# A decentralized Grid-based Random Search Algorithm for Locating Targets in Three dimensional Environments by a Mobile Robotic Network

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## Abstract

In this paper, we study the problem of locating targets in a bounded 3D space by a group of mobile robots. We present a bio-inspired decentralized grid-based random search algorithm for finding randomly located objects in 3D area. Using this algorithm, the mobile robots do the search task by moving randomly to the vertices of a common 3d covering grid from any initial position. The proposed algorithm uses some simple consensus rules for building a common covering grid also it uses a random walk search pattern drawn by a Levy flight probability distribution to do the search task. Performance of the proposed random search algorithm has been evaluated by extensive simulations and comparison to other random search methods. Furthermore, we give mathematically rigorous proof of convergence with probability 1 of the proposed algorithm.

## 1 Introduction

The problem of search in both two and three-dimensional spaces is a common task in biological and engineering systems. To do the search task a team of mobile robots equipped with sensing peripherals capability is deployed in the search area to locate targets like sea mines, black boxes from downed aircraft or ships, hazardous chemicals, fire spots in the jungle or to measure a concentration of hazardous materials [Sutantyo *et al.*, 2013; Nurzaman *et al.*, 2009; Bernard *et al.*, 2011]. Finding targets in large environments through the use of a single robot is inefficient, and using a team of mobile robots with local communication capability can succeed more quickly [Cheng and Savkin, 2009; Cheng *et al.*, 2011; Cheng and Savkin, 2011b; 2011a; 2013; Savkin *et al.*, 2015]. Moreover, the success of robot teams searching for targets where targets availability is unknown and the condition of the environment is unpredictable depends on the efficiency of the search strategy [Fricke *et al.*, 2013]. In environments where the distribution of search targets is unknown a priori or change over time randomized search strategies are more effective than deterministic search [Stephens and Krebs, 1986; Acar *et al.*, 2003].

Animals perform foraging activities when searching for food sources in nature with no knowledge of the environment and a limited sensory range. Biological creatures, marine predators, fruit flies, and honey bees who search for sources of food, for a new site and for a mate are believed to use the Levy flight random motion [Viswanathan *et al.*, 2002; Yang and Deb, 2009]. Levy flight is a random walk mechanism that has the Levy probability distribution function in determining the length of the walk. Using this random search method, the forager perform the search task by searching a small area of the space, and then jump to an area that is likely previously unexplored. This behaviour is similar to that of a foraging animal who searches for food in a given area with a series of small movements, then travels a larger distance to another area to search again. In fact, by performing Levy flight random walk mechanism, forager optimizes the number of targets encountered versus the travelled distance. The idea is that the probability of returning to the previous site is smaller compared to other random walk mechanism. The process of evolution and natural selection has led animals to optimize their foraging strategies. As a result, researchers in this new emerging area are finding much inspiration from biology.

In [Fricke *et al.*, 2013; 2015] authors proposed a search strategy for a team of robots based on the T cell movement in lymph nodes and they showed that the distributions of the step-sizes taken by T cells are best described by a random walk with the Levy-like distribution. In [Sutantyo *et al.*, 2013], a bio-inspired random search strategy was proposed for the multi-robot system to efficiently localize targets in underwater search scenarios. Furthermore, it was shown a novel adaptation strategy based on the firefly optimization algorithm can improve the Levy flight performance particularly when targets are clustered. In [Sutantyo *et al.*, 2010], authors combined the Levy flight bio-inspired search algorithm to an

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artificial potential field to improve robots dispersion in the environment to optimize search task by a team of mobile robots [Sutantyo *et al.*, 2010]. In [Keeter *et al.*, 2012], a randomized algorithm, based on the Levy flight, for locating sparse targets in a three-dimensional bounded aquatic or air environments environments was proposed.

In all of the mentioned works robots search an area using a random walk with a Levy-flight distribution. Most of the previous works were based on the assumption that targets are sparsely distributed in search areas. For many real-world search applications targets are clustered in a group such as schools of fish, underwater mines clustered in fields and human victims in disaster areas. Furthermore, some of the previous works were restricted to two dimensions, and some of them did not consider cooperation and communication amongst robots [Ghaemi *et al.*, 2009]. In this paper, we propose a novel decentralized grid-based Levy flight random search algorithm. Here, we combine the bio-inspired Levy flight random search mechanism with a 3D covering grid proposed in [Nazarzehi *et al.*, 2015] to optimize the search procedure. In previous works the Levy flight random walks took place on a continuous space but here the random walk occurs on a discrete grid. Unlike [Nazarzehi *et al.*, 2015], in this paper the mobile robots move randomly on the vertices of the covering grid with the length of the movement following a Levy flight distribution. Based on this method the Levy flight algorithm will generate the length of the movement, while the vertices of the covering truncated octahedral grid will improve the dispersion of the deployed mobile robot as a result it optimizes the search task by reducing the search time. In order to stop search and reduce the cost of operation, each robot communicate to other robots in its vicinity to broadcast information regarding detected targets to other robots within its communication range. Performance of this grid-based random search method are verified by simulations also to evaluate the performance of the proposed grid-based Levy flight algorithm we compare it to other random search strategies for locating sparse and clustered distribution of targets. Furthermore, we give a mathematically rigorous proof of the convergence of the proposed algorithm with probability 1 for any number of mobile robots and targets.

The rest of the paper is structured in the following way. Section 2 describes the problem of grid-based random search in 3D environments. The proposed algorithm is explained in Section 3. Section 4 are devoted to evaluate the proposed algorithm by simulations. Section 5 concludes this work.

## 2 Problem formulation

The objective of this paper is to design a decentralized grid-based random search algorithm to drive a group of robots for target search in a bounded 3D area. We assume that the mobile robots have no information about the the positions of the targets $T_1, T_2, ...T_l$ and search area ($M \subset \mathbb{R}^3$) , but they can detect the boundaries of the search area. Let $p_i(.) \in \mathtt{R}^3$ be

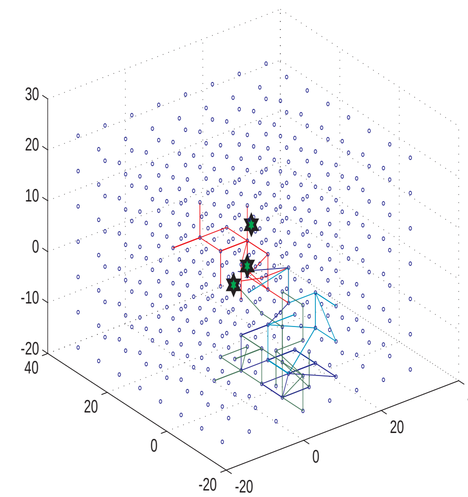



Figure 1: Vertices of the grid denoted by $o$, Mobile robots search trajectories represented by $-$ and targets by star

the Cartesian coordinates of the mobile robot $i$ and each mobile robot has a sensing range of $\mathtt{R}_s > 0$ and a communication range of $\mathtt{R}_c > \frac{4}{\sqrt{5}}\mathtt{R}_s$. We assume a spherical sensing model such that each mobile robot has a sensing range of $\mathtt{R}_s > 0$ can reliably detect any object that is located within a distance of $\mathtt{R}_s$ from the robot. In other words, sensor $i$ has the ability to identify objects in a sphere of radius $\mathtt{R}_s$ defined by:

$$\mathtt{S}_{i,\mathtt{R}_s} = \{p \in \mathtt{R}^3; \|(p - p_i(\mathtt{K}))\| \leq \mathtt{R}_s\} \quad (1)$$

We assume a spherical communication model where each mobile robot has a communication range of $\mathtt{R}_c > 0$ can reliably communicate with any mobile robots located within a distance of $\mathtt{R}_c$ from the mobile robot. It means that mobile robot $i$ has the ability to obtain information on its neighbours in a sphere of radius $\mathtt{R}_c$ defined by:

$$\mathtt{S}_{i,\mathtt{R}_c} = \{p \in \mathtt{R}^3; \|(p - p_i(\mathtt{K}))\| \leq \mathtt{R}_c\} \quad (2)$$

Our problem is to develop a distributed grid-based random search algorithm to drive the mobile robots and to efficiently spread the mobile robots in the search area for identifying all targets in a bounded 3D space. We take advantage of vertices of a 3D truncated octahedral grid proposed in [Alam and Haas, 2006; 2014]. Unlike [Alam and Haas, 2006], here we assume that there is not a common coordinate system among all mobile robots, therefore we use consensus variables to build a common coordinate system for the mobile robots. One advantage of using vertices of a truncated octahedral grid for the search is that using this grid for search leads to fewer number of vertices to be searched and this minimizes the time of search.

*Definition 2.1*: Consider a truncated octahedral grid cutting M into equal truncated octahedrons with the sides of $\frac{2R_s}{\sqrt{10}}$. Let V be the infinite set of centres of all the truncated octahedrons of this grid. The set $\hat{V} = V \cap M$ is called a truncated octahedral covering set of M (Fig.1).

At first, the mobile robots build a common covering truncated octahedral grid. Then, they perform the search task by moving randomly with a Levy flight probability distribution on the vertices of the grid from different initial locations to locate all randomly located objects in the 3D area. During the search, the mobile robot keep the information of the previously detected targets and communicates with its neighbours in the communication range at the discrete sequence of times $k = 0, 1, 2, ...$, to exchange the search information.

A target is said to be detected by a mobile robot if it is located within the sensing region of the mobile robot, which is the sphere of radius $R_s > 0$ centred at the mobile robot.

We take advantage of the notion of graph to define the relationships between neighbouring mobile robots. As a result, the vertices $i$ and $j$ of the graph $g(k)$ are considered connected if the mobile robots $i$ and $j$ are neighbours at time $k$. In the following, we impose a condition on the connectivity of the graph.

*Assumption 2.1:* There exists non-empty, an infinite sequence of contiguous, bounded time-intervals $[k_m, k_m + 1)$, $m = 0, 1, 2, ...$, such that across each $[k_m, k_m + 1)$, the graph $g(k)$ is connected [Savkin *et al.*, 2012; Cheng and Savkin, 2012].

## 3 Decentralized grid-based random search algorithm

In this section, we present a two-stage decentralized grid-based random search algorithm for finding targets in a bounded 3D area. Based on this algorithm, the mobile robots build a common 3D truncated octahedral grid. Then, they perform the search task by moving randomly on the vertices of the 3D grid using random walks with a Levy-flight probability distribution.

Levy flight is a renowned bio-inspired random search mechanism with step lengths taken from a heavy-tailed probability distribution. Using this distribution the probability of returning to a previously visited site is smaller, therefore advantageous when target sites are sparsely and randomly distributed [Viswanathan *et al.*, 2000]. To optimize the search task, we supplement the bio-inspired Levy flight random search strategy to a covering truncated octahedral grid. It means that mobile robots do the search task by moving randomly to the vertices of the covering grid with random movements with Levy-flight probability distribution. At first we assume that the robots do not have a common coordinate system and a common covering grid therefore the robots use consensus variables to build a common covering grid 3D [Nazarzehi *et al.*, 2015]. The covering grid is a set that consists of the centres of the truncated octahedrons which are used to tessellate the search area.

Vector $V_2 = (x + (2\alpha_1 + \alpha_3)\frac{2R_s}{\sqrt{5}}, y + (2\alpha_2 + \alpha_3)\frac{2R_s}{\sqrt{5}}, z + \alpha_3\frac{2R_s}{\sqrt{5}})$

represents the position of the centres of the truncated octahedrons which are used to tessellate the search area. Here, $\alpha_1, \alpha_2$ and $\alpha_3 \in Z$ and $Z$ is the set of all integers. The inputs to our algorithm are the sensing range $R_s$ and the coordinates of the point $(x, y, z)$, which act as a seed for growing the 3D grid. To build a common covering grid, we assume that each robot has consensus variables $x_i(k)$,$y_i(k)$ and $z_i(k)$ in its own coordinate system. It is obvious that, any 3D grid is uniquely defined by a point $q_i(k) = (x_i(k), y_i(k), z_i(k))$ and $R_s$. Thus, any $q_i(k)$ and $R_s$ uniquely define a 3D grid in $M$. The scalar parameters $\alpha_1$, $\alpha_2$,$\alpha_3$ and $R_s$ along with the 3D consensus variables $q_i(k) = [x_i(k) \quad y_i(k) \quad z_i(k)]$ characterize the coordinates of the vertices of the grid. The robots will start with different values of the coordination variables $x_i(0)$,$y_i(0)$ and $z_i(0)$, then eventually converge to some consensus values $x_0$, $y_0$ and $z_0$ which define a common coordinate system for the mobile robotic network.

Let $p_i(k)$ be the coordinate of the mobile robot $i$ at time $k$, and $C[q, R_s](p)$ defines the closest vertices of the 3D grid to p. The following rules for updating the consensus variables and the mobile robots coordinates are proposed:

$$x_i(k+1) = \frac{x_i(k) + \sum_{j \in N_i(k)} x_j(k)}{1 + |N_i(k)|} \quad (3)$$

$$y_i(k+1) = \frac{y_i(k) + \sum_{j \in N_i(k)} y_j(k)}{1 + |N_i(k)|}$$

$$z_i(k+1) = \frac{z_i(k) + \sum_{j \in N_i(k)} z_j(k)}{1 + |N_i(k)|}$$

$$p_i(k+1) = C[q_i(k), R_s](p_i(k)) \quad (4)$$

Where $N_i(k)$ is the set of mobile robots $j$'s neighbour ($j \neq i$ and $j \in 1, 2, ..., n$) located within its communication range and $|N_i(k)|$ is the number of neighbours at time $k$.

Based on the rules (3) and (4), each mobile robot updates its consensus variables using the average of its own and the consensus variables values of its neighbours located within its communication range.

*Theorem 3.1:* Suppose that assumptions 2.1 hold and the mobile robots move according to the laws (3), (4). Then, there exists a truncated octahedral covering set $\mathscr{V}$ such that: $\forall$ $i = 1, 2, ..., n$, $\exists$ $v \in \mathscr{V}$ ; $lim_{k \to \infty} p_i(k) = v$.

*Proof*: The proposed update law (3) and the assumption 2.1 mean that there exist $x_0$, $y_0$, $z_0$ such that:

$x_i(k) \to x_0$, $y_i(k) \to y_0$ and $z_i(k) \to z_0$.

(see [Jadbabaie *et al.*, 2003] for the proof of convergence of consensus variables to some constant values). Since $R_s$ ,$\alpha_1$ ,$\alpha_2$ and $\alpha_3$ are common among all robots, as a result:

$$(x_i + (2\alpha_1 + \alpha_3)\frac{2R_s}{\sqrt{5}}, y_i + (2\alpha_2 + \alpha_3)\frac{2R_s}{\sqrt{5}}, z_i + \alpha_3\frac{2R_s}{\sqrt{5}}) \to$$

$$(x_0+(2\alpha_1+\alpha_3)\frac{2R_s}{\sqrt{5}}, y_0+(2\alpha_2+\alpha_3)\frac{2R_s}{\sqrt{5}}, z_0+\alpha_3\frac{2R_s}{\sqrt{5}}). \quad (5)$$

This relation implies that after a while the mobile robots build a truncated octahedral grid which is common among all of them. Moreover, the rule (4) guaranties that all mobile robots move to the vertices of this grid and this complete the proof of theorem [Nazarzehi *et al.*, 2015].

In our previous algorithm robots performed the search task by moving randomly on the unvisited vertices in their neighbourhood. To optimize the search task and to minimize the time of search in this section, we propose a novel random search strategy for the mobile robots. Using this method, the robots do the search task by moving randomly to the vertices of the covering grid based on the random walk generated by the Levy flight distribution. A Levy flight random walk pattern uses a Levy probability distribution as follows [Viswanathan *et al.*, 2008]:

$$P_{\alpha,\gamma}(l) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-\gamma q^{\alpha}} cos(ql)dq \quad (6)$$

Where $\gamma$ denotes the scaling factor and $\alpha$ represents the shape of the distribution. Moreover, the distribution is symmetrical around $l$. In this paper, the Levy flight provides a random walk with the random step length is drawn from an approximated Levy distribution proposed in [Sutantyo *et al.*, 2013] as follows:

$$P_{\alpha}(l) = (l)^{-\alpha} \quad (7)$$

Where $l$ is a random number, $\alpha > 0$ defines the length of walk and $1 < \alpha < 3$. Let $\hat{\mathbb{V}}$ be the set of all vertices of the 3D covering grid and $\hat{v} \in \hat{\mathbb{V}}$ is a randomly selected element of $\hat{\mathbb{V}}$ with Levy flight probability distribution. We propose the following random algorithm for the mobile robots to search the 3D area:

$$p_i(k+1) = \hat{v} \quad (8)$$

Here we assume that the Boolean variables $b_T(k)$ defines the states of the targets at the time $k$. If target $T$ has been detected by any of the mobile robots before time $k$ then, $b_T(k) = 1$ and $b_T(k) = 0$ otherwise. Throughout search, each mobile robot communicates to other robots in the communication range at the discrete sequence of times $k = 0, 1, 2,$ to exchange the information about detected targets. The search process should be stopped after finding all targets.

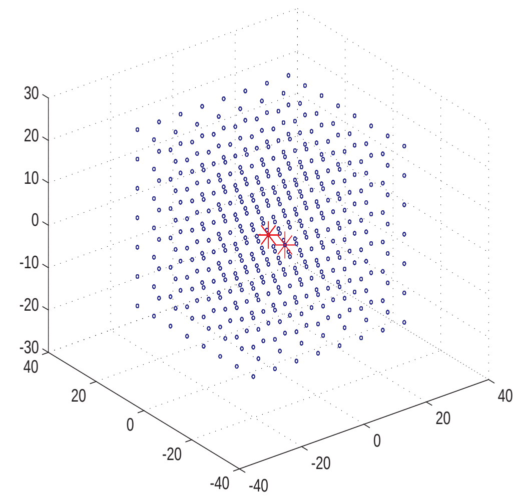
In other words,

$$p_i(k+1) = p_i(k) \quad (9)$$

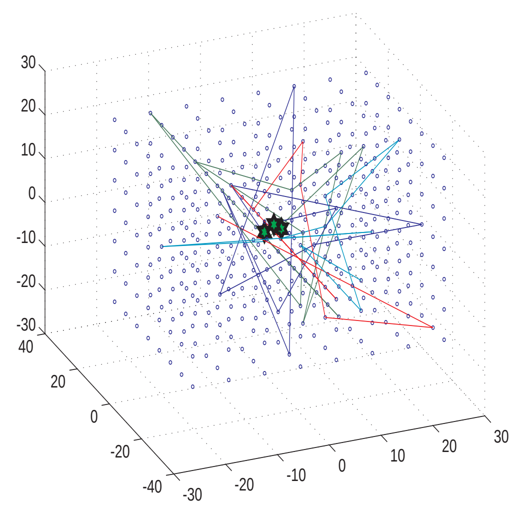
$$if \quad \forall \quad T = \{T_1, T_2, ...T_l\}, \exists \quad k, i \quad ; \|(P_T - p_i(k))\| \leq R_s$$

Where, $T = \{T_1, T_2, ...T_l\}$ and $P_T = \{P_{T_1}, P_{T_2}, ...P_{T_l}\}$, are the sets of targets and their positions, respectively.

*Theorem 3.1*: Suppose that the mobile robots move according to the law (8),(9). Then for any number of mobile robots and




(a) Mobile robots' positions after convergence to the vertices of the common covering truncated octahedral grid




(b) Mobile robots' trajectories after 15 steps

Figure 2: Trajectories of four mobile robots performing grid-based Levy flight random search for locating clustered targets. Mobile robots' trajectories denoted by -, vertices of the common grid by o, Targets by star.

Table 1: Comparison of different search styles for finding sparsely located targets

| Type of random walk | Average search time |
|---|---|
| Levy flight | 1300 |
| Grid-based Levy flight | 87 |
| Grid-based normal distribution | 125 |
| Neighboring random search | 300 |

any number of targets, with probability 1 there exists a time $k_0$ such that:

$$\forall \quad t \in T, \quad b_T(k_0) = 1$$

*Proof*: The algorithm (8),(9) describes an absorbing Markov chain that consists of a number of absorbing states (that are impossible to leave) and many transient states. Vertices where mobile robots stop are considered as absorbing states (9), also the vertices of the grid which are visited during search are considered as transient states (8). Relation (8) implies that the mobile robots randomly move to visit unvisited vertices, this continues until all targets are detected (an absorbing state). It is obvious that, from any initial state with probability 1, one of the absorbing states will be reached.

## 4 Simulation results

In this section, we conduct several simulations using MATLAB R2012b to evaluate the performance of the proposed grid-based Levy flight search algorithm for different number of mobile robots and for clustered and sparsely distributed targets in bounded three dimensional environments. At the first set of simulations, we assume that three targets are randomly distributed in the search area. The initial positions of all mobile robots and targets are generated randomly within the search area. For this case, a team of four mobile robots are deployed in the area to locate clustered targets. As shown in Fig.2(a), using the update law (3),(4), first the mobile robots build a common covering truncated octahedral grid and after that all mobile robots move to the vertices of this grid. Then, the mobile robots perform the search task by moving randomly by the random walk taken by a Levy-flight probability distribution on the vertices of the grid. Fig.2(b) shows the trajectories of the mobile robots during the search. For this case, the mobile robots have detected the targets after 15 steps.

For the second case, we assume that targets are sparsely distributed at the search environment. Fig.3 shows trajectories of the mobile robots doing the search task by moving randomly to the vertices of the common truncated octahedral grid to find the targets. For this case, the mobile robots have detected the targets after 47 steps. Simulation results in Fig.2

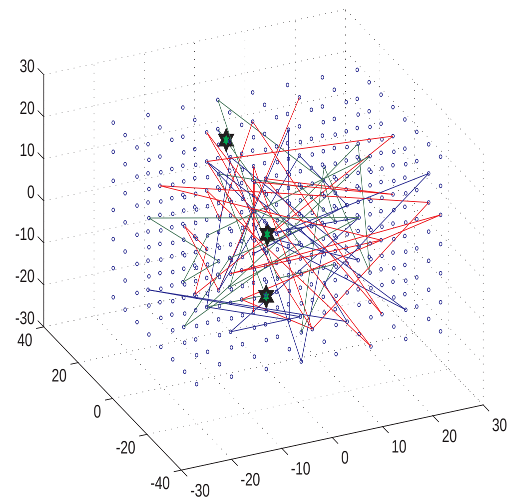
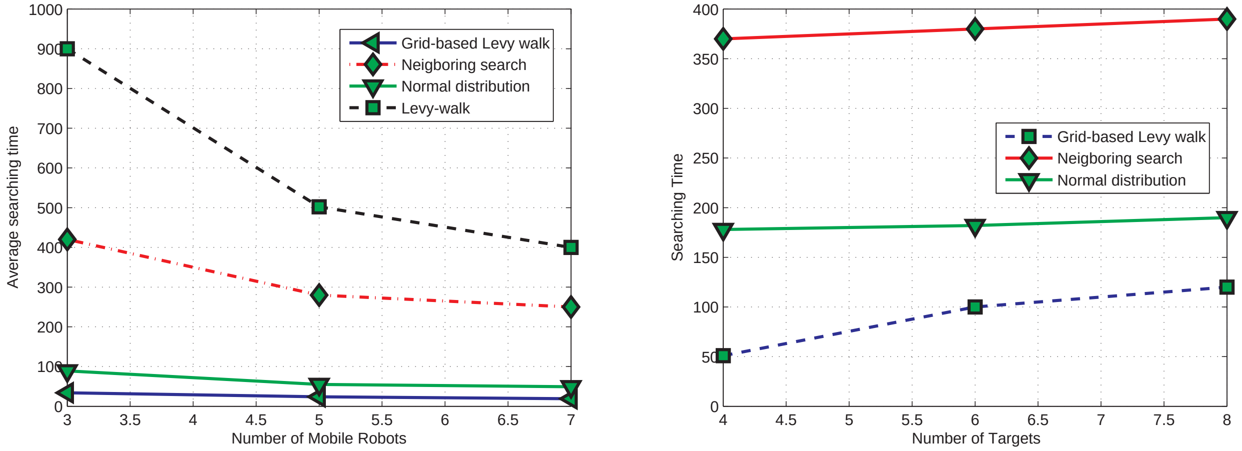


Figure 3: Trajectories of mobile robots performing grid-based Levy flight random search for locating sparsely located targets. Mobile robots' trajectories denoted by -, vertices of the common grid by o, Targets by star.

and Fig.3 show that for both cases the mobile robots have detected the given targets after a reasonable amount of time (15 and 47 steps) by moving randomly to the vertices of the common covering grid. It means that, simulations verify the effectiveness of the proposed decentralized grid-based random search algorithm for finding targets in both distributions of targets.

The next set of simulations are carried out to compare the proposed grid-based Levy flight search algorithm to the Levy flight search algorithm without using grid, our random search algorithm proposed in [Nazarzehi *et al.*, 2015] and to a grid-based random search with the step length taken by a normal distribution for detecting targets in a bounded 3D environment. In the following figures and table, for notational convenience we call the proposed grid-based Levy flight search algorithm, the Levy flight search algorithm without using grid, our random search algorithm proposed in [Nazarzehi *et al.*, 2015] and the grid-based random search with the step length taken by a normal distribution as grid-based Levy flight, Levy flight, neighboring search and normal distribution, respectively. As these algorithms are stochastic we run each simulation in 10 trials. Moreover, we compare these random search methods using different number of mobile robots and targets with different initial positions. Performance of the simulations is measured by the total steps, which is indicative of the total time spent to locate targets. Figure 4.(a) shows simulation results for locating four clustered targets in a bounded 3D area using 3,5 and 7 mobile robots and four random search methods. As shown, the proposed grid-based Levy flight method outperforms the other random search strategies for




(a) Searching time versus number of mobile robots

(b) Searching time versus number of targets

Figure 4: Comparison of random search methods using different number of mobile robots and targets

locating the targets. Figure 4.(b) shows the simulation results for locating 4, 6 and 8 clustered targets in 3D area using three mobile robots. This simulation demonstrates that using our proposed algorithm the mobile robots detected the given number of targets in the least time.
To evaluate the proposed algorithm for locating sparsely located targets, we did several simulations with different number of targets and compared our proposed grid-based Levy walk random search strategy to the others. Table I demonstrates that the proposed grid-based Levy flight strategy outperforms other random walk strategies in locating targets.

## 5 Conclusions

In this paper, we proposed an efficient grid-based random search algorithm to drive mobile robots for locating clustered and sparsely distributed targets in bounded 3D areas. The proposed method combines the bio-inspired Levy flight random search mechanism for determining the length of the walk with vertices of a covering truncated octahedral grid to optimize the search procedure. The proposed approach has the advantage that it does not need centralized control system and also it is scalable. Moreover, we showed that moving randomly on the vertices of the covering grid can minimize the time of locating randomly located objects in the search area by improving dispersion of the robots. Simulation results demonstrated that the proposed algorithm outperforms the other random walk search methods. Also, we gave mathematically rigorous proof of convergence with probability 1 of the proposed algorithm for any number of mobile robots and targets.

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