

# MODELLING OF AN AUTONOMOUS AMPHIBIOUS VEHICLE

T.H. Tran<sup>+</sup>, Q.P. Ha<sup>+</sup>, R. Grover<sup>++</sup>, S. Scheduling<sup>++</sup>  
ARC Centre of Excellence in Autonomous Systems (CAS)

<sup>+</sup> Faculty of Engineering,  
University of Technology, Sydney,  
PO Box 123 Broadway NSW 2007  
E-mail: {ttran|quangha}@eng.uts.edu.au

<sup>++</sup> Australian Centre for Field Robotics  
School of Aerospace, Mechanical and Mechatronic Engineering J07  
The University of Sydney NSW 2006  
E-mail: {r.grover|scheduling}@acfr.usyd.edu.au

## Abstract

This paper presents the modelling of an autonomous amphibious vehicle developed at the Australian Centre for Field Robotics, Sydney. All parts of the vehicle's driveline from the engine, CVT, gearbox to wheels are analysed and modelled. Results from simulation, compared with data from characterising experiments, show that these models are valid and can be used for the control purpose.

**Keyword:** automotive, CVT, differential, skid-steering

## List of symbols:

$N_e$ :	Engine speed
$N_c$ :	CVT output speed
$N_G$ :	Gearbox output speed
$N_d$ :	Speed of differential case ( $=N_c$ )
$N_{dR}$ :	Differential's output speed on right side
$N_{dL}$ :	Differential's output speed on left side
$N_{wR}$ :	Right wheel speed
$N_{wL}$ :	Left wheel speed
$T_e$ :	Engine torque
$T_{fric,e}$ :	Engine friction torque
$T_{ec}$ :	Load on engine
$T_C$ :	Load on CVT
$T_{fric,G}$ :	Gearbox friction torque
$T_G$ :	Load on gearbox
$T_{fric,D}$ :	Differential friction torque
$T_d$ :	Load on differential's case
$T_{dR}$ :	Load on differential's right output
$T_{dL}$ :	Load on differential's left output

$T_{wR}$ :	Load from right wheels
$T_{wL}$ :	Load from left wheels
$T_{fric,w}$ :	Wheel friction torque

$K_1$ :	CVT gear ratio
$K_2$ :	Gearbox gear ratio
$K_3$ :	Chain system gear ratio
$r$ :	Wheel radius

## 1 Introduction

Serving as an outdoor demonstrator for the research programme at CAS is the ARGO, an autonomous amphibious vehicle shown in Figure 1. The vehicle is retrofitted on the platform of the Conquest 8x8, a 20hp, 3m x 1.45m x 1.1m, 0.5 tonne automotive vehicle that can achieve 30km/h on land and 3km/h on water. Figure 2 shows the driveline of this vehicle. The power transmission system includes a water-cooled V-2 Kawasaki engine, continuous variable transmission (CVT), gearbox, differential and a chain system.



Figure 1: ARGO - an autonomous amphibious vehicle

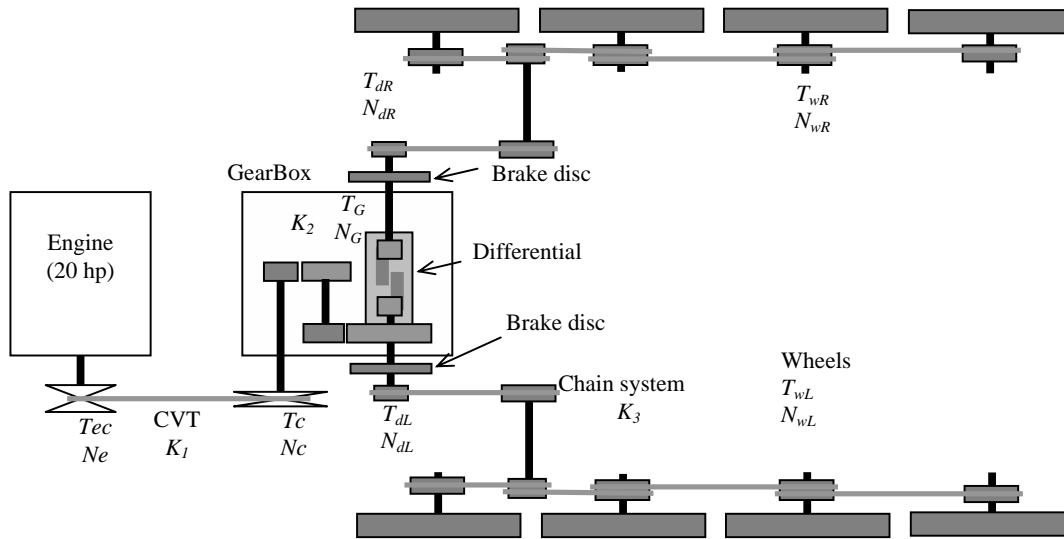


Figure 2: Driveline of the vehicle

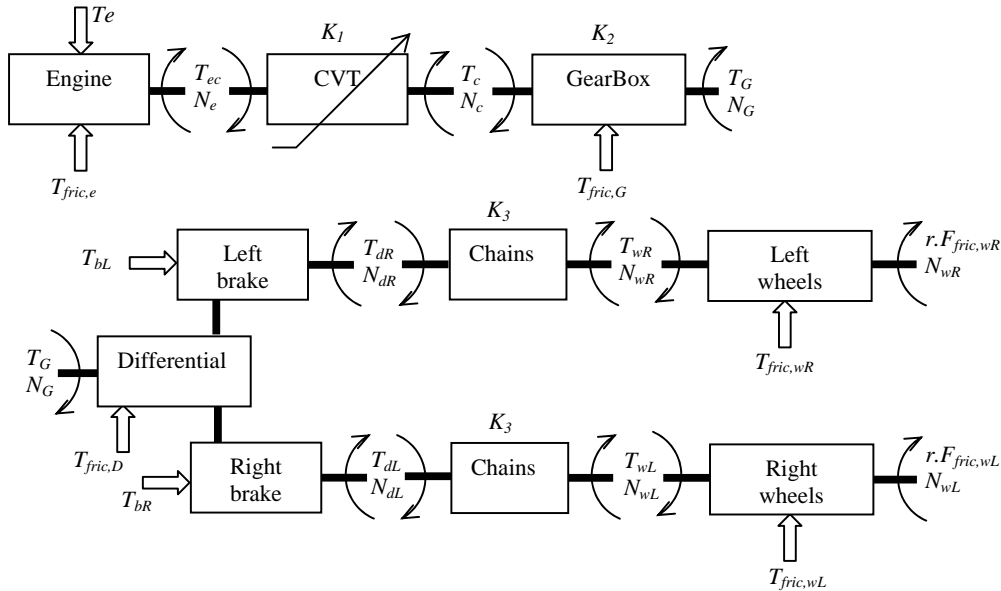


Figure 3: Subsystems of the driveline

The vehicle has eight 22x10.0" wheels, connected together by the chain system and driven by the left and right outputs of the differential. Two brake discs are attached to the outputs of the differential and can be operated separately. The differential and the brake system decide the turning of the vehicle (skid-steering).

This driveline of the current vehicle is equipped with speed sensors to allow for measurements of the engine speed, gearbox input speed, and left and right wheel speeds. The engine throttle, choke, left brake and right brake can now be controlled separately by suitable actuators. Inputs to these actuators are deliberately set from 0 to 100%.

The robotic vehicle represents a highly nonlinear and dynamically coupled complex system. For the control

purpose, it is therefore essential to find a simplified and useful model for the vehicle, which is the objective of this paper. Some parts of the modelling have to be based on trial results. The model is tested on Matlab/Simulink and the simulation results are also compared with data obtained from the characterising experiments.

## 2 Driveline modelling

The driveline of the vehicle consists of the engine, CVT, gearbox, differential (in gearbox), chains, and eight wheels. Figure 3 shows the subsystems of the driveline with their distributed torques and speeds, respectively. Note that the models of the engine, CVT, gearbox, and chains are independent to braking. To derive the wheel speeds, the model of the differential-wheel system should however take into account the brakes applied.

## 2.1 Engine

The engine can be modelled by combining dynamics of its components including throttle body, intake manifold, mass flow rate, compression and torque generation [Crossley and Cook, 1991]. For the control purpose, we believe that it is not necessary to look into details of the engine dynamics. Indeed, experiments show that the generating torque,  $T_e$ , of a combustion engine can be modelled as a first-order transfer function [Zanasi *et al.*, 2001].

$$T_e = \frac{K}{\tau_p s + 1} \theta, \quad (1)$$

where  $\theta$  is throttle position, and  $K$  and  $\tau_p$  are respectively the engine gain and time constant.

The engine motion equation is then:

$$J_e \dot{N}_e = T_e - T_{fric,e} - T_{ec}, \quad (2)$$

where  $J_e$  is its moment of inertia.

## 2.2 Transmission

The vehicle uses an automatic torque converter known as the continuous variable transmission (CVT). It consists of a driver clutch located on the engine output shaft, a driven clutch located on the input shaft of the transmission, and a drive belt. The driver clutch radius increases on acceleration, resulting in an increase of the output speed. On the other hand, when the vehicle is under load, the driven clutch increases its radius and more torque can be transmitted to the wheels. Therefore, the CVT can be modelled as a variable gear ratio, which depends on the engine speed and load torque [Setlur *et al.*, 2003]:

$$K_1 = f(N_e, T_c), \quad (3)$$

where  $K_1$  is estimated from experiments. Here it is modelled as a linearised function of the CVT torque  $T_c$  followed by a nonlinear deadzone with a threshold of 1200 rpm for the engine speed.

The torque and speed at the output of the CVT are thus given by:

$$\begin{aligned} T_c &= T_{ec} / K_1, \\ N_c &= N_e K_1. \end{aligned} \quad (4)$$

## 2.3 Gearbox

The gearbox has four positions, namely Reverse - for backing up the vehicle; Neutral - for starting the engine or idling; Low - for use when extra pulling power or very low speed is required on rough terrain; and High - for general use at normal operating speeds. The output of gearbox engages directly to the case (ring gear) of the differential. Therefore, the differential's case can be considered as the output of the gearbox when calculating the gearbox ratio,  $K_2$ .

$$K_2 = \begin{cases} -0.1295, & \text{Reverse} \\ 0, & \text{Neutral} \\ 0.1295, & \text{Low} \\ 0.2655, & \text{High.} \end{cases} \quad (5)$$

The torque and speed at the output of the gearbox are

$$\begin{aligned} T_G &= (T_c - T_{fric,G}) / K_2 \\ N_G &= N_c K_2. \end{aligned} \quad (6)$$

## 2.3 Chains

The chain system can be simplified as a gear ratio  $K_3$ , calculated from the vehicle pulley system.

$$K_3 = 0.2483. \quad (7)$$

The derivation of the wheel torque and speed depends on the brake applied on the right and left wheels.

## 2.4 Differential-Wheels

Figure 4 shows the structure of the differential. It is a planetary gear system consisting of two sun gears, six planet gears and a case. The left sun gear engages in three left planet gears, which, in turn, engage in right planet gears. The right sun gear engages in right planet gears. The case spins at speed  $N_d$  (or  $N_G$  because the case is considered as the output of gearbox). If load torques on left and right sun gears are equal, the two outputs of the differential will have the same speed with the case. Otherwise, they are different.

The differential distributes the torque from the gearbox to the left and right wheels. The two driving shafts of the differential are attached to the left and right brake discs. The torque difference enables the vehicle to turn. It is necessary to consider the differential in two cases when no brake applied and when applying brake.

### 2.4.1 No brakes applied

Torque from engine is distributed equally for the left and right wheels. Assuming that load on left wheels and right wheels is the same, all wheels have the same speed and therefore, are modelled as one wheel. One can derive then

$$\begin{aligned} T_{dL} &= T_{dR} \\ T_{dL} + T_{dR} &= T_d = T_G - T_{fric,D} \\ N_{dR} &= N_{dL} = N_d = N_G. \end{aligned} \quad (8)$$

The wheel torque and speed, obtained after the chain system, are as follows

$$\begin{aligned} T_w &= T_d / K_3 \\ N_w &= N_d K_3. \end{aligned} \quad (9)$$

Now let us assume that the vehicle with mass  $m$  and velocity  $v$  (at the centre of mass) is travelling on a flat road of a slope angle  $\chi$ , as shown in Figure 5. The force equation can be derived from the Newton's second law in the longitudinal direction as:

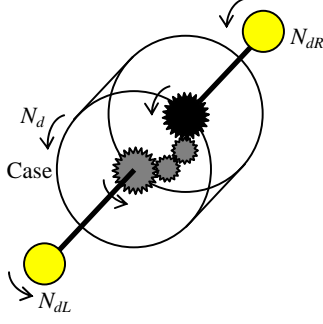


Figure 4. Differential

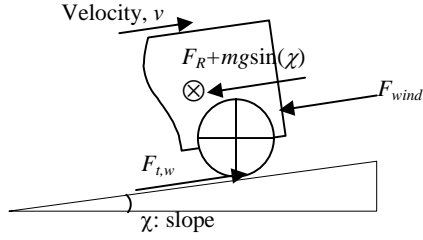


Figure 5. Longitudinal forces acting on the vehicle

$$F_{t,w} = m\dot{v} + F_{wind} + F_R + mg \sin(\chi), \quad (10)$$

where  $F_{t,w}$  is the driving force at the wheel,  $F_{wind}$  is the air drag force,  $F_R$  is the rolling resistance determined by

$$F_R = m(c_{r1} + c_{r2}v), \quad (11)$$

in which  $c_{r1}, c_{r2}$  are coefficients depending on the tires and road conditions [Kiencke and Nielson, 2000], and  $g$  is the gravitational acceleration. Ignoring  $F_{wind}$  as the vehicle runs at low speed, the vehicle motion equation is written as:

$$J_w \dot{N}_w = T_w - T_{fric,w} - rF_{t,w}, \quad (12)$$

where  $J_w$  is the wheel moment of inertia. Substitution of equation (10) into equation (12) and noting that  $v = N_w r$  gives

$$(J_w + m.r^2) \dot{N}_w = T_w - T_{fric,w} - mr(c_{r1} + c_{r2}rN_w) - mgr \sin(\chi). \quad (13)$$

From equations (1-13) above, a flow diagram for simulation is suggested in Figure 6, where the engine speed is transferred through CVT, gearbox, differential, chains to wheels and the load torques from the wheels are referred backward to the engine shaft. Note that friction torques of the engine, gearbox, differential and wheels are modelled in our simulation as due to viscosity.

$$\begin{aligned} T_{fric,e} &= b_e N_e \\ T_{fric,G} &= b_G N_G \\ T_{fric,d} &= b_D N_G \end{aligned}$$

$$T_{fric,w} = b_w N_w, \quad (14)$$

where  $b_e, b_G, b_D$  and  $b_w$  are corresponding damping coefficients.

#### 2.4.2 Brakes applied

When applying brakes, the left and right wheel speeds are different. The difference takes place at the outputs of the differential, chains and wheels.

It is assumed that longitudinal forces acting on the vehicle are distributed equally to the left wheels and right wheels. Applying equations (10), (12), and (13) to the left wheels and right wheels gives correspondingly

$$\begin{aligned} T_{wL} &= \frac{1}{2}(J_w + mr^2) \dot{N}_{wL} + T_{fric,wL} + \\ &+ \frac{1}{2}mr(c_{r1} + c_{r2}rN_{wL}) + \frac{1}{2}mgr \sin(\chi), \end{aligned} \quad (15a)$$

$$\begin{aligned} T_{wR} &= \frac{1}{2}(J_w + mr^2) \dot{N}_{wR} + T_{fric,wR} + \\ &+ \frac{1}{2}mr(c_{r1} + c_{r2}rN_{wR}) + \frac{1}{2}mgr \sin(\chi) \end{aligned} \quad (15b)$$

where  $T_{fric,wL}$  and  $T_{fric,wR}$  are respectively friction at the left and right wheel. Torques and speeds for the chains in this case are

$$\begin{aligned} T_{dL} &= T_{wL} K_3 \\ T_{dR} &= T_{wR} K_3 \\ N_{wL} &= N_{dL} K_3 \\ N_{wR} &= N_{dR} K_3. \end{aligned} \quad (16)$$

For the differential, let us first define the speed difference,  $x = N_d - N_{dR} = N_G - N_{dR}$ . The speeds at left and right differential output are respectively:

$$N_{dL} = N_G + x, \quad N_{dR} = N_G - x. \quad (17)$$

When brake torques,  $T_{bL}$  and  $T_{bR}$ , applied, the total load torques on the left and right sun gears are

$$\begin{aligned} T_{sL} &= T_{dL} + T_{bL} \\ T_{sR} &= T_{dR} + T_{bR}. \end{aligned} \quad (18)$$

The total load on the case is then

$$T_d = T_{sR} + T_{sL} + T_{sd}, \quad (19)$$

where  $T_{sd}$  is the torque caused by turning due to speed difference  $x$ . Although depending also on the road condition,  $T_{sd}$  can be simplified as

$$T_{sd} = Kx. \quad (20)$$

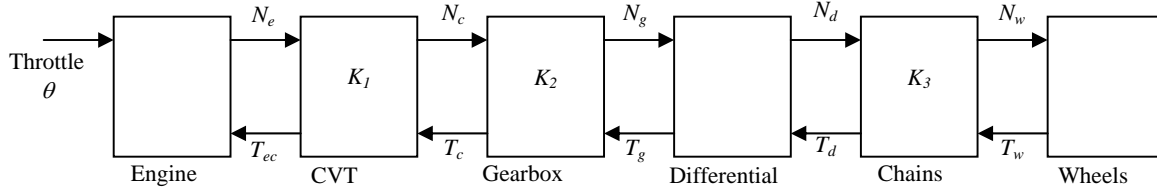


Figure 6. Simulation flow diagram- no brakes applied

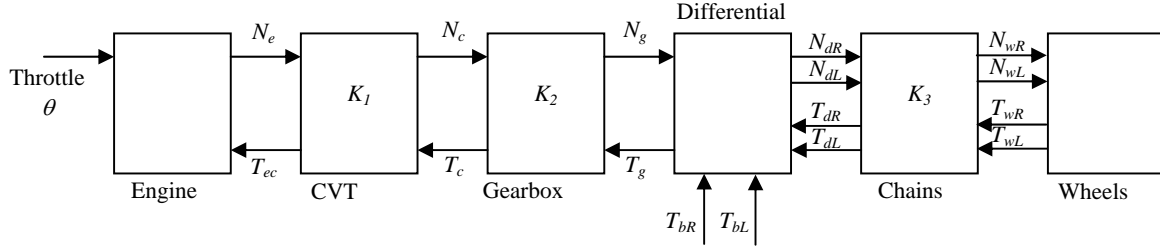


Figure 7. Simulation flow diagram- brakes applied

The speed difference causes a friction component, which can be modelled as viscous damping. This component is resulted basically from the difference between the left and right load of the sun gears:

$$T_{sR} - T_{sL} = b_{D,in}x. \quad (21)$$

Hence, one can approximate the speed difference as

$$x = \frac{T_{sR} - T_{sL}}{b_{D,in}}, \quad (22)$$

where  $b_{D,in}$  represents viscosity inside the differential's case.

The simulation of the driveline in the case of applying brakes is now based on the set of equations (1-7) and (15-22), according to the flow diagram shown in Figure 7. Note that this model can be reduced to the previous one when there are no brakes applied.

### 3 Results

#### 3.1 Simulation

In this section simulation results are shown for some interesting cases: step response of the throttle input, and turning the vehicle left/right. Table 1 lists the numerical values for parameters used in the simulation.

Figure 8 presents the vehicle's engine torque, and engine and gearbox speeds when running the vehicle with 100% throttle for ten seconds, turning left by applying 50% braking torque to the left brake for ten seconds, then turning right by applying 50% braking torque to the right brake according to the pattern shown in Figure 9. It can be seen that the gearbox speed is more load sensitive than the engine speed, as expected in automotive engineering.

The transient of the vehicle speed and torque to a step response of the throttle exhibits a time delay of approximately 0.6 sec due to the CVT speed threshold and a time constant of 0.3 sec due to the engine time constant.

The right and left wheel speeds are shown corresponding to the braking pattern in Figure 9. It can be seen that the difference between the right and left wheel speed enables the turning of the vehicle. The dynamic torque distribution of the right/left wheels, gearbox and engine over the period of 50 sec is depicted in Figure 10, where the wheel-environment interactions are ideally negligible.

#### 3.2 Field trials

The vehicle endured a number of laboratory and field tests before achieving successfully the current status of a fully autonomous ground vehicle operating outdoors. Upon testing on field – at Marulan, three hours driving southern from Sydney city – the vehicle dynamic behaviour was very close to the desired one that the research team had expected. Figure 1 shows the ARGO on the first field characterizing test at Marulan in June 2004. Data collected were then compared with the results obtained in the modelling presented in this paper.

Figure 11 shows the vehicle responses to a step input of the throttle, where data from the throttle sensor are used as an input to the simulated model. It can be seen that the simulated speed looks close to the practical development of the speed at the engine but not much coincides with that at the gearbox. The difference may be explained by some integration drift in the gearbox encoder data, and another pole omitted in the system when using a simplified first-order model for the engine (1). Again, a time delay can be observed and should be taken into account when designing closed loop controllers for the vehicle.

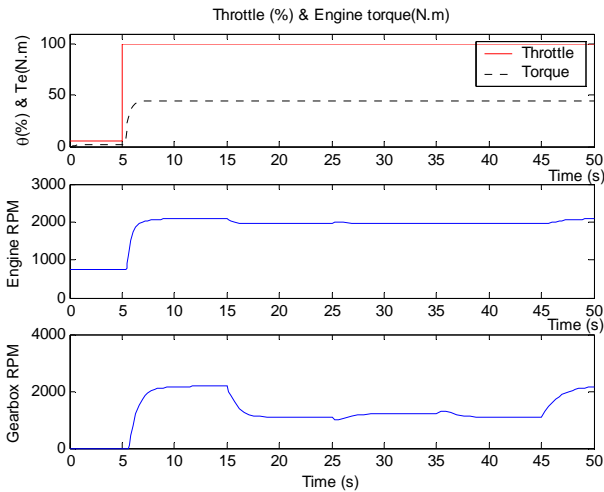


Figure 8. Responses to throttle step input

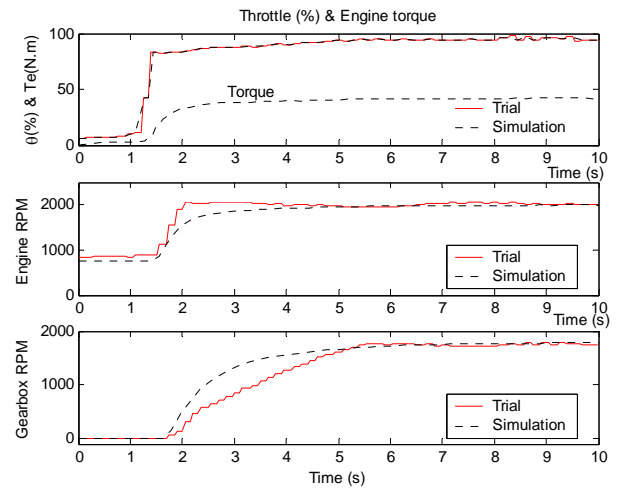


Figure 11. Field test: responses to throttle step input

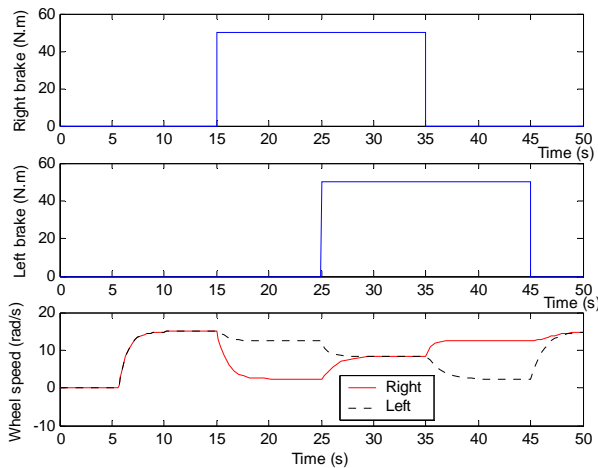


Figure 9. Braking pattern and wheel speeds

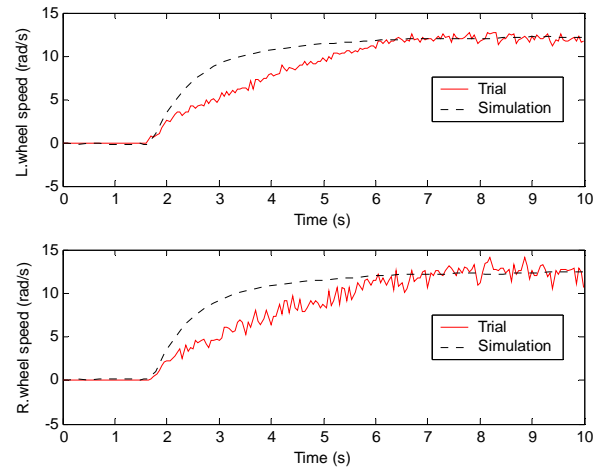


Figure 12. Field test: wheel speed responses

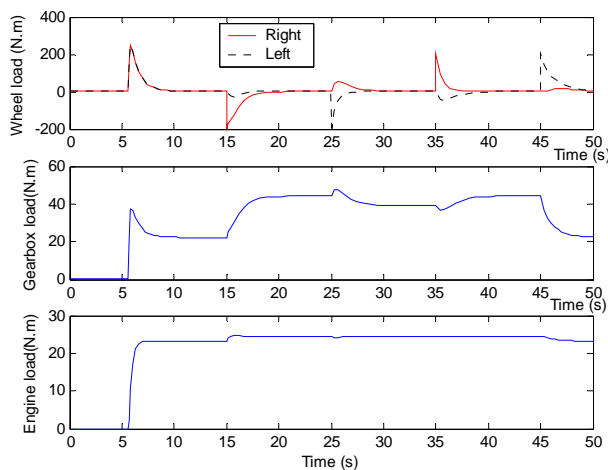


Figure 10. Load distribution

Figure 12 shows a comparison between the simulated and the experimental responses of the left/right wheel speeds under the same throttle input. Some noisy spikes are observed in the right wheel encoder data, perhaps because

of the rough road encountered during the trial. In general, however, there is a coincidence between the simulation results from our modelling and the experimental ones obtained from the trial. Similar conclusion can be made for the braking tests.

In transient processes, the difference between trial and simulation results is accounted for by several factors. First, the CVT is modelled as a linear function of speed and load, but in fact, it is highly nonlinear. In addition, the model did not consider weight and deadzone of the gears in gearbox, differential, brake discs, and chains. More importantly, complicated interactions between the vehicle and terrains were not taken into account. Future work will be directed to a better model to include these factors when designing suitable control strategies, in particular for the system braking system.

## 4 Conclusion

We have presented the modelling of the ARGO, an autonomous amphibious vehicle developed at the Australian Centre for Field Robotics (ACFR). The

vehicle's driveline, including the engine, CVT, gearbox, differential, chains and wheels, is analysed and modelled. Simulation results are provided. Data set obtained from field tests are then used to verify the model validity. Applying data from the throttle and brakes as inputs, the simulated responses are somehow close to the experimental ones. Discussion on the results is included. It is believed that the models proposed will be helpful when designing closed loop low-level controllers for the vehicle.

### Acknowledgement

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Driveline Component	Nomenclature	Symbol and value
Engine	Engine gain Time constant Moment of inertia Damping coefficient	$K = 0.44$ $\tau_p = 0.3 \text{ sec}$ $J_e = 0.07 \text{ kgm}^2$ $b_e = 0.01 \text{ Nms}$
CVT	CVT gear ratio	$K_1 = f(N_e, T_c) = g(N_e)h(T_c)$ $g(N_e) = \begin{cases} 1/1200(N_e - 1200), & \text{if } N_e \geq 1200 \text{ rpm} \\ 0, & \text{if } N_e \leq 1200 \text{ rpm} \end{cases}$ $h(T_c) = \begin{cases} 1/40(80 - T_c) & \text{if } T_c \leq 80 \text{ Nm} \\ 0 & \text{if } T_c \geq 80 \text{ Nm} \end{cases}$
Gearbox	Damping coefficient Gearbox gear ratio	$b_G = 0.01 \text{ Nms}$ $K_2 = \begin{cases} -0.1295, & \text{Reverse} \\ 0, & \text{Neutral} \\ 0.1295, & \text{Low} \\ 0.2655, & \text{High.} \end{cases}$
Differential	Damping coefficient Damping coefficient inside Turning load ratio	$b_d = 0.001 \text{ Nms}$ $b_{D,in} = 0.25 \text{ Nms}$ $K = 0.375$
Chain	Chain gear ratio	$K_3 = 0.2483$
Wheel-body	Vehicle weight Wheel radius and weight Damping coefficient Rolling resistance Rolling resistance	$m = 490 \text{ kg}$ $r = 0.25 \text{ m}, m_w = 6.6 \text{ kg}$ $b_w = 0.04 \text{ Nms}$ $c_{r1} = 0.01$ $c_{r2} = 0.08$

Table 1. Parameters used in simulation