD+NL+AD control.

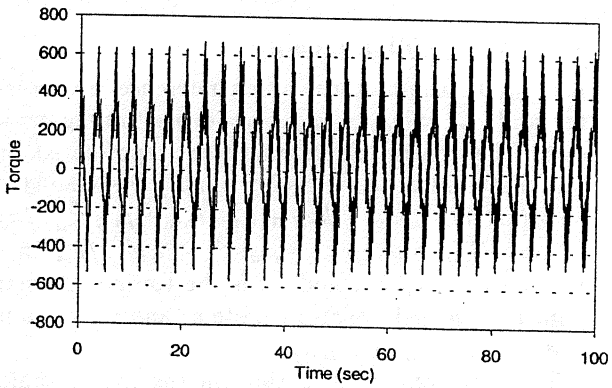

Figure 5: Computer output torque τ_{c1} in PD+NL+AD control.

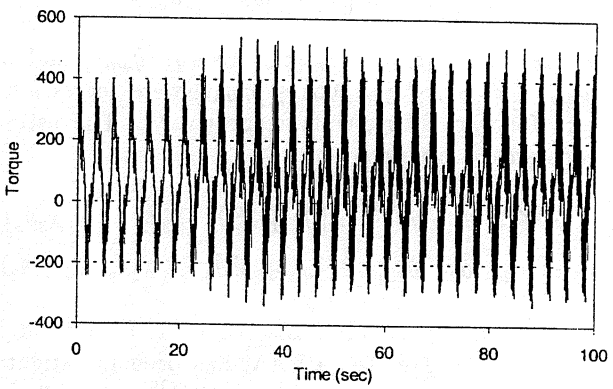

Figure 6: Computer output torque τ_{c2} in PD+NL+AD control.

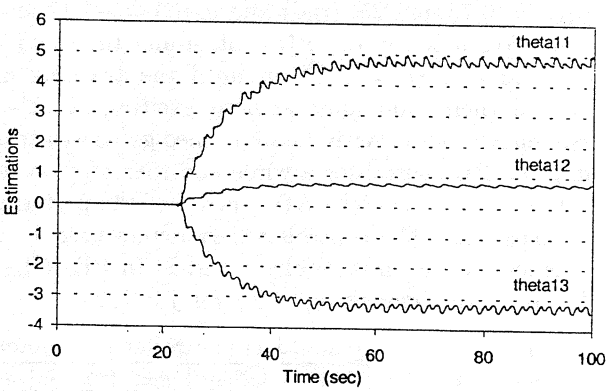

Figure 7: Profiles of $\hat{\theta}_1/k_{c1}$ in PD+NL+AD control.

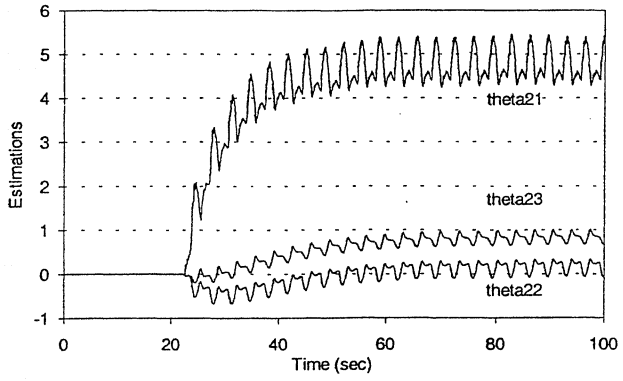

Figure 8: Profiles of $\hat{\theta}_2/k_{c2}$ in PD+NL+AD control.

References

[Arimoto and Miyazaki, 1984] S. Arimoto and F. Miyazaki. On the Stability of PID Feedback with Sensory Information. In *Robotics Research,* Cambridge: MIT Press, 1984.

[Craig, 1988] J. J. Craig. *Adaptive Control of Mechanical Manipulator,* New York: Addison-Wesley, 1988.

[Desoer and Vidyasagar, 1975] C. A. Desoer and M. Vidyasagar. *Feedback Systems: Input-output Properties.* New York: Academic Press, 1975.

[Fu and Fu, 1988] L. C. Fu and T. Y. Fu. Robust Adaptive Independent Joint Control of Robot Manipulator. In *Proc. 12th Symposium on Automatic Control,* Taiwan, 1988, pp. 363-371.

[Gavel and Hsia, 1987] D. T. Gavel and T. C. Hsia. Decentralized Adaptive Control of Robot Manipulators. In *Proc. IEEE Intl. Conf. Robotics Automat.,* 1987, pp. 1230-1235.

[Hill and Moylan, 1976] D. Hill and P. Moylan. The Stability of Nonlinear Dissipative Systems. *IEEE Trans. Automat. Contr.,* vol. AC-21, pp. 708-711, 1976.

[Ioannou, 1986] P. A. Ioannou. Decentralized Adaptive Control of Interconnected Systems. *IEEE Trans. Automat. Contr.,* vol. AC-31, pp. 291-297, 1986.

[Kawamura *et al.*, 1988] S. Kawamura, F. Miyazaki and S. Arimoto. Is a Local Linear PD Feedback Control Law Effective for Trajectory Tracking of Robot Motion? In *Proc. IEEE Intl. Conf. Robotics Automat.,* 1988, pp. 1335-1340.

[Khorrami and Ozguner, 1988] F. Khorrami and U. Ozguner. Decentralized Control of Robot Manipulators via State and Proportional- Integral Feedback. In *Proc. IEEE Intl. Conf. Robotics Automat.,* 1988, pp. 1198-1203.

[Liu, 1999] M. Liu. Decentralized Control of Robot Manipulators: Nonlinear and Adaptive Control Approaches. *IEEE Trans. on Automatic Control,* vol.44, pp. 357-363, 1999.

[Liu *et al.*, 1998] M. Liu, T. Nguyen, N. Le and E. Kwan. An Open-Structure Robot Controller. In *Proc. IFAC Sym. on 5th Low-cost Automation,* Shenyang, 1998, pp. TS10 13-17.

[Liu, 1998] M. Liu. Stability of PD/PID Control for Robot Arm Tracking: Lyapunov Analysis and Input-output Interpretations. In *Proc. 3rd Intl. Conf. Systems Science and Systems Eng.,* Beijing, 1998, pp. 462-467.

[Liu, 1997] M. Liu. Decentralized PD and Robust Nonlinear Control for Robot Manipulators. *Journal of Intelligent and Robotics Systems,* vol. 20, no. 2-4, 1997, pp 319-332.

[Liu, 1996] M. Liu. Decentralized PD and Robust Nonlinear Control of Robot Manipulators. Monash Univ. Tech. rep. ECSE-TR-96-5,1996.

[Liu, 1991] M. Liu. PUMA 560 Robot Arm Analogue Servo System Parameter Identification. *Tech. Report ASR-91-2, Mech. Dept., Melbourne Univ.,* 1991.

[Ortega and Spong, 1989] R. Ortega and M. W. Spong. Adaptive Motion Control of Rigid Robots: A Tutorial. *Automatica,* vol. 25, pp. 877-888, 1989.

[Seraji, 1988] H. Seraji. Adaptive Independent Joint Control of Manipulators: Theory and Experiment. In *Proc. IEEE Intl. Conf. Robotics Automat.,* 1988, pp. 854-861.

[Seraji, 1989] H. Seraji. Decentralized Adaptive Control of Manipulators: Theory, Simulation, and Experiment. *IEEE Trans. Robotics Automat.,* vol. 5, pp. 183-201, 1989.

[Sezer and Siljak, 1981] M. E. Sezer and D. D. Siljak. On Decentralized Stabilization and Structure of Linear Large Scale Systems. *Automatica,* vol. 17, pp. 641-644, 1981.

[Shi and Singh, 1992] L. Shi and S. K. Singh. Decentralized Adaptive Controller Design for Large-Scale Systems with Higher Order Interconnections. *IEEE Trans. Automat. Contr.,* vol. AC-37, pp. 1106-1118, 1992.

[Siljak, 1978] D. D. Siljak. *Large-Scale Dynamical Systems: Stability and Structure.* New York: North-Holland, 1978.

[Slotine and Li, 1987] J. J. Slotine and W. Li. On the Adaptive Control of Robot Manipulators. *Int. Journal of Robotic Research,* vol. 6, pp. 49-59, 1987.