Path Planning and Force Control of a 4WD4WS Vehicle

Penglei Dai and Jay Katupitiya
University of New South Wales, Australia
p.dai@student.unsw.edu.au, j.katupitiya@unsw.edu.au

Abstract
The aim of this paper is to develop a system where forces at the wheels of a ground vehicle are controlled to guide the vehicle along a predetermined path. The 7-order Bézier curves are applied to path planning and path tracking, and provide profiles of velocities and accelerations for the mass centre of a 4WD4WS vehicle. The vehicle is considered to be a rigid body with known inertia and mass. The dynamic model used for force control is developed for the determination of the reference forces and steering angles at four independent wheels. The slip angles are also considered in the dynamic model built. The desired forces and steering angles obtained are then used to control the drive forces and the steering angles at individual wheels to guide the vehicle along the planned path. Simulation results are provided to validate the proposed methodology.

Keywords: Force control, ground vehicles, modelling, Bézier curve, path planning.

1 Introduction
One of the key complexities associated with the guidance of large scale ground vehicles that operate in semi- or unstructured environments is the significant amount of lateral and longitudinal slip present. Most methodologies developed for non-holonomic vehicles are not acceptable due to the non-holonomic constraint not being satisfied. The demand for high accuracy navigation at high speeds is ever increasing, especially in agriculture and highway maintenance such as lane marking. Developing error models or offset models to achieve accurate path tracking does not deliver the expected performance due to poor quality of the kinematic or dynamic models used. Despite the considerable efforts put towards the design of controllers using such models, the final achievements in terms of the controller performance are less than satisfactory. This work presents a platform and a methodology to achieve a more robust solution.

There is an abundance of literatures that present kinematic modelling of ground vehicles [Yalcin et al., 2006] [Tham et al., 1999] [Maalouf et al., 2006], in which the control inputs are velocities and the vehicles are subjected to low accelerations (to minimize the inertial effects) and low speeds (to minimize the radial accelerations during cornering). The majority of the kinematic models deal with non-holonomic systems - the systems that are not subjected to lateral slip. In terms of dynamics, some researchers have proposed traction control of robotic vehicles by the optimization of power consumption or ground contact forces [Iagnemma and Dubowsky, 2004] [Waldron and Abdallah, 2007]. However, the skid steering mechanism of vehicles in these studies restrict the mobility and flexibility of vehicles, especially in occasions where high speed is required. Meanwhile, as either lateral forces or lateral slips of wheels were ignored, the algorithms proposed are not applicable where the accurate path following at relatively high speed is desired.

Satisfying the non-holonomic constraint in the path tracking of off-road vehicles is very challenging due to the existence of longitudinal and lateral slips at the four independent wheels. Some methods were proposed to restrict steering angles within small ranges for minimizing the slips at the wheels [Langson and Alleyne, 1997] [Yavin, 2003]. However, this leads to limited maneuverability of the vehicle due to the restricted steering angles. In the case of kinematic models, for a chosen instantaneous centre of rotation (ICR), the steering angles of all four wheels can be geometrically determined. However, in a slip situation, maintaining this ICR becomes impossible and the desired steering angles obtained through Ackermann formula becomes unusable [Selekwa and Nistler, 2011]. An attempt to locate the ICR using the yaw rate is presented in [Connette et al., 2009]. The complexity of the controller development is evident in many works available in the literature. In this
2 Path Planning

The forces for controlling the vehicle are to be obtained using the desired path and the associated kinematic parameters along the path. In this work, Bézier curves are used to obtain such path and the desired forces.

2.1 Bézier Curves

Bézier curve was devised by Pierre Bézier in 1962 for the design of car bodies in the automobile industry [Joan-Arinyo, 1998]. Different from other kinds of 2D curves, Bézier curves only pass through the start and final control points, and the intermediate control points define the start and the final orientation and the shape of the curve as per [Zhou et al., 2011]. Bézier curve always stay within the convex hull comprised by control points and is smoother than the cubic splines.

The parametric function of a Bézier curve is:

\[ P(u) = \sum_{j=0}^{n} K_j^n(u)P_j, \quad 0 \leq u \leq 1 \]

where \( K_j^n(u) \) is a Bernstein polynomial and \( P_j \) indicates the \( j^{th} \) control point.

Bézier curves are much more easier to define in contrast to some other kinds of curves. Given a start point \( S(A_0, B_0) \) and an end point \( E(A_3, B_3) \), by selecting the control points \( C1(A_1, B_1) \) and \( C2(A_2, B_2) \), a cubic Bézier curve can be obtained and is shown in Fig. 2.

The curvature \( \kappa(u) \) at any point on the Bézier curve can be expressed in terms of the first and second derivatives of \( x(u) \) and \( y(u) \) with respect to \( u \) as in [Guechi et al., 2009].

\[ \kappa(u) = \frac{1}{R(u)} = \frac{x'(u)y''(u) - y'(u)x''(u)}{[x'(u)^2 + y'(u)^2]^{3/2}} \]

2.2 Order Determination for Bézier Curve

Due to outstanding properties of Bézier curves, many studies have been carried out in applying Bézier curves in the path planning of ground robots. K. G. Jolly used 3-order Bézier curve in Robot Soccer System [Jolly et al., 2009]. The cubic Bézier curves were also applied in the path planning of AGV and mobile robots [Petrinec and Kovacic, 2005] [Niu et al., 2008]. One significant limitation of the above studies is that only a single Bézier curve was generated to guide the robot at cornering. These methods will not be applicable when it comes to long path planning, which is normally composed by many successive straight lines and corners.

One important constraint in path planning is that the curvature along the path needs to be continuous. Ji-Wung developed path planning algorithm by 5-order Bézier curves [wung Choi et al., 2010].
As the curvature reflects the change rate of tangential angles along the path, the path defined by 5-order Bézier curves can realize the continuity of yaw rate which is essential for ground vehicles controlled by wheel velocity at kinematic level. However, in this work, force control will be applied on wheels to guide a 4WD4WS vehicle, and the angular acceleration along the path needs to be continuous accordingly. It means that, for applying Bézier curves in path planning, the order of Bézier curve segments needs to be at least increased to 7, and 8 control points should be specified for each segment.

### 2.3 Path Planning by Bézier Curve

In this work, one of the aims is to generate a number of successive Bézier curve segments. These segments are linked end-to-end to compose the desired path that has continuity in position, tangent angle and curvature for the vehicle to track while its wheels are under force control. After determining the order of the Bézier curves, there are still two issues that need to be addressed. The first issue is that, as shown in (1), the Bézier curve is parameterized by \( u \), which should be replaced by \( t \) in time domain for the vehicle to track the predefined path. The second one is that, as discussed above, 8 control points need to be specified for each segment.

\[ u(t) = \frac{\int_0^t V(t) \, dt}{S_B} \]  

(3)

where the curve length \( S_B \) can be calculated by the control points predefined. If the vehicle was supposed to complete the path with constant velocity \( v_0 \), the travel time \( t \) can be obtained by \( S_B/v_0 \). As the constant velocity \( v_0 \) has been used instead of the inherent velocity of Bézier curve characterized by the first derivative of (1), there were original errors existed between the predefined path and the tracking path under velocity profile \( v_0 \) before the implementation of control.

In this study, the main idea of introducing \( t \) into Bézier curve is to replace \( u \) by \( t/T^i \), where \( T^i \) is the time for completing the current Bézier curve segment with desired average tangent velocity \( V^i \), which is specified as \( 3m/s \) in this work, and \( t \) varies from 0 to \( T^i \). Hence, the problem becomes how to estimate segment length \( S^i \).

Fig. 4 shows an original path, in which the curvature is not continuous at junction points between straight lines and arcs. There is also sharp corner exists, which is difficult for the vehicle to track. This study will use 7-order Bézier curve segments to track the original path and achieve curvature continuity. To estimate the curve length \( S^i \), 2-order Bézier curve is applied. As shown in Fig. 4, for corner \( i \), the start point \( P_0^i \) and the end point \( P_2^i \) can be selected on straight lines around junction points, and \( P_1^i \) is the intersection of two adjacent straight
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Figure 5: Planned path with Bézier curve segments

Figure 6: Curvature of original path and planned path

(1)\[ P^i(t) = (1 - u)^2 P^i_0 + 2(1 - u)u P^i_1 + u^2 P^i_2 \] (4)

\[ T^i = \frac{S^i}{V_o} = \int_0^1 \frac{P^i_x(u)^2 + P^i_y(u)^2}{V_o} \, du \] (5)

As discussed above, for the path planning used for dynamic model, 7-order Bézier curve segments need to be built to realize the continuity of angular acceleration. After obtaining \( T^i \), the 7-order Bézier curve function with variable \( t \) can be expressed by (6).

\[ P^i(t) = \sum_{j=0}^{7} K^j_s \left( \frac{t}{T^i} \right) P^i_j, \quad 0 \leq t \leq T^i \] (6)

To determine the equation in (6), 8 control points from \( P^i_0 \) to \( P^i_7 \) need to be specified for each segment.

**Choosing 8 control points**

For segment \( i \), the start control point \( P^i_0 \) and the end control point \( P^i_7 \) can be located by the start point and desired end point within the segment. The equations for \( P^i_0 \) and \( P^i_7 \) are presented in (7).

\[
\begin{cases}
P^i_0 = P^i_{\text{start}} \\
P^i_7 = P^i_{\text{end}}
\end{cases} \rightarrow P^i_0, P^i_7 \] (7)

The control point \( P^i_1 \) can be calculated by solving the equations composed by longitudinal and lateral velocities. In this work, for every start point within segment, the longitudinal velocity \( (V_{ls}) \) should always be 0. \( P^i_1 \) can be obtained by (8).

\[
\begin{cases}
V_{ls} = V_a \\
V_{ls} = 0 \rightarrow P^i_1
\end{cases} \] (8)

By using the second and third derivatives of 7-order Bézier curve equation and making \( t = 0 \), the expressions for longitudinal acceleration \( (a_s) \) and curvature \( (C_s) \) at the start point can be obtained. Solving these two equations by defining \( a_s \) and \( C_s \) equal to 0, the control point \( P^i_2 \) can be obtained by (9).

\[
\begin{cases}
a_s = 0 \\
C_s = 0 \rightarrow P^i_2
\end{cases} \] (9)

In the same way, the expressions for angular acceleration \( (\alpha_s) \) and change rate of longitudinal acceleration \( (J_s) \) can also be deduced, and \( P^i_3 \) can be calculated by solving (10).

\[
\begin{cases}
\alpha_s = 0 \\
J_s = 0 \rightarrow P^i_3
\end{cases} \] (10)

The detailed expressions for (8), (9) and (10) are given in the Appendix I.

The last 4 control points can be calculated in the same manner as above, and the sequence for getting \( P^i_4 \) to \( P^i_7 \) is listed in (11).

\[ P_7 \rightarrow P_6 \rightarrow P_5 \rightarrow P_4 \] (11)
After computing all 8 control points for Bézier curve segments in the corners of original path, the planned path can be generated and shown in Fig. 5, in which the planned path is very close to the original path. Fig. 6 shows the curvature distribution along the original path and planned path. Obviously, the curvature becomes continuous after path planning.

To track the planned path shown in Fig. 5, the above path planning algorithm can be applied regardless of where the start point of vehicle locates. Fig. 7 shows the predefined tracking path of vehicle, in which the straight lines on planned path are tracked by 7-order Bézier curves as well.

As all Bézier curve segments have been obtained, the profiles for vehicle velocity, longitudinal and lateral acceleration, yaw rate and angular acceleration can be generated along the whole tracking path. These parameters are all continuous and shown in Fig. 8. As these reference parameters are for the mass center, to figure out the steering angles and drive forces for the control of the vehicle, the dynamic model of 4WD4WS vehicle needs to be built.

3 Dynamic Model of 4WD4WS Vehicle

The model used in this study is a 4WD4WS vehicle, which is equipped with four independent force sensors on four driving/steering modules for detecting drive forces and lateral forces acting on the wheels.

Drive forces and lateral forces acting on wheels are the two main external forces that govern the dynamics of ground vehicles, and contribute to the majority of the efforts for path tracking of vehicles. This work will focus on these two types of forces at each wheel to guide the vehicle along the planned path.

3.1 Lateral Forces

When a vehicle is cornering, tires generate appropriate lateral forces to guide the vehicle along a certain path. The lateral forces cause the tyres to deform and as a result, the actual travelling direction of the tire differs from the wheel centre plane by the slip angle [Koo et al., 2004].

The relationship between lateral forces and slip angles is illustrated by tire lateral characteristic curve in [Baffet et al., 2006]. From this characteristic curve, when slip angle is under 5 degrees, the lateral force \( F_L \) has a linear relationship with slip angle \( \alpha \):

\[
F_L = C_L \alpha, \quad -5^\circ \leq \alpha \leq 5^\circ
\]

The cornering stiffness \( C_L \) varies considerably depending on the vertical loads on the vehicle and different types of tires. Note that positive \( \alpha \) causes negative \( F_L \).

3.2 Dynamic Model of 4WD4WS Vehicle

The forces acting on the 4WD4WS vehicle are illustrated in Fig. 9. As the vehicle is considered as a rigid body, the ICR is determined by wheel velocities as well as the slip velocities. All resultant velocities are perpendicular to the lines joining those points to the ICR.

For wheel \( i \), the relationship among steering angle \( \delta_i \), slip angle \( \alpha_i \) and sideslip angle \( \beta_i \) is expressed as:

\[
\delta_i = \beta_i - \alpha_i, \quad i = 1, \ldots, 4
\]

In (13), when wheel \( i \) turn left, \( \delta_i \) and \( \beta_i \) are defined as positive values, and \( \alpha_i \) is defined as a negative value. Inversely, if wheel \( i \) was in right posture, \( \delta_i \) and \( \beta_i \) are negative, and \( \alpha_i \) is positive.

To let the vehicle travel along the desired path, the longitudinal velocity at centre of gravity (CG) should always be tangent to the path. In other words, the velocity at CG is perpendicular to the line between ICR and CG. Based on geometric relationships shown in Fig. 9,
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\[ \dot{\beta}(t) = \tan^{-1} \left( \frac{\dot{v}(t) L_r}{\dot{v}(t) L_f} \right) \]

\[ \dot{\beta}_2(t) = \tan^{-1} \left( \frac{\dot{v}(t) + \theta(t) h}{\dot{v}(t) + \theta(t) h} \right) \]

\[ \dot{\beta}_3(t) = -\tan^{-1} \left( \frac{\dot{v}(t) + \theta(t) h}{\dot{v}(t) + \theta(t) h} \right) \]

\[ \dot{\beta}_4(t) = -\tan^{-1} \left( \frac{\dot{v}(t) + \theta(t) h}{\dot{v}(t) + \theta(t) h} \right) \]

where \( \dot{\theta}(t) \) and \( V(t) \) can be obtained by path planning algorithm presented in Section 2.

By dynamic analysis, for tracking the desired path, the vehicle should meet the requirements of longitudinal acceleration \( a_t \), radial acceleration \( a_r \) and angular acceleration \( \dot{\theta} \). In this work, the system inputs are \( F_d, \delta_f \) and \( \delta_r \). \( F_d \) is the algebraic sum of rolling resistance and drive force acting on each of front wheels, and is called the residual drive force. The steering angles of front wheels are considered to be equal and denoted by \( \delta_f \), \( \delta_r \) is the steering angle of the rear wheels. The residual drive forces of rear wheels will be kept zero in this work.

The dynamic equations for 4WD4WS vehicle are listed as follows:

\[ 2F_d \cos \delta_f - \sum F_{L12} \sin \delta_f - \sum F_{L34} \sin \delta_r = ma_t \]

\[ 2F_d \sin \delta_f + \sum F_{L12} \cos \delta_f + \sum F_{L34} \cos \delta_r = mV_t^2 \sin \left( \frac{\kappa}{R} \right) \]

\[ 2F_d \sin \delta_f L_f + \sum F_{L12} \cos \delta_f L_f - \sum F_{L34} \cos \delta_r L_r + \Delta F_{L12} \sin \delta_f h - \Delta F_{L34} \sin \delta_r h = I \dot{\theta} \]

Table 1: The constant parameters of 4WD4WS vehicle

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>100</td>
<td>kg.</td>
<td>Vehicle weight</td>
</tr>
<tr>
<td>( m_d )</td>
<td>10</td>
<td>kg.</td>
<td>Driving unit weight</td>
</tr>
<tr>
<td>( I )</td>
<td>4</td>
<td>kg.m^2</td>
<td>Inertia</td>
</tr>
<tr>
<td>( ar_{max} )</td>
<td>6</td>
<td>m/s^2</td>
<td>Max. radial acceleration</td>
</tr>
<tr>
<td>( L_r )</td>
<td>0.6</td>
<td>m.</td>
<td>Length: CG to rear</td>
</tr>
<tr>
<td>( L_f )</td>
<td>0.8</td>
<td>m.</td>
<td>Length: CG to front</td>
</tr>
<tr>
<td>( h )</td>
<td>0.3</td>
<td>m.</td>
<td>Half width</td>
</tr>
</tbody>
</table>

where \( \sum F_{L12} = (F_{L1} + F_{L2}), \sum F_{L34} = (F_{L3} + F_{L4}), \Delta F_{L12} = (F_{L1} - F_{L2}) \) and \( \Delta F_{L34} = (F_{L3} - F_{L4}) \).

According to (13) and the constraints that \( \delta_1 = \delta_2 = \delta_f \) and \( \delta_3 = \delta_4 = \delta_r \), the relationship between slip angles can be expressed by:

\[
\begin{align*}
\alpha_2 &= \alpha_1 + \beta_1 + \beta_2 \\
\alpha_3 &= \alpha_4 + \beta_3 - \beta_4
\end{align*}
\]

To apply the linear relationship between lateral force and slip angle, it is assumed that all slip angles are small and specified as \( |\alpha_i| \leq 5^\circ, i = 1, \ldots, 4 \). The expression for lateral force in (12) can be substituted into (15). Considering (16) and removing \( F_d \) using (15), the relationship between \( \alpha_1 \) and \( \alpha_4 \) can be expressed as:

\[
C_L \Delta \beta_{12} \sin (\beta_1 - \alpha_1) h - C_L (2 \alpha_1 + \Delta \beta_{34}) \cos (\beta_4 - \alpha_4) (L_f + L_r) - C_L \Delta \beta_{34} \sin (\beta_4 - \alpha_4) h = I \dot{\theta} - ma_r L_f
\]

where \( \Delta \beta_{12} = (\beta_1 - \beta_2), \Delta \beta_{34} = (\beta_3 - \beta_4) \) and \( a_r = V_t^2 \sin(\kappa)/R \).

For small angles, \( \alpha_1 \) can be expressed by \( \alpha_4 \) as:

\[
\alpha_1 = C \alpha_4^2 + D \alpha_4 + E
\]

where coefficients \( C, D, E \) are given in the Appendix II.

Using the first two equations of (15), another relationship between \( \alpha_1 \) and \( \alpha_4 \) can be obtained as follows:

\[
2C_L \alpha_1 - C_L \Delta \beta_{12} + C_L (2 \alpha_1 + \Delta \beta_{34}) \cos (\beta_1 - \alpha_1) \sin (\beta_1) = ma_r (\cos \beta_1 + \alpha_1 \sin \beta_1)
\]

where \( \Delta \beta_{34} = (\beta_1 - \beta_4) \).

Considering small angles and substituting (18) into (19),

\[
F_\alpha^3 + G \alpha_4^4 + H \alpha_4 + K = 0
\]

where \( F, G, H \) and \( K \) are coefficients which contain constant parameters of the system and the kinematic parameters obtained from path generation. Expressions of \( F, G, H \) and \( K \) are listed in the Appendix II.

The value of \( \alpha_4 \) can be obtained as the smallest real root of all potential solutions to (20). Then, \( \alpha_1 \) can be
3.3 Force Sensor Model

In this work, the control of drive forces on wheels is based on the forces sensed by force sensors. Hence, the force profile acting on force sensors needs to be calculated in advance. Fig. 12 shows the forces acting on one driving unit of the vehicle, in which the force sensed by force sensor can be obtained by (21).

Figure 10: Reference steering angles and slip angles

Figure 11: Reference driven forces and lateral forces

obtained from (18), $\alpha_2$ and $\alpha_3$ can be obtained from (16), steering angles $\delta_f$ and $\delta_r$ can be calculated from (13), and the residual drive force of front wheels $F_d$ can be obtained using the first equation of (15).

To verify the feasibility of dynamic model built, simulations were carried out. Based on the constant parameters of 4WD4WS vehicle listed in Table 1, input the profiles shown in Fig. 8 into the dynamic model in (15), the reference values of steering angles and drive forces on four wheels can be calculated and shown in Fig. 10 and Fig. 11 respectively. In this work, the planned path is composed by 10 successive Bézier curve segments, in which 10 corresponding cornering stiffness $C_L$ vary from -1000 to -4200 were selected in the simulation. In the future work, $C_L$ will be obtained by measuring the lateral forces and slip angles on tires of vehicle and inputted into control system in real-time. In Fig. 10, it can be seen that all slip angles locate within the range from -5 degrees to +5 degrees. Hence, the assumption that slip angles are small is valid.

3.3 Force Sensor Model

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Figure 12: Force sensor model of 4WD4WS vehicle

$$F_s = F_d - m_d a_d$$ (21)

Where $F_d$ is the residual drive force in (15), $m_d$ and $a_d$ are the mass and acceleration of driving unit. As drive force control will be applied on the outputs of drive motors, the reference data for drive forces of motors needs to be calculated. The motor drive force $F_m$ is expressed in (22)

$$F_m = F_d + F_{rr}$$ (22)

Where $F_{rr}$ is the rolling resistance on wheel which can be calculated by equation in (23).

$$\begin{cases} F_{rr} = K_{rr} V_w & 0 \leq V_w \leq 5 \\ F_{rr} = f_{cons} & 5 < V_w \end{cases}$$ (23)

Where $V_w$ is the wheel speed, $K_{rr}$ is the linear coefficient that is specified as 0.008 in the simulation, and constant $f_{cons}$ is 0.04. Force sensor is normally modeled as a spring with Stiffness $K_s$ and damping coefficient $C_s$. In this study, the equation for force sensor model is presented in (24).

$$F_s = K_s x + C_s d x$$ (24)

Based on the force sensor model built in this section, the profile of forces acting on force sensors can be generated. Fig. 13 shows the $F_s$ sensed by force sensors on driving units. Note that $F_s$ was calculated based on the condition that the residual drive force is $F_d$ for front wheels, and is 0 for the rear wheels.

4 Controllers Design for 4WD4WS Vehicle

To guide the 4WD4WS vehicle to track the predefined path by maintaining the profiles of kinematic parameters generated, the controllers for steering angles and drive forces on wheels need to be designed. In this work,
the control parameters include $\delta_f$ (front steering angle) and $\delta_r$ (rear steering angle) shown in Fig. 10, and $F_{sf}$ (force sensed by front sensors) and $F_{sr}$ (force sensed by rear sensors) shown in Fig. 13. The forces felt by all four wheels may also differ, yet, as the force control is individually carried out at each wheel, this does not pose a problem.

### 4.1 P-Controller for Drive Forces

The output torque of DC motor is directly proportional to the electrical current in the rotor armature [Dimitry M. Gorinevsky et al., 1997]. The equation for output torque can be expressed in (25).

$$M_G = \frac{M_s}{U_c} E_{ff} u$$  \hspace{1cm} (25)

where $M_s$ is the starting torque, $U_c$ is the nominal voltage, $u$ is the voltage supplied, and $M_s/U_c$ is specified as 3.275 according to motor parameters. $E_{ff}$ is the efficiency of motor which is evaluated as 0.86. $M_G$ is the output torque which can be expressed by motor drive force $F_m$ and wheel radius $R_w$ in (26).

$$M_G = F_m R_w$$  \hspace{1cm} (26)

Considering the proportional relationship between motor torque and current supplied, P-control was applied in controller design and shown in (27)

$$\Delta u(k) = K_{Pf} (e_f(k) - e_f(k - 1))$$  \hspace{1cm} (27)

where $K_{Pf}$ is the gain of P-controller and is designed as 0.03. $e_f(k)$ denotes the error between force sensed and reference value.

### 4.2 PI-Controller for Steering Angles

The steering unit is modeled as a DC motor together with a gearbox which is used to achieve the steering motions of driving unit by the position control of motor. The position control can be realized by regulating the speed of motor based on the position error emerged. As

the output speed of DC motor has non-linear relationship with voltage, this work applied PI-control, in which I-control can be used to eliminate the steady-state deviation accumulated.

According to parameters of motor and gearbox, the system transfer function of steering unit used in this study can be obtained and presented in (28).

$$G_p(s) = \frac{\omega(s)}{U(s)} = \frac{1.785}{0.0021s^2 + 0.17s + 1}$$  \hspace{1cm} (28)

After discretization, the rotary speed of motor can be expressed by:

$$\omega(k) = 1.4302\omega(k - 1) - 0.4618\omega(k - 2) + 0.0318u(k - 1) + 0.0246u(k - 2)$$  \hspace{1cm} (29)

Given sample time $p$, the steering angle can be calculated by:

$$\delta(k) = \delta(k - 1) + \omega(k - 1)p$$  \hspace{1cm} (30)

For PI-control, the controller for steering motion can be designed as:

$$\Delta u(k) = K_{Ps}(e_d(k) - e_d(k - 1)) + K_{I_d} e_d(k)$$  \hspace{1cm} (31)
Appendix I

\[ V_{Lx} = 7 \left( \frac{1}{T} \right) (K_{1x} \cos(\theta_0) + K_{1y} \sin(\theta_0)) \]
\[ V_{Lx} = 7 \left( \frac{1}{T} \right) (K_{1x} \sin(\theta_0) - K_{1y} \cos(\theta_0)) \]
\[ a_s = 42 \left( \frac{1}{T} \right)^2 (K_{2x} \cos(\theta_0) + K_{2y} \sin(\theta_0)) \]
\[ C_s = 294 \left( \frac{1}{T} \right)^3 (K_{1x} K_{2y} - K_{1y} K_{2x})/W_{xy}^2 \]
\[ J_s = 210 \left( \frac{1}{T} \right)^3 (K_{3x} \cos(\theta_0) + K_{3y} \sin(\theta_0)) \]
\[ \alpha_s = 1470 \left( \frac{1}{T} \right)^4 (K_{1x} K_{3y} - K_{1y} K_{3x})/W_{xy} - 172872 \left( \frac{1}{T} \right)^6 \]
\[ (K_{1x} K_{2y} - K_{1y} K_{2x}) (K_{1x} K_{2x} + K_{1y} K_{2y})/W_{xy}^2 \]

Where,
\[ K_{1x} = P_{1x}^i - P_{0x}^i \]
\[ K_{1y} = P_{1y}^i - P_{0y}^i \]
\[ K_{2x} = P_{2x}^i - 2P_{1x}^i + P_{0x}^i \]
\[ K_{2y} = P_{2y}^i - 2P_{1y}^i + P_{0y}^i \]
\[ K_{3x} = P_{3x}^i - 3P_{2x}^i + 3P_{1x}^i - P_{0x}^i \]
\[ K_{3y} = P_{3y}^i - 3P_{2y}^i + 3P_{1y}^i - P_{0y}^i \]
\[ W_{xy} = 49 \left( \frac{1}{T} \right)^2 ((P_{1x}^i - P_{0x}^i)^2 + (P_{1y}^i - P_{0y}^i)^2) \]

Note that \( \theta_0 \) indicates the orientation at start point.

Appendix II

\[ A = I \ddot{\theta}_d - m \frac{V_k^2}{R} \sign(\kappa) L_f \]
\[ B = C_L (\beta_1 - \beta_2) \cos(\beta_1 h) \]
\[ C = -2 C_L (L_f + L_r) \sin(\beta_1) / B \]
\[ D = \left( -2 C_L (L_f + L_r) \cos(\beta_1 + C_L h (\beta_3 - \beta_4) \cos(\beta_4)) / B \right. \]
\[ - C_L (L_f + L_r) (\beta_3 - \beta_4) \sin(\beta_4) \]
\[ E = (C_L h (\beta_1 - \beta_2) \sin(\beta_1 - C_L h (\beta_3 - \beta_4) \sin(\beta_4) \]
\[ - C_L (L_f + L_r) (\beta_3 - \beta_4) \cos(\beta_4 - A) / B \]
\[ F = 2 C_L \sin(\beta_1 - \beta_4) C \]
\[ G = 2 C_L C_L \sin(\beta_1 - \beta_4) (2 - 2D - (\beta_3 - \beta_4) C) \]
\[ - ma_c \sin(\beta_1 - ma_C \cos(\beta_1) \]
\[ H = 2 C_L D + 2C_L \cos(\beta_1 - \beta_4) \]
\[ - C_L \sin(\beta_1 - \beta_4) (\beta_3 - \beta_4 - 2E - \beta_3 D + \beta_4 D) \]
\[ - ma_c \sin(\beta_1 - ma_d \cos(\beta_1) \]
\[ K = 2 C_L E - C_L (\beta_1 - \beta_2) + C_L \cos(\beta_1 - \beta_4) (\beta_3 - \beta_4) \]
\[ + C_L \sin(\beta_1 - \beta_4) (\beta_3 - \beta_4) E - ma_c \cos(\beta_1) \]
\[ - ma_e E \sin(\beta_1 + ma_c \sin(\beta_1 - ma_c E \cos(\beta_1) \]

5 Conclusion

The paper has presented a method that plan the path with curvature continuity by 7-order Bézier curve segments. In the same way, the tracking segments can be generated to provide the profiles of kinematic parameters. Considering the lateral forces acting on wheels, the dynamic model used for independent force control was built, and the reference drive forces and steering angles for each segment can be calculated by inputting the obtained kinematic profiles into this model. Finally, P-controller and PI-controller were designed for the drive forces and steering angles of four independent wheels. The relevant simulation results show that force and position controllers can be implemented at each of the wheels independently and the accurate navigation of the vehicle can be achieved. In future work, methods will be developed to estimate the slip stiffness in real-time, and experiments will be carried out on the 4WD4WS vehicle developed at authors’ laboratories to verify the validity of methods proposed in this work.
Note that $\ddot{a}_d$ is the desired acceleration from path generation.

References


