Towards Large-scale Occupancy Map Building using Dirichlet and Gaussian Processes

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Abstract
This paper proposes a new method for building occupancy maps using Dirichlet and Gaussian processes. We consider occupancy map building as a classification problem and apply Gaussian processes. The main drawback of Gaussian processes, however, is the computational complexity of $O(n^3)$ related to the matrix inversion, where $n$ is the number of data points.

To enable large-scale occupancy map building, we propose to use Dirichlet process mixture models which cluster input data without fixing the number of clusters a priori and to apply a mixture of Gaussian processes for the clustered data. This approach also has an advantage of dealing with local discontinuities better than one global Gaussian process model. Simulation results will be provided demonstrating the benefits of the approach.

1 Introduction
Mapping is a fundamental problem of mobile robots which have to navigate through the environment, but generating an accurate and informative environmental map is still a challenging problem both theoretically and practically.

Since occupancy grid maps [Moravec and Elfes, 1985] have been developed, they have been widely used for mapping with various sensors such as sonar sensors, laser range finders and stereo cameras [Thrun et al., 2006; Murray and Little, 2000]. The popularity of occupancy grid maps owes to its simplicity and accuracy compared with other map representations such as feature maps and topological maps.

At the core of its simplicity, however, there exists a strong assumption that the occupancy of a grid cell is independent of neighboring cells’. Thus, many researchers have tried to relax the strict assumption and to enhance the accuracy, for example, by utilising forward sensor models [Thrun, 2001]. Another drawback of occupancy grid maps is that the map resolution should be fixed a priori, and the occupancy is calculated on the discretised input space, which cannot provide exact occupancy for arbitrary positions.

Recently, some researchers consider occupancy map building as a classification problem and applied Gaussian processes [O’Callaghan et al., 2009]. By doing that, they were able to hold the dependency between observations and to expand the map into a continuous space.

However, the computational complexity of matrix inversion during training and inference of Gaussian processes is $O(n^3)$, where $n$ is the number of data points. Thus, it is not applicable for large amount of data which is common in robotic applications. Moreover, global hyperparameters are poor to deal with input-dependent properties such as local discontinuities of occupancy in the environment.

Therefore, in this paper we propose a new method to build occupancy maps using a mixture of Gaussian processes [Tresp, 2001]. Particularly, we apply Dirichlet process mixture models [Neal, 2000] to cluster input data without fixing the number of clusters in advance and apply Gaussian processes to each cluster.

The benefits of this approach are two-folds. First, thanks to the reduced size of each clustered data, the computational complexity for building occupancy maps is dramatically decreased. Second, the accuracy of occupancy maps is enhanced due to using a mixture of local models rather than a global one.

We will provide experimental results using simulation data in 2D with a laser range finder. However, notice that our method is not restricted to that specific case, but can be extended to 3D occupancy mappings using relevant sensors such as stereo cameras.

The structure of the paper is as follows. In Section 2 related work will be summarised. We outline three steps of our occupancy map building method in Section 3 and show experimental results in Section 4. We conclude the paper with future work in Section 5.
2 Related Work

A Gaussian process [Rasmussen and Williams, 2006] is a collection of random variables, any finite number of which have joint Gaussian distributions. Since it is a Bayesian non-parametric model, overfitting and model selection problems can be avoided and thus, it is often used for non-linear regression and classification.

Recently, some researchers viewed building elevation maps as a regression problem and applied Gaussian processes [Lang et al., 2007; Hadsell et al., 2010]. They are, however, unable to discriminate vertically overlapping objects such as tunnels and bridges because basically they estimate the height of the surface at each point of the ground which is thus called a 2.5D map.

For full 3D map representation, O’Callaghan et al. proposed occupancy map building using Gaussian process classification [O’Callaghan et al., 2010]. They used laser hit points as occupied points and discretised laser beam segments for unoccupied points, which are stored in different kd-trees.

In order to reduce the size of unoccupied points they only used the perpendicular feet on the near laser beam segments which are found with the k-nearest unoccupied points to the query points. Similarly, they only used k-nearest occupied points to the perpendicular feet. The main drawback of this approach is that the sampled training data are unique for each query point and thus, a new covariance matrix should be generated and inverted each time a query point is evaluated.

Later, they have addressed the problem of discretising laser beam segments into unoccupied points by introducing integral kernel [O’Callaghan and Ramos, 2011]. They integrated the point-to-point kernel over the parameterised line for the line-to-point kernel, and double integrated for the line-to-line one. By doing that, they were able to use laser beam segments as they are and thus, to reduce the size of data and reuse the same covariance matrix for every query point. However, the number of data is still so large that applying the method for large scale mappings is not feasible.

In the machine learning community, the Mixture of Experts [Jacobs et al., 1991] scheme is commonly used as a divide-and-conquer strategy. With this concept, a mixture of Gaussian processes [Tresp, 2001] has been proposed to reduce the size of data and improve the performance. However, the number of Gaussian process experts should be fixed in advance.

3 Occupancy Mapping using a Mixture of Gaussian Processes

Our method for occupancy map building consists of three steps. First, we cluster input data into several groups by using Dirichlet process mixture models. For each cluster we apply a Gaussian process and build a local occupancy map based on its own observations. Finally, local occupancy maps are merged into one by using a mixture of Gaussian processes. Each step will be explained in detail in the following subsections.

3.1 Clustering Data via Dirichlet Process Mixture Models

In order to cluster data, we apply Dirichlet process mixture models which is a nonparametric Bayesian approach to clustering. The major advantage of this method is that we do not need the number of clusters before clustering like k-means clustering.

Given $n$ observations $\{x_i\}_{i=1}^{n}$, we assume that $x_i$ belongs to the component $z_i$ whose distribution $F$ is parameterized with $\theta_{z_i}$. Each parameter $\theta_k$ is drawn independently and identically from a distribution of parameters, $G$ which has a Dirichlet process (DP) prior [Ferguson, 1973];

$$x_i \mid z_i, \{\theta_k\} \sim F(\theta_{z_i})$$

$$\theta_k \mid G \sim G$$

$$G \mid \alpha, G_0 \sim DP(\alpha, G_0),$$

where $\alpha > 0$ is the concentration parameter which determines the variance of the Dirichlet process, and $G_0$ is the prior distribution over the component parameters $\theta$.

The probability for assigning an observation $x_i$ to either an existing component $z_k$ or a new one $z_{new}$, given other component assignments $z_{-i}$ is

$$p(z_i = z_k \mid z_{-i}) = \frac{n_{-i,k}}{n - 1 + \alpha}$$

$$p(z_i = z_{new} \mid z_{-i}) = \frac{\alpha}{n - 1 + \alpha},$$

where $n_{-i,k}$ denotes the number of instances assigned to component $z_k$ excluding $x_i$.

Here, we set $F$ as a Gaussian distribution so that the parameters are a mean vector and a covariance matrix, $\theta_k = \{\mu_k, \Sigma_k\}$. We set $G$ to the Gaussian-Wishart conjugate prior distribution and $\alpha$ to 1. Note that since the joint distribution $p(x, z)$ is analytically intractable, we apply Gibbs sampling and take the configuration as final when it converges.

The laser beam segments acquired from a laser range finder can be categorized as return (the measured range is less than the maximum) or no-return (otherwise), and the end point is called a laser hit point when it is returned.

Thus, the observation data are laser beam segments and laser hit points which are considered as an empty space and occupied point, respectively. Note that a laser beam segment will be associated with a target value of +1 and a laser hit point with −1 for supervised learning in Section 3.2.
Here, we first cluster laser hit points via Dirichlet process mixture models and add laser beam segments with return to the same cluster of corresponding laser hit points. Laser beam segments with no-return are added to the clusters which have the most similar laser beam segments with return. The similarity between laser beam segments is defined as an integral kernel which will be discussed in Section 3.2.

### 3.2 Building Local Occupancy Maps via Gaussian Processes

We model the occupancy map, \( f(x) \) with Gaussian noise \( \varepsilon \) with variance \( \sigma_n^2 \) for regression;

\[
y = f(x) + \varepsilon, \quad \varepsilon \sim N(0, \sigma_n^2)
\]  

(3)

Given \( n \) observations \( \{(x_i, y_i)\}_{i=1}^n \), a Gaussian process with its zero mean function and covariance function \( k(x, x') \) assumes the joint Gaussian distribution of the observed target values \( y \) and the function values \( f \) of query points \( X_q \):

\[
\begin{bmatrix} y \\ f \end{bmatrix} \sim N \left( \mathbf{0}, \begin{bmatrix} K(X, X) + \sigma_n^2 I & K(X, X_q) \\ K(X_q, X) & K(X_q, X_q) \end{bmatrix} \right)
\]  

(4)

The conditional distribution of Eq. 4 gives the predictive equations in closed form for Gaussian process regression;

\[
E(f_q) = K_q^* (K_q + \sigma_n I)^{-1} y, \\
V(f_q) = K_{q*} - K_q(K_q + \sigma_n I)^{-1} K_q^*,
\]  

(5)

where \( K_q = K(X_q, X) \), \( K_{q*} = K(X_q, X_q) \) and \( K^* = K(X, X_q) \).

In case of classification, however, the posterior becomes non-Gaussian and exact analytical inference is intractable. Therefore, approximateimations on non-Gaussian posteriors can be applied such as Laplace approximation [Williams and Barber, 1998] and Expectation Propagation [Minka, 2001]. Here, we use the latter.

For point-to-point similarity between two laser hit points, we use the squared exponential covariance function,

\[
k(x, x') = \sigma_f^2 \exp \left( -\frac{1}{2} \sum_{d=1}^{D} \frac{(x_d - x'_d)^2}{l_d^2} \right).
\]  

(6)

For line-to-point and line-to-line similarity, we apply integral kernel which integrates a point-to-point kernel. Thanks to the integral kernel, it is allowed to use laser beam segments instead of discretising them into points, thus reducing the data size dramatically.

The integral kernels for line-to-point and line-to-line similarity are

\[
k_I(l(u), x) = \int_0^1 k(l(u), x) du, \quad k_{II}(l(u), l'(v)) = \int_0^1 \int_0^1 k(l(u), l'(v)) du \, dv,
\]  

(7)

(8)

where \( l(u) \) and \( l'(v) \) are line segments parameterised by \( u \) and \( v \). Eq. 8 does not have a closed form formula and thus we applied Simpson quadrature [Gander and Gautsch, 2000] to numerically evaluate the integral.

### 3.3 Merging Local Maps via Mixture of Experts

The Mixture of Experts model consists of several expert networks and one gating network both having access to the input vector \( x \). The output of a Mixture of Experts \( y(x) \) is the weighted mean of the expert outputs \( y_j(x) \),

\[
y(x) = \sum_{j=1}^{M} p_j(x) y_j(x),
\]  

(9)

where \( p_j(x) \) is the probability that the input \( x \) is attributed to the expert \( j \) and \( M \) is the number of experts.

The mixture of Gaussian processes applies another Gaussian process classification to infer the probability that the query point \( x \) belongs to each cluster.

One might consider a Bayesian committee machine [Tresp, 2000] as an alternative for merging local maps. It combines different kinds of estimators which are trained on different data sets. In case of using Bayesian committee machine the expectation and variance given all data are factorised as

\[
E(f_q | X, y) = \sum_{i=1}^{M} \sum_{i=1}^{M} V(f_q | X_i, y_i)^{-1} E(f_q | X_i, y_i),
\]  

(10)

\[
V(f_q | X, y) = \left( \sum_{i=1}^{M} \sum_{i=1}^{M} V(f_q | X_i, y_i)^{-1} \right)^{-1}.
\]

In Section 4.4 occupancy maps generated with a mixture of Gaussian process and a Bayesian committee machine will be compared.

### 4 Experimental Results

#### 4.1 Simulation Data

The robot used for simulation is equipped with a laser range finder which sweeps 180 degrees with 17 beams and of which maximum range is 8m. It scanned the 22m × 18m environment at 26 different poses. Totally, 696 observations are obtained; 254 laser hit points, 254 laser beam segments with return, and 188 laser beam segments with no-return.
The data obtained from simulation are shown in Fig. 1. The red circles stand for robot’s poses, black line segments for laser beam segments, and blue asterisks for laser hit points on the wall.

### 4.2 Clustered Data

The clustered data by Dirichlet process mixture model are shown in Fig. 2. It is clustered as 8 groups which are distinguished with different colors. The number of data in each cluster is summarized in Table 1, where H stands for laser hit points, R for laser beam segments with return, and N for laser beam segments with no-return.

In order to check the quality of clustering we drew a pair-wise similarity matrix for all observations in Fig. 3 where the darker is the higher similarity. In Fig. 3a repeated peaks with a period of 17 are found in each row. This is because a laser scan is similar to the next one. Note that another robot trajectory started from the bottom in Fig. 1, and this caused separate similarities on the last few rows. On the other hand, in Fig. 3b dark rectangles are found along the diagonal. But there still exist correlations between clusters. This is because each region of clusters are partially overlapped.

### 4.3 Local Occupancy Maps

Local occupancy maps are generated by local models which trained with clustered data. Occupancy values are predicted at every 20cm in the environment. Fig. 6 shows local occupancy maps with their clustered data and uncertainty for some clusters. It is found that local occupancy maps fit clustered data very accurately.

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**Table 1: The Number of Data in Each Cluster (H, R and N stand for laser hit points, laser beam segments with return and no-return, respectively)**

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<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
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<th>5</th>
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<td>41</td>
<td>44</td>
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<td>51</td>
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<td>145</td>
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<td>115</td>
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</table>
4.4 Final Occupancy Map

The final occupancy map is built by merging local maps. For comparison we built occupancy maps in three ways; a mixture of Gaussian processes (our method), Bayesian committee machine, and single Gaussian process (without clustering). Each occupancy map is shown in the first row of Fig. 4. The second row depicts the uncertainty of the corresponding occupancy maps.

Fig. 5 depicts Receiver Operating Characteristic (ROC) curves of three occupancy maps. From Fig. 4 and 5, it can be said that the accuracy of a mixture of Gaussian processes (our method) is as good as that of single Gaussian process (previous method) and is better than Bayesian committee machine. It seems that the reason our local models is a little bit lower in performance than the global one is because of some mis-clustering and the simplified method for calculating weights of each experts to query points.

However, the computational time of our method is much faster as described in Table 2. A single Gaussian process takes about 320 minutes to generate a occupancy map, but a mixture of Gaussian processes about 40 minutes which is about 8 times faster. It is measured on a computer with a Intel Core 2 Duo 3.0 GHz CPU and 3.25 GB RAM.

5 Conclusions

In this paper, we proposed a new method to build occupancy maps using a mixture of Gaussian processes. Particularly, we clustered training data using Dirichlet process mixture models which enabled to reduce the data size in each cluster, and thus reducing the computational complexity significantly. Another benefit of our approach is that the local models can better capture the local properties, such as terrain variations, than one global model.

We demonstrated our method with simulation data showing that our method is applicable for large data size without loss of accuracy. Experiments with real data captured in 3D would be more impressive.

One of the limitations of our approach is that the clustering and training parts are separated. Trained Gaussian process experts can be used again to cluster data and thus, the procedures iterate until converge. This kind of combined approach would generate more accurate occupancy maps without clustering errors, even though the computation becomes more complex.

Another drawback of a mixture of Gaussian processes is to apply a Gaussian process classification to calculate the relevancy of a query point to each cluster. That is a global model which is against the objective of this pa-
Figure 5: Receiver Operating Characteristic (ROC) of occupancy maps built by three different methods

Table 2: Computational time to learn hyperparameters and infer occupancy values of query points in minutes where GP and MGP stand for Gaussian process and mixture of Gaussian process (our method), respectively

<table>
<thead>
<tr>
<th></th>
<th>Learning</th>
<th>Inference</th>
<th>Total</th>
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<tr>
<td>GP</td>
<td>312.8</td>
<td>7.4</td>
<td>320.1</td>
</tr>
<tr>
<td>MGP</td>
<td>36.9</td>
<td>2.1</td>
<td>39.1</td>
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The authors will address both limitations by using infinite mixture of Gaussian processes [Rasmussen and Ghahramani, 2002] in the future work.

6 Acknowledgement

The authors wish to thank Edwin Bonilla for his comments and Simon T. O'Callaghan for his simulation codes.

References


Figure 6: Clustered data (laser hit points and laser beam segments) and local occupancy maps with uncertainty for each cluster where red and blue colors represent high and low values, respectively.