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A Study of Soft Contact Models in Simulink

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Abstract

This paper investigates the use of different soft contact and friction models, in Simulink, in the context of legged locomotion. The main criterion for assessing the models is the number of integration steps that are taken using variable step integration methods. The variables that are considered for the comparisons are the type of contact and friction models, the integration method, the desired integration accuracy, and mass of the colliding body.

1 Introduction

Foot-ground interaction is an important part of all legged machine simulations. Generally, this interaction can be studied in terms of the normal and tangential forces (friction) that act on a body while making contact with the ground. Among the two general types of normal force models, i.e., rigid models and compliant models [Gilardi and Sharf, 2002], the performance of compliant contact models are investigated here, on the grounds that many contact phenomena involve a hard and a soft body or two soft bodies, and rigid body models are inherently incapable of describing accurately the phenomenon of compliant contact behaviour [Brach, 2007; Kim, 1999]. Other advantages of compliant models over the rigid ones are that they allow the inclusion of more accurate friction models in the contact surface [Ciavarella, 1998; Lim and Stronge, 1999] and can also cover multi-contact conditions [Riley and Sturges, 1996; Brach, 2007; Meriam *et al.*, 1987].

The main criterion for assessing the models is the number of integration steps that are taken using variable step integration methods in Simulink, MATLAB. By varying the step size during the simulation, variable-step solvers reduce the total number of steps in the simulation, hence reducing the simulation time. Simulink achieves this by reducing the step size to increase accuracy when a model's states are changing rapidly, and increasing the step size to avoid taking unnecessary steps when the model's states are changing slowly. If a contact or friction model causes Simulink to take small steps then it slows down the whole simulation; and the same would be true for any other simulator that uses variable-step integration.

Three of the main normal force models for soft contacts are described in the next section and are compared against each other based on this criterion. All the models and their corresponding parameter values are obtained from the literature. The effect of other variables such as integration method, integration accuracy, mass of the colliding body and parameters of the selected normal force model on the integration step size are also investigated. These are all done by simulating a point mass with one degree of freedom in the vertical direction colliding with the ground. Further in the paper, friction force is added to the contact force and its effect on the integration step size is investigated. Two general types of friction force models are described in the next section which are combined with the normal force models later on. Simulations for both a point mass with two degrees of freedom, and a rigid bar with three degrees of freedom (general planar motion) colliding with the ground are carried out and compared against each other in terms of integration steps taken. All models and simulations are in 2D.

2 Background

In this section, the description and formulation for the soft contact normal force and friction force models that are used in this paper are presented.

Normal force

In this context, we only deal with explicit normal force models, i.e., contact models in which normal contact force is described as an explicit function of local indentation x and its rate $\dot{x}$, formalized by [Gilardi and Sharf, 2002; Brach, 2007]:

$$F_n \equiv F_n(\dot{x}, x) = F_{\dot{x}}(\dot{x}) + F_x(x). \quad (1)$$

Three such compliant contact models have been considered here. The first one is the Linear Spring-Damper with one spring [Goldsmith, 2001] (Figure 1), given as:

$$F_n = b\dot{x} + kx, \tag{2}$$

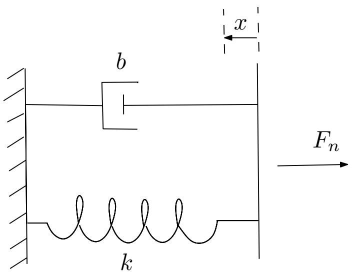


Figure 1: Linear S-D

Being one of the simplest normal force models, this model has a number of drawbacks [Marhefka and Orin, 1999; Gilardi and Sharf, 2002]: the contact force is not continuous at the beginning of impact due to the form of the damping term; and at the point of separation, a negative force holding the objects together is present and causes a sticking effect. Furthermore, the equivalent coefficient of restitution obtained for this model does not depend on impact velocity, which is against empirical results of [Goldsmith, 2001]. We can deal with the sticking effect by adding a state variable and monitoring it to detect sticking [Featherstone, 2008] (Figure 2). The equations governing this model (referred to as Linear S-D 1 throughout this paper) are:

$$F_n = \begin{cases} 0 & \text{if } x > z \\ max(0, kz + b\dot{z}) & \text{if } x = z \end{cases} \tag{3}$$

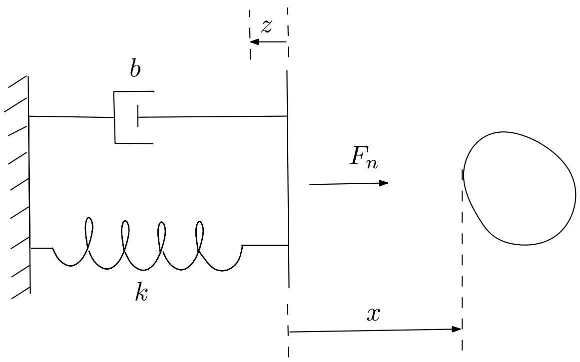


Figure 2: Linear S-D 1

The second model, which is still linear but with more complexity, is the Linear Spring-Damper with two springs, which is referred to as Linear S-D 2 throughout the paper (Figure 3). As opposed to Linear S-D 1, this model has the advantage of having a continuous contact force due to the existence of the extra series spring. The equations governing this model are given as:

$$F_n = k_2(x - y), \tag{4}$$

where the value of y is the solution to the differential equation:

$$b\dot{y} + k_1 y - k_2(x - y) = 0. \tag{5}$$

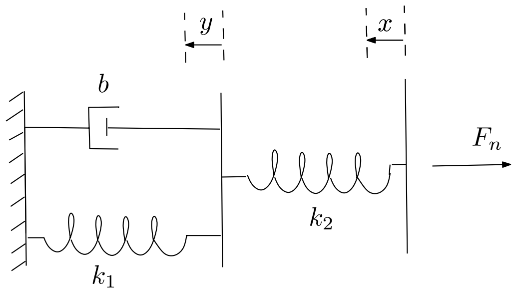


Figure 3: Linear S-D 2

The third model, which was developed by Hunt and Crossley [Hunt and Crossley, 1975], uses a Nonlinear Spring-Damper. The equation governing this model (which is also described in the works of Marhefka and Orin [Marhefka and Orin, 1999] and Lankarani and Nikravesh [Lankarani and Nikravesh, 1990]) is given as:

$$F_n = bx^n\dot{x} + kx^n. \tag{6}$$

The advantage of this model over the linear one is that while the computational requirements are similar, the contact forces evolve continuously with time, they are able to handle impacts in the sense that the equivalent coefficient of restitution is dependent on the impact velocity, and moreover, no sticking effects are present in these models.

Friction force

Among the tangential force (friction) models, we have investigated the performance of two general types [Olsson *et al.*, 1998; Armstrong-Helouvry *et al.*, 1994]. The first type has two separate equations to describe the two friction regimes of sticking and sliding. The model used in this paper is a combination of Coulomb friction and a Linear Spring-Damper where the former captures the sliding and the latter the pre-sliding regime [Featherstone, 2008] (p. 239). It is expressed as:

$$F_f = \begin{cases} \mu F_n & \text{if } f_{sick} < -\mu F_n \\ \mu F_n & \text{if } f_{sick} > \mu F_n \\ f_{stick} & \end{cases} \tag{7}$$

where,

$$f_{stick} = -k_t x - b_t \dot{x} \tag{8}$$

and k_t and b_t are the spring and damping coefficients in the tangent direction, and μ is the coefficient of kinetic friction.

The second type uses a single equation to represent both the sliding and pre-sliding regimes. Examples of such a model are the Dahl [Dahl, 1968], the LuGre [de Wit *et al.*, 1995], and the Bliman-Sorine [Bliman and Sorine, 1991] models. For example, the Dahl model is described as:

$$\begin{aligned} F_f &= \sigma_0 z, \ \sigma_0 > 0, \\ \dot{z} &= \dot{x}(1 - \frac{\sigma_0}{F_c} sgn(\dot{x}) z) \end{aligned} \tag{9}$$

where F_f is the friction force, σ_0 is the contact stiffness, $z(t)$ specifies the state of elastic strain in the frictional contact (bristle deflection), x is the rigid body displacement, and F_c is the Coulomb's friction term.

Due to the dependence of the state variable on the normal force, in applications such as legged robot simulations, these models cannot be used in their original form (e.g. as given in Eq. (9)). In fact, the differential equation Eq. (9) will have singularities when the normal force is near zero. A remedy to this problem is to modify these models so that the differential equations would represent the coefficient of friction rather than the friction force itself, which results in a differential equation for the state variable that is independent of the normal force. An example of such a model is an adapted version of the Bliman-Sorine model expressed as [Wight, 2008]:

$$\begin{aligned} \dot{u} &= \frac{-3|\dot{x}|}{s_p} u + \frac{3 f_k}{s_p} \dot{x} \\ F_f &= u\, F_n, \end{aligned} \tag{10}$$

where $\dot{x}$ is the tangential velocity of the contact point relative to the surface, f_k is the coefficient of kinetic friction, s_p is a rough estimation of the displacement before saturation of the friction occurs, and u is the state variable representing the coefficient of friction. This is the model we shall use in this paper.

3 Results

To derive our results, we ran several sets of experiments with incremental addition of complexity. In the first set of experiments, we investigate only the three normal contact force models described in Section 2. This is done by simulating a point mass falling in the vertical direction with an initial velocity of zero and a height of 0.3 m colliding with the ground (a height of 0.3 m would be a realistic initial condition for the leg of a robot). Variables that can be changed in this setting are: the integration method, the integration accuracy (controlled by relative and absolute tolerance in Simulink), mass of the colliding body, and the ground model. Throughout this paper, wherever the values of these variables are not mentioned, the default values are assumed, i.e., integration method: ODE 45, integration accuracy: 1e-3, ground model: Linear S-D 1 with parameter values as shown Table 1, initial velocity: 0 m/sec, initial height: 0.3 m, colliding mass: 2 kg, simulation time period: 2 sec.

Integration method and accuracy

In this part, the effects of integration accuracy and integration method on the integration step size are investigated for a point mass of 2 kg falling from a height of 0.3 m with an initial velocity of zero colliding with a surface which is represented by the Linear S-D 1 model with parameter values given in Table 1. There are two parameters than can be altered to change the integration accuracy in Simulink: the relative and the absolute tolerance. Table 2 shows the effect of changing both these parameters at the same time on the integration step count for three experimental runs, two integration methods (ODE45, and ODE23s [Shampine and Reichelt, 1997; Dormand and Prince, 1980; Bogacki and Shampine, 1989]), and for two ground stiffness values (5×10^4 and 5×10^5 N/m). ODE45 is an ordinary integration method whereas ODE23s is an integration method used for solving stiff differential equations. As can be seen from Table 2, for low integration accuracies (e.g. 1e-3) and for very stiff surfaces, using stiff integration methods such as ODE23s results in a faster simulation with longer integration steps (Table 2). However, for higher integration accuracies (e.g. 1e-6), a stiff integrator takes more steps than an ordinary integrator such as ODE45. Therefore, for high integration accuracies, it is better to choose an ordinary integration method such as ODE45 rather than a stiff one even if the stiffness of the surface is very high.

Normal Contact Model	Parameter Values
Linear S-D 1	k = 50000 N/m b = 180 Nsec/m
Linear S-D 2	k1 = 50000 N/m k2 = 250000 N/m b = 250 Nsec/m
Nonlinear S-D	k = 50000 N/m b = 37500 Nsec/m n = 1

Table 1: Parameter values used for the three contact models of Section 2. Parameter values for the Nonlinear S-D model are taken from [Marhefka and Orin, 1999], and values for the other two models are synthesized such that the energy loss after one collision is similar to the Nonlinear S-D model.

Integration accuracy (rel.& abs. tolerance)	Time steps count ODE 45	ODE 23s
(stiffness: 5×10^4 N/m)		
e-3	136	185
e-6	259	6423
e-9	624	76677
(stiffness: 5×10^5 N/m)		
e-3	1471	355
e-6	1718	7994
e-9	2810	192507

Table 2: Effect of integration accuracy and ground stiffness on integration step count for the model Linear S-D 1, t = 2 sec, m = 2 kg

Ground models

In this part, the effect of the three normal ground force models mentioned in Section 2 on the integration step size is investigated. For the ground models to be comparable to each other, we need to set a criterion; for example, finding parameter values for the various models which result in the same maximum penetration depth for a standard impulse, or parameter values which result in the same amount of energy loss after one collision. Parameter values for the Nonlinear S-D model are taken from [Marhefka and Orin, 1999], and parameters for the other two models are synthesized such that the energy loss after one collision is similar to the Nonlinear S-D model (Table 1).

The plots in Figure 4 show the vertical position of the colliding mass vs. time for these three models, and the plots in Figure 5 show the step size as a function of time. Figure 5 shows that Simulink starts the simulation by setting the integration step time to a small value, and then quickly increases it to the default maximum value of $\frac{T}{50}$, where T is the total run time of the simulation. At time $t = 0.4$ sec, where the first collision occurs, it is detected by Simulink and the integration step size is suddenly reduced to near zero. Then it starts to rise to a higher value (less than $\frac{T}{50}$) until the second collision occurs where it nears zero once again. This sequence continues until the mass comes to a rest in a state of sustained contact. While in sustained contact, the integration step size stays bounded around an average value. The variations of the step size around this value depend on variables such as the ground model, the integration accuracy, and the integration method.

Another look at the plots of Figure 5 shows that for a sustained contact scenario, the Linear S-D 1 takes the least integration steps among the three models. This is particularly true when the integration accuracy is set to higher values which cause the integration method to take very small steps for the Nonlinear S-D model (Figure 5 vs. Figure 6). However, in conditions where there are frequent transitions between contact and non-contact states, the Nonlinear S-D model works best. This can be attributed to the fact that there are no state variables used in modeling the Nonlinear S-D model. Having two state variables for describing it, the Linear S-D 2 model takes the most integration steps among the three.

According to what was said in this section and the advantages of the Nonlinear S-D model over the Linear ones mentioned in Section 2, depending on the desired integration accuracy and whether the simulation involves sustained contact conditions, one can decide which model to choose for a specific application.

Mass of the colliding body

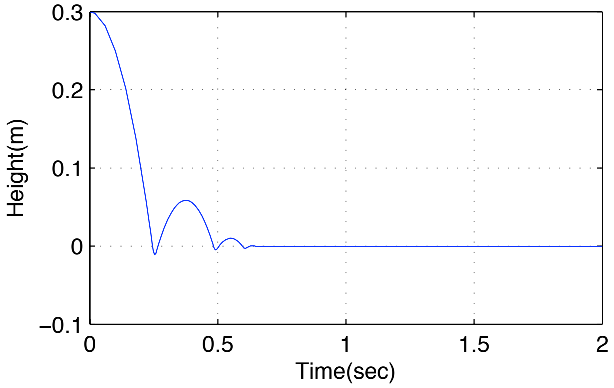
Next, we have investigated the effect of the colliding mass, both for sustained contacts and transitions from contact to non-contact for two point masses of 2 kg and 5 kg. Plots of Figure 7 show the position and step size for a mass of 5 kg colliding with the Linear S-D 1 model. Comparing these plots with those in Figures 4(a) and 5(a), where a mass of 2 kg is used with the same settings, reveals that a lighter mass comes to a rest quicker than a heavier one, and the integration routine takes smaller steps during sustained contact. This shows that a light foot will be more expensive to simulate than a heavy one.

Servo effect

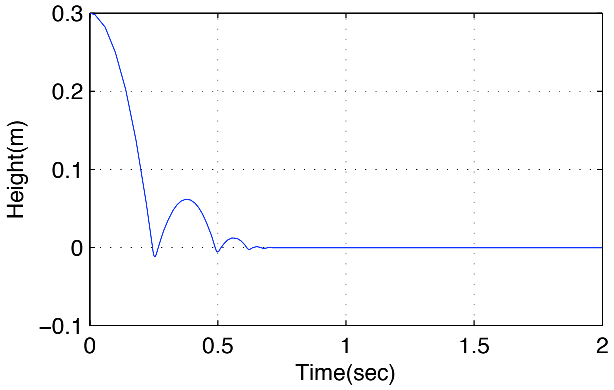
Usually, a servo system is present in a legged robot simulation. To include the effect of a servo in a system (e.g. a feedback controller), we add a unit delay block to the Simulink model, with its sampling period set to the desired servo rate. The effect of this block is to force Simulink to truncate any integration step that would otherwise pass over a sample time. For example, if our simulation period is 2 sec and the sampling period is 0.001 sec, there will be at least $\frac{2}{0.001} = 2000$ integration steps taken. Depending on the model that is used, adding a servo system will have different effects on the integration step size. Table 3 shows the effect of adding a servo with a servo rate of 0.001 or 0.1 seconds to the system for 2 sec of simulation time. As can be seen from the table, low servo rates do not have much effect on the integration step count; however, for high servo rates, the overall integration steps taken are overshadowed by the servo rate. Hence keeping the servo rate as low as possible has a considerable effect on the number of integration steps that are taken by Simulink.

2DOF point mass

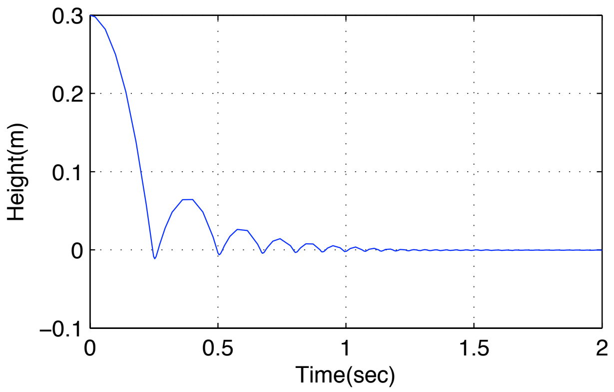
In this section, 2 degrees of freedom are allowed for a point mass colliding with the ground, i.e., movement in the x and y directions. This allows the inclusion of friction in the contact model. Here, we investigate the effect



(a) Linear S-D 1

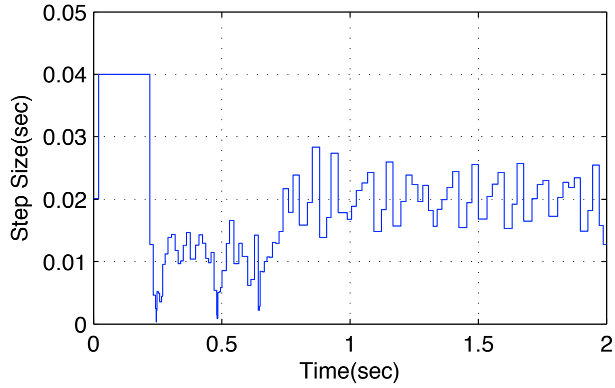


(b) Linear S-D 2

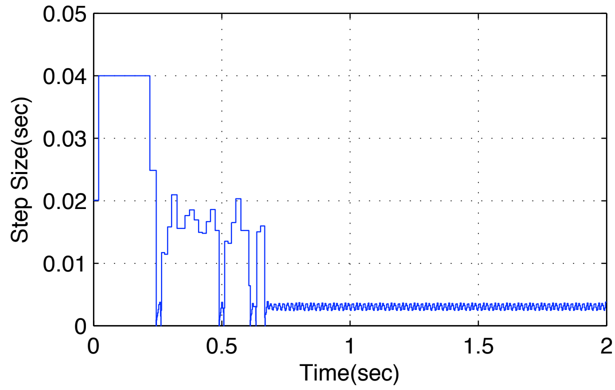


(c) Nonlinear S-D

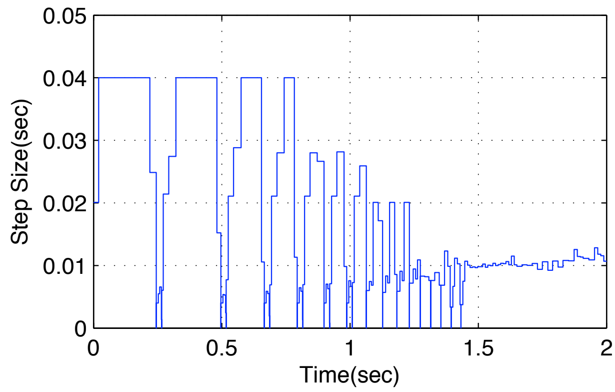
Figure 4: Vertical position vs. time for a point mass falling vertically onto three contact normal force models



(a) Linear S-D 1



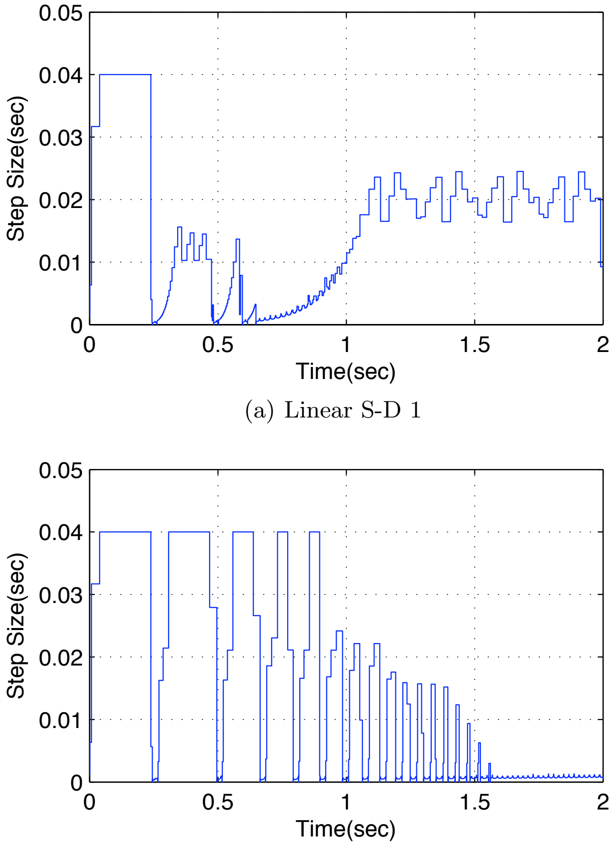
(b) Linear S-D 2



(c) Nonlinear S-D

Figure 5: Step size vs. time for the three experiments shown in Figure 4. Integration accuracy is 1e-3.

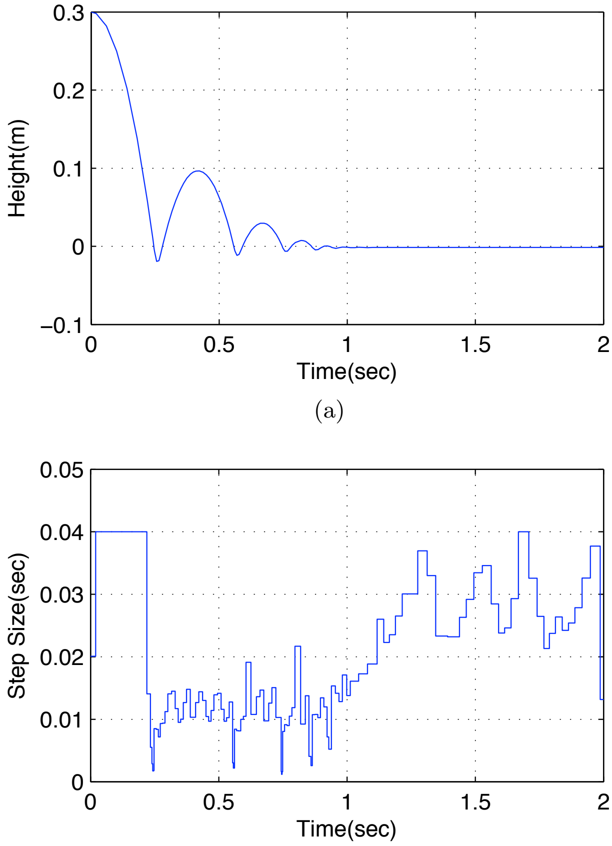
of the two friction force models described in Section 2, combined with the two normal force models Linear S-D 1 and Nonlinear S-D, on the integration step size. The computational complexity of both of the friction models are very similar since both of them are described in terms of a first order differential equation. Basically, the integration steps taken for these models depend on the parameters in their corresponding model. The first model is represented in terms of its tangential stiffness and damping coefficients k and b, and the coefficient of kinetic friction μ. The second model is described in terms of parameter s_p and coefficient of kinetic friction f_k. Table 4 shows the parameter values chosen for these two models. For the first model, assigning the tangential



(a) Linear S-D 1

(b) Nonlinear S-D

Figure 6: Step Size vs. Time for the Linear S-D 1 and Nonlinear S-D models. Integration accuracy is 1e-9.



(a)

(b)

Figure 7: Vertical Position and Step Size vs. Time for a mass of 5 kg colliding with the Linear S-D 1 model (Compare to Figure 4(a) and 5(a) where the colliding mass is 2 kg)

stiffness to twice the stiffness value in the normal direction would be close to reality. The coefficient of kinetic friction is chosen to be the same for the two models. The value $s_p = 0.0045$ is chosen such that the number of integration steps for the two models used along with Linear S-D 1 normal force model would almost be the same for 2 seconds of simulation.

Figure 8 shows the curve of step size vs. time for a 2 second simulation of a point mass falling from 0.3 m, with an initial velocity of 1 m/sec in the horizontal direction. Table 5 shows the integration steps taken by Simulink for the same setting with different combinations of normal and friction forces. Comparing Table 5 and the first column of Table 3 (which shows the simulation of the normal force without the servo effect), reveals the additional integration steps Simulink takes if friction is present. For example, the number of integration steps for 2 seconds of simulation for the Linear S-D 1 model is 136 whereas this number has increased to 800 for the 2D case where the Linear S-D 1 is combined with the Double State Friction, almost a factor of 6 increase in integration step count. Another observation from Table 5 is that for the specified simulation setting, the number of integrations steps for all the combinations are almost the same.

3DOF rigid bar

Last, we investigate the additional integration steps required if another degree of freedom is present. We achieve this by simulating a planar rigid bar colliding with the ground. The bar can move along the x and y axis and rotate around the z axis (the center of mass for this rigid bar is set to 0.6 m with the two end points located 0.3 m from its center of mass). For a rigid bar, there are two points of contact at each end as compared to a point mass. Comparing Figure 9 and 8 shows that for transitions between contact and non-contact, the integration steps are almost the same except that for the rigid bar, due to the existence of two points of contact, the number of collisions are higher. However, for a sustained contact, smaller integration steps are taken for the rigid bar since the bar is making contact at two points.

Model	Time steps count		
	without servo	with servo	
		0.1 sec	0.001 sec
Linear S-D	136	141	2012
Nonlinear S-D	189	196	2004

Table 3: Effect of adding a servo on time steps count

Friction Model	Model Parameters
Double State Friction	$k_t = 100000$ $b_t = 100$ $\mu = 0.4$
Bliman-Sorine	$s_p = 0.0045$ $f_k = 0.4$

Table 4: Parameter values for the friction models of Section 2

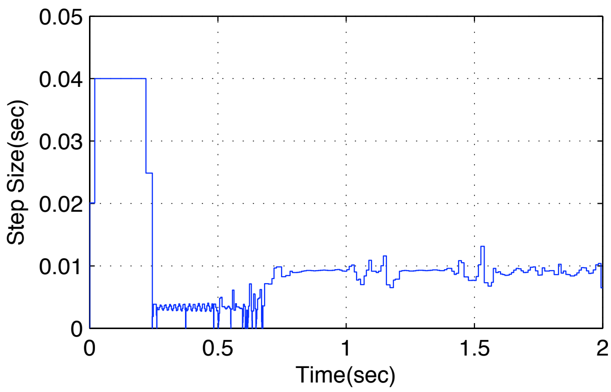


Figure 8: Point mass colliding with ground (normal force: Linear S-D1 , friction force: Double state friction)

Friction Model	Integration steps	
	Linear S-D	Nonlinear S-D
Double State Friction	800	871
Bliman-Sorine Friction	802	813

Table 5: Step counts for a 2 sec simulation of a point mass falling, bouncing, sliding and coming to rest

4 Conclusions

In this paper, we investigated the use of a number of soft contact and friction models in the context of legged machines in Simulink and explained how Simulink varies the integration step size for simulating the collision of a rigid body colliding with, sliding over, and resting on the ground. We showed the effects of variables such as integration method, integration accuracy, mass of the colliding body, type of the ground model and its corresponding parameter values on the integration step size and gave suggestions on values and settings for these variables which result in a faster simulation run time. We also showed the effect of adding a servo to the system and how it overshadows the regular step size Simulink takes. In addition to that, adding friction to the contact force was also investigated. We also showed the variations of the integration step size for a 2 DOF mass and 3 DOF rigid bar colliding with, sliding over, and resting on the ground with the presence of friction in the surface.

In a nutshell, it was found that the two-spring model is relatively inefficient compared with the other two contact normal force models. The nonlinear spring-damper model is more efficient than the linear one-spring model during periods of non-contact, due to the fact that the latter has a state variable and the former does not, but the linear one-spring model is more efficient during periods of contact. At low accuracies, the integration method ODE23s is more efficient than ODE45, but ODE45 is more efficient at higher accuracies. If the simulation includes a servo (a sampled-data control system), and the servo rate is high, then the number of integration steps taken by the numerical integrator is dominated by the servo. When friction models were combined with contact normal models, the combination is approximately a factor of 6 less efficient than when friction is not present.

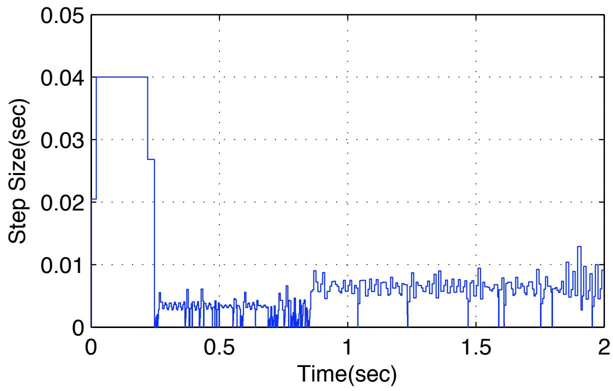


Figure 9: Rigid bar colliding with ground (normal force: Linear S-D 1 , friction force: Double state friction)

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